Structural Estimation for Political Science

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The overwhelming majority of empirical research in political science is "reduced form", in that the goal is to estimate quantities whose interpretation does not depend on a particular model of the underlying political process. The goal is just to recover a conditional correlation, or if we're ambitious, an average treatment effect. From there we can argue about which theories that estimate gives us evidence in favor of or against.

Structural estimation is an alternative—and in my view, complementary—statistical paradigm. We start with a formal model of the political process under study, and then we use data about that process to estimate the parameters of the model.

Why estimate structural models? I think about it this way. Formal theorists are accustomed to studying political phenomena *qualitatively* through the lens of a model. In case studies, we try to identify which region of the parameter space the case falls into, then interpret historical evidence in terms of the model's key mechanisms. Structural estimation is a *quantitative* variant of this interpretive process. We take data about some complicated political process, and reduce it down to the model parameters that help us make sense of it.

Structural estimation is controversial and (in my opinion) poorly understood in the discipline. Empiricists object to the model-dependence of the statistical procedures. Theorists say they build their models to simplify complex phenomena and highlight specific strategic tradeoffs, not to fully capture the data-generating process. We'll discuss the validity of these objections and try to sort out which research questions we do—and don't—want to answer with structural models. But even if you never publish a structural paper, I think learning this stuff makes you a better theorist *and* empiricist.

Structural modeler *par excellence* Matias Iaryczower told me he advises his students to "write for the gods"; i.e., do the work that's truly best, without worrying about how the audience will react. I'm more pragmatic than that. You won't be writing for the gods very long if you don't land a tenure-track job. To that end, while I encourage you to learn about structural estimation and possibly even to dabble in structural work, under no circumstances will I sign off on you writing a structural job market paper.

Expectations

If you're taking this for credit, here's what I expect from you.

1. **Participation.** Show up prepared every week. Ask questions when you're confused. You will be confused often, so you should ask questions often.
2. **Problem sets.** As with any methods skill, you learn structural estimation best through practice. I'll assign occasional problem sets to help build these skills and expand your knowledge. You can work together on these, but turn in your own writeup.
3. **Final project and presentation.** The capstone project for the course will be a final paper applying structural estimation to a substantive question in your area of interest. An ideal final paper will execute a full analysis on real data. An acceptable one will present a compelling proof of concept. (What that looks like depends on what you're doing, so make sure to chat with me about it early and often.) The final week of class will be devoted to presentations of your projects.

   You don't have to turn in the final paper until the last day of classes in whatever semester you end up getting your independent study credit for this. However, for your own sake, I strongly recommend getting it done before fall 2024 classes start.

Summary of Topics and Readings

Most of these readings are just here for reference. Unless I tell you otherwise, the lecture notes I circulate the Friday before we meet are the only thing you *must* read each week. Refer to the readings here if you get stuck on something, want to learn more, or want to see how the modeling technology we're using is employed—and, importantly, pitched to a political science audience—in practice.

At the end of the syllabus, there's an annotated section with some intellectual background on each reading.

1. **Maximum likelihood and numerical optimization.** Greene 2003, chapter 17; Davidson and MacKinnon 1993, chapter 8; Judd 1998, chapter 4.
2. **Random utility models.** McFadden 1974; Maddala 1983, chapter 3; Manski 1975; Horowitz 1992.
3. **Discrete extensive form games.** McKelvey and Palfrey 1992; McKelvey and Palfrey 1998; Haile, Hortaçsu and Kosenok 2008; Signorino 2003; Bas, Signorino and Walker 2008; Leblang 2003.
4. **Discrete strategic form games.** Bresnahan and Reiss 1991; Bajari et al. 2010; Bajari, Hong and Ryan 2010; Holt and Palfrey 2024; Gibilisco, Kenkel and Rueda 2022; Gibilisco and Montero 2022; Jia 2008.

5. **Bargaining games.** Diermeier, Eraslan and Merlo 2003; Silveira 2017; Kenkel and Ramsay 2024.
6. **Contests.** Erikson and Palfrey 2000; Greene 2003, chapter 18; Gordon and Hartmann 2016; Kenkel and Meisels 2024; Kang 2016; König et al. 2017.
7. **Discrete dynamic decision problems.** Blackwell 1962; Stokey, Lucas Jr. and Prescott 1989, chapter 4; Ljungqvist and Sargent 2018, chapter 4; Rust 1987; Rust 1988; Iaryczower, López-Moctezuma and Meirowitz 2024; Christensen and Gibilisco 2024.
8. **Discrete dynamic games.** Hotz and Miller 1993; Pesendorfer and Schmidt-Dengler 2008; Egesdal, Lai and Su 2015; Crisman-Cox and Gibilisco 2018; Frey, López-Moctezuma and Montero 2023.

