PSCI 8350: Mathematical Foundations of Political Analysis

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This course will introduce you to the essential mathematical techniques necessary to understand and undertake research in political science. The focus will be on mathematical topics with applications to statistics and causal inference, though many of the techniques covered here are also useful for formal modeling.

Why do political scientists need to learn about math? Certainly not because human behavior can be reduced to a set of equations—we're not doing physics or chemistry here. Instead, the first reason we need math is for precise communication with other researchers. When you report the results of an analysis in a conference paper or publication, you must describe to your readers what you did and why you did it. Mathematical notation and equations allow you to do this both concisely and precisely. You will be much better able to communicate what you did and understand what others have done if you can speak the language of math. The main goal of this course is to help you develop that sort of fluency.

We also use mathematical reasoning as a check on the logic of our arguments. At its core, math isn't about numbers or equations—it's about constructing arguments where your conclusions follow from your premises. Virtually everything you write in your career as a political scientist, from your first term paper to your valedictory address as the president of the American Political Science Association, will be trying to get your readers to agree with an argument you're making. You will be able to do that better if you have some practice making arguments that a reader *has to* accept, where the conclusions follow inexorably and unquestionably from the premises. To that end, throughout the course, we will practice working through and writing mathematical proofs.

Topics and schedule

Your main source for readings will be the lecture notes at `https://bkenkel.com/mfpa`. I will also assign some chapters from *Mathematics: A Very Short Introduction* by Timothy Gowers (Oxford, 2002).

Some useful, but optional, additional references:

- *Mathematics for Economists* by Carl P. Simon and Lawrence E. Blume (W. W. Norton, 1994).
- *The Nuts and Bolts of Proofs* by Antonella Cupillari (Academic Press, 2011).
- *How to Prove It: A Structured Approach* by Daniel J. Velleman (Cambridge, 2019).
- *The Cartoon Guide to Algebra* and *The Cartoon Guide to Calculus* by Larry Gonick (William Morrow, 2015 and 2011).
- *The No Bullshit Guide to Linear Algebra* by Ivan Savov (Minireference Co., 2017).
- *A First Course in Optimization Theory* by Rangarajan K. Sundaram (Cambridge, 1996).

We will cover a variety of topics from calculus, linear algebra, and real analysis. The schedule of dates is approximate and may change depending on how quickly or slowly we get through each topic—we won't move on from a technique until you understand it well and feel comfortable using it in practice.

1. **Formal logic (August 21 and 26).** The key operators—negation, and, or. De Morgan's Laws. Implication and equivalence. Truth tables. Proof by contradiction.

2. **Set theory (August 28 and September 2).** Basic language and operations—element, inclusion, subset, intersection, union, set difference. De Morgan's Laws (yes, again!). Special sets like the naturals, integers, rationals, and reals. Set builder notation. Functions as mappings between sets.

3. **Sequences and series (September 4 and 9).** Intuitive definition of convergence. Basics of predicate logic—"for all" and "there exists" statements. Formal definition of convergence. Series as a sequence of partial sums. Geometric series.

4. **Derivatives of one-variable functions (September 11, 13, and 18).** Rates of change and the tangent curve. Derivative as a limit of small differences. Essential rules of differentiation—power rule, constant multiple rule, sum rule, product rule, quotient rule, chain rule. Higher-order derivatives. Concavity and convexity.

   First exam September 23.

5. **Optimization in a single dimension (September 25 and 30).** Connections between differentiation and optimization. First- and second-order conditions. Local versus global optima. Optimization of concave functions under an interval constraint.

6. **Integration in a single dimension (October 2, 7, and 14).** Area under the curve and Riemann sums. Integral as the reverse of a derivative. Integration of polynomial and exponential functions. Change of variables technique. Integrals with infinite limits.

7. **Vectors and matrices (October 16 and 21).** Vector arithmetic. Dot products and norms. Linear independence. Matrix transposition, addition, and multiplication. Writing linear systems as matrix formulas.

8. **Matrix algebra (October 23, 28, 30).** Matrix inversion. Necessary and sufficient conditions for invertibility. Determinants. Matrix inverses as solutions to linear systems. Cramer's rule.

   Second exam November 4.

9. **Derivatives of multivariate functions (November 6, 11, and 13).** Partial derivatives and the gradient. Convexity and concavity of multivariate functions. Optimization over multiple inputs. Implicit functions and their derivatives.

10. **Constrained optimization (November 18, 20, December 2 and 4).** Optimization under equality constraints and non-negativity constraints. Method of Lagrange multipliers. General inequality constraints. Constraint qualifications.

    Final exam December 12.

Grading

Grades in graduate school are weird. The PhD is a terminal degree, so you don't need to keep up good grades to get into some future program. No employer, whether in academia or outside of it, will care about your grades or GPA in a PhD program. And yet you *should* care about your grades—just not in the same way, or for the same reasons, as you (perhaps) did when you were an undergraduate.

In graduate school, a grade below A represents incomplete mastery of the material. We're teaching you this material because we think you need to understand it deeply in order to produce high-quality research in political science. So while a low grade on an assignment or in a course as a whole isn't a career catastrophe, it is a signal light flashing at you to say "You need to spend more time on this material to master it fully." Take that signal seriously.

Problem sets. There will be a weekly problem set, with a target time of 3–4 hours to complete. These problem sets will comprise 50% of your course grade.

I strongly recommend trying problems on your own before seeking help from peers. The whole point of this course is to build your—yes, I mean you personally, the individual person reading this syllabus—fluency and facility with the language and techniques of mathematics. You'll do that best if you spend some time struggling on your own.

Same goes for seeking help from AI tools like ChatGPT. There's no formal restriction on your use of AI in this course. Just make sure to use it as a *complement to* your learning, not as a *substitute for* learning on your own.

Exams. There are three in-class exams. The first two will each count for 15% of your course grade, and the final will count for 20%.