Annotated Reading List

1 Maximum likelihood and numerical optimization

Greene 2003, chapter 17. This chapter provides an applied practitioner's overview to maximum likelihood estimation with some simple examples and advice for real-world empirical work. If your question is "What would any empirical economist be expected to know about [insert econometrics topic here] in the pre–*Mostly Harmless* era?", then Greene typically has your answer.

Davidson and MacKinnon 1993, chapter 8. Whereas Greene is econometrics for people who want to do econometrics, Davidson and MacKinnon is econometrics for people who prize theoretical rigor and find actual data analysis mildly distasteful. They are better writers than Greene, and their notation is better, but they assume more background knowledge in math and statistics. I go to this textbook when I need to remind myself just how much I *don't* know about estimation and inference.

Judd 1998, chapter 4. The numerical optimization techniques we'll study are elementary and used across many disciplines, so there is no shortage of introductions to them. I like Judd's coverage because it's written for economists with examples from problems akin to the ones we want to solve.

2 Random utility models

McFadden 1974. The paper—actually, book chapter—that popularized random utility and won McFadden a Nobel Prize. In some sense, nothing here is original, as the very useful intellectual history in footnote 7 makes clear. Statisticians and econometricians had already begun to use multinomial logit models about a decade earlier, while theorists in economics and psychology had begun working out random utility around the same time. The marriage of utility theory with econometric practice is what makes this paper a work of genius.

Maddala 1983, chapter 3. This chapter summarizes the developments in discrete choice models that came in the decade following McFadden's introduction of conditional logit. The main issue with the baseline conditional logit is the IIA (independence of irrelevant alternatives) assumption, which is implausible in many applications. Maddala gives good coverage of the multinomial probit and nested logit models, which are two ways to estimate utility functions without imposing IIA.

Manski 1975. Like most of Manski's work—which is well worth reading in general—this paper is about statistical inference under minimal assumptions. But unlike the contemporary *Econometrica* style of "let's prove this in as abstract a topological space as possible, comprehensibility be damned" assumption relaxation, Manski's work strives for simplicity and transparency. This paper makes a first stab at estimating random utility models without imposing a type 1 extreme value distribution (the hidden backbone of all too much structural work) on the error terms.

Horowitz 1992. Manski's maximum score estimator is theoretically nifty, but finicky enough in practice that you wouldn't really want to try to estimate it, let alone test hypotheses about coefficients. This is a common problem problem with robust statistical procedures based on medians or other non-smooth functions of the sample data. This paper introduces a variant of the Manski model that can be estimated with standard optimization routines and is amenable to inference via bootstrap.

3 Discrete extensive form games

McKelvey and Palfrey 1992. An extensive form game with sequential actions typically has a unique subgame perfect equilibrium. Yet in experiments like the one McKelvey and Palfrey run here, we see subjects behave differently even when the parameters are exactly the same. This paper illustrates how to "rationalize" variation in play by introducing uncertainty, as well as how to structurally estimate the extent of that uncertainty.

McKelvey and Palfrey 1998. The previous paper's proposed solution to divergence between theory and experimental results—namely, the existence of altrustic player types—is pretty specific to the centipede game. In this paper, McKelvey and Palfrey propose a new solution concept for extensive form games called agent quantal response equilibrium, in which all outcomes are reached with positive probability along the path of play. The "logit" form of AQRE is closely related to the conditional logit proposed by McFadden 1974.

Haile, Hortaçsu and Kosenok 2008. Structural models using the QRE solution concept typically assume that the payoff shocks are independent and identically distributed across the actions available to each player. This paper shows just how critical that assumption is: if we relax independence *or* identicality, then a QRE can rationalize any distribution of outcomes given any set of mean utilities.

Signorino 2003. This paper provides a kind of user's manual on different ways to take an extensive form game and derive a corresponding statistical model. The conceptual discussion on different types of uncertainty is also helpful, though less important in practice for estimation than the paper makes it out to be.

Bas, Signorino and Walker 2008. How do we actually go about computing estimates for these structural models? This paper's ambition was to popularize structural estimation by showing you could obtain consistent (albeit inefficient) estimates using regular old logit commands. In a sense this was a failure—16 years later, the paper has less than 100 citations, and the intersection of "people who want to estimate structural models" and "people uncomfortable deriving the likelihood and using `optim`" is basically empty. But I still find this paper useful for demystifying structural estimation, and we'll use the method here to obtain starting values for our optimizers.

Leblang 2003. Back in the summer of 2010, when Signorino was still hoping to make structural estimation great again, he hired me to write the (now defunct) `games` R package to implement some simple estimators of discrete extensive form games. When going through the literature at the time to find replication data to illustrate the package's use, I ended up deciding Leblang's work on speculative attacks was the best political science application available. I'm going to go ahead and assume that's still true 14 years later.

4 Discrete strategic form games

Bresnahan and Reiss 1991. Using models of market entry as the jumping-off point, this paper clearly illustrates the problems that arise estimating parameters from strategic form games. The key difference from extensive form games is the presence of multiple equilibria, which creates conceptual problems in defining statistical problems and practical problems in implementing estimators.

Bajari et al. 2010. To solve the problems identified in Bresnahan and Reiss 1991, this paper proposes a two-step estimator for utility parameters in strategic form games. The estimator is easy to implement (akin to the "statistical backward induction" procedure of Bas, Signorino and Walker 2008), but depends critically on an equilibrium selection assumption.

Bajari, Hong and Ryan 2010. A more sophisticated approach that places fewer assumptions on equilibrium selection, but whose implementation is considerably more involved. Key quote on this point: "In a Monte Carlo study, we find it takes *less than a day of CPU time* to construct estimates and standard errors for our model" (emphasis added).

Holt and Palfrey 2024. An experimental study where the payoffs are known, so the goal is to estimate the extent of agent error in the strategic form variant of QRE. (Thereby skirting the complications that arise when the goal is to estimate utility parameters.) A nice thing about this setting is that the Nash equilibrium of the underlying game is in mixed strategies, so variability in the game outcomes doesn't trivially falsify the hypothesis that players are behaving according to the Nash equilibrium.

Gibilisco, Kenkel and Rueda 2022. Structural estimation is particularly useful to study whether a particular interaction has strategic complements (the more I do X, the more you want to do X too) or strategic substitutes (the more I do X, the less you want to do X). Here we identify that disparate theories of civilian victimization in civil war imply strategic complements in this process, and we use a two-step estimator a la Bajari et al. 2010 to estimate the extent of complements in a simple game-theoretic model of victimization.

Gibilisco and Montero 2022. Another search for complementarities versus substitutes, this time in the context of major-power interventions into civil wars. The two-step estimator's assumption of "same equilibrium under same parameters" is less plausible here because of the temporal and geographical scope, so this paper adopts the more sophisticated Bajari, Hong and Ryan 2010 approach to estimation.

Jia 2008. Not a political science application, but so cool I couldn't keep it off the syllabus. Structural models of discrete choice typically assume independence across observations in order to apply traditional maximum likelihood estimators, but that's not plausible when the players are two major chain stores deciding which markets to open stores in. Jia cleverly exploits the structure of the interaction to estimate the model without implausible independence assumptions. (The only sad thing is that her method is specific to games that either have strategic complements, or else two or fewer players.)

5 Bargaining games

Diermeier, Eraslan and Merlo 2003. One of the earliest structural models of a bargaining game, with an application to the formation of coalition governments in Western Europe. Legislative bargaining is a convenient area of application for structural estimation, as the "rules of the game" are often constitutionally fixed, and we observe a good amount about the proposers, the offers, and the responses to them.

Silveira 2017. This paper estimates a model of bargaining between prosecutors and criminal defendents, where there are observability problems that don't arise in the legislative setting. In particular, we don't observe the counterfactual sentence length for the cases that are pleaded out, and we don't observe the rejected plea offer for the cases that go to trial. Besides the sophisticated econometric approach to a tricky problem, I appreciate this paper because it's very clearly written and explained.

Kenkel and Ramsay 2024. We estimate a model of international crisis bargaining, where the observability problems are yet more severe than in Silveira 2017. We don't observe the value of even the accepted offers—we only know whether they're accepted or rejected, along with an estimate of each side's expected value from war. Those latter estimates come from a contest model we also estimate (again with an observability problem, namely that we don't see effort), which nicely leads into the next section of the course.

6 Contests

Erikson and Palfrey 2000. Empirical analyses of campaign spending dating back to the 1970s had identified a puzzling lack of correlation between spending and electoral success in congressional elections. Erikson and Palfrey nicely show that this observation isn't so puzzling if you think of spending as the outcome of a contest between the two sides, in which the marginal value of spending is highest for both sides when the election is expected to be close. They estimate a sort of proto-structural simultaneous equations model, but didn't have the technology at the time to estimate utility parameters.

Greene 2003, chapter 18. We need another detour into pure econometrics in order to understand the estimators for structural models of contests. It's tricky to derive and program a maximum likelihood estimator for a contest model, as we'd have to calculate the joint probability density of any conceivable set of effort choices in a given observation of the contest. So structural models of contests instead typically work with moment conditions—i.e., restrictions on observables implied by the assumption of equilibrium play—and use the generalized method of moments for estimation.

Gordon and Hartmann 2016. This paper takes on the Erikson and Palfrey 2000 problem, but estimates a proper structural model of campaign advertising as a contest. In the equilibrium of a contest, if both players' efforts are nonzero, then their first-order condition must hold with equality for both of them. Gordon and Hartmann figure out how to put the stochastic component of the model in just the right place, allowing them to derive an estimator from the first-order conditions for optimal advertising spending. The estimator is valid even in the presence of multiple equilibria.

Kenkel and Meisels 2024. Mellissa and I adapt the Gordon and Hartmann 2016 estimator to study outside spending in congressional elections. We model these races as contests between coalitions of potentially arbitrary numbers of players, in which spending from one outside group might encourage or dissuade spending by other supporters of the same candidate. (Strategic complements and substitutes appear again!) We use data from electoral outcomes to estimate the parameters of the contest success function, and then we estimate the other utility parameters off of the first-order conditions for optimal spending.

Kang 2016. Similar to Kenkel and Meisels 2024 in terms of modeling a contest between coalitions, but with a slightly different model (with an upfront entry cost for participation) and a substantially different estimator. Kang's model has a unique equilibrium, so she uses a least-squares estimator to match predicted spending as closely as possible to actual spending, rather than identifying off of first-order conditions as in the other contest papers.

König et al. 2017. Yet another model of a contest between coalitions, this one with an application to the Second Congo War. The key innovation here is to write the underlying game-theoretic model so as to make each player's best response a linear function of the the other players' efforts, thereby allowing use of an Erikson and Palfrey 2000–style simultaneous equations estimator to recover utility parameters.

7 Discrete dynamic decision problems

Blackwell 1962. A technical, but concise and clear, statement and proof of the most important results for single-player dynamic programming problems with finite state and action spaces.

Stokey, Lucas Jr. and Prescott 1989, chapter 4. The canonical, and considerably more general, textbook treatment of dynamic programming problems with economic applications. Always the pragmatist, I recommend setting up your models to have finite state and action spaces so as not to worry about the infinite-dimensional issues tackled here.

Ljungqvist and Sargent 2018, chapter 4. Except under the most special circumstances, dynamic programs cannot be explicitly solved with pen and paper. Ljungqvist and Sargent provide helpful practical advice on computational methods. These computational solutions turn out to be remarkably easy to program—much simpler than solving for Nash equilibria, for example—which makes it less daunting than you'd think to implement Rust's nested fixed point algorithm.

Rust 1987. Perhaps the best-titled paper in all of econometrics. More importantly, a remarkably clear and compelling explanation of the nested fixed point algorithm for estimating the parameters of a stochastic dynamic programming problem.

Rust 1988. Companion paper to Rust 1987. Less intuitive presentation of the underlying econometric ideas, but a bit more general and comprehensive—and thus a bit more useful for practical purposes. More to the point, when I'm coding up an NFXP estimator, this is the paper I have open in the other window.

Iaryczower, López-Moctezuma and Meirowitz 2024. A clearly written and substantively important use of the NFXP technology to estimate the weight U.S. senators place on policy versus office benefits.

Christensen and Gibilisco 2024. Another NFXP application, this time to the relationship between budget shocks and power-sharing coalitions in autocracies.

8 Discrete dynamic games

Hotz and Miller 1993. Two-step estimation with conditional choice probabilities, akin to the Bajari et al. 2010 estimator we saw for strategic form games. This paper is actually about single-player dynamic programming problems, which raises the question of why I didn't assign it in the previous section. With the programming and computing technology of the early '90s, it was very costly to implement and execute an NFXP estimator, making a simpler approach appealing. Three decades later, those concerns aren't relevant for decision problems unless you're working with a truly enormous state space. But when we move to dynamic *games*, potential multiplicity makes NFXP less straightforward to implement, making the two-step approach appealing once again.

Pesendorfer and Schmidt-Dengler 2008. In the late '00s, there was a bit of a cottage industry of econometrics papers introducing new estimators for dynamic games. This paper is part of that literature, proposing a least squares estimator—but that's not the part I care about. The important part of this paper is Section 5, which lays out conditions for identification of parameters in dynamic games. These results are relevant regardless of which estimator you use.

Egesdal, Lai and Su 2015. Coming a bit after the craze for new estimators had died down, this paper provides a summary and comparison of the various techniques. Along the way, it introduces the mathematical program with equilibrium constraints (MPEC) estimator, which is a conceptually beautiful (though computationally demanding) approach to the problem.

Crisman-Cox and Gibilisco 2018. There were various Signorino-style finite-horizon structural models of international war published in the '00s and early '10s. I see this paper as the first "modern" structural model of interstate conflict. There is an important substantive contribution here estimating audience costs, which had been a bedeviling problem in IR for two decades due to selection bias issues. Technically, the paper is also nice in its application of the MPEC technology.

Frey, López-Moctezuma and Montero 2023. Another nice substantive application of a structural dynamic model, this time to the formation of electoral coalitions in Mexican municipal elections. This paper uses a conditional choice probability approach, but with an additional wrinkle due to the presence of unobserved heterogeneity—candidate "valence" shocks that the players observe but the analysts don't.

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