The Effective Power of Military Coalitions: A Unified Theoretical and Empirical Model

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September 8, 2020

Abstract

We develop a unified model of crisis bargaining and war fighting between military coalitions. We argue that the effective power of a coalition depends not only on the raw capabilities of its members, but also how much effort they choose to exert. Free-riding may make a coalition amount to less than the sum of its parts. Using data on the escalation and outcomes of international disputes, we structurally estimate the model parameters, allowing us to identify determinants of countries' force multipliers and the degree of prior uncertainty. We find that demographic and economic characteristics are the most important determinants of military effectiveness, with regime type and geopolitical considerations playing lesser roles. The structural model also allows us to simulate counterfactual outcomes in disputes between coalitions. This allows us to test historical claims like that earlier American involvement would have made a substantial difference in both World Wars, even in the face of strategic free-riding by its allies.

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From the Thirty Years' War in the 17th century to the wars in Afghanistan and Iraq in the 21st, the most destructive and historically significant conflicts have drawn in large coalitions of states. Many smaller conflicts also involve more than two states. Fourteen percent of Militarized Interstate Disputes have two or more countries on a single side, which rises to 36 percent among disputes that resulted in at least 25 fatalities. International relations scholars widely accept the idea that war is the result of crisis bargaining in the shadow of military power (Fearon 1995; Powell 1999; Reiter 2003). Therefore, to understand why these conflicts occur—and who prevails when they do—we must understand both the aggregation of power *within* a coalition of states and the shape of negotiations that occur *between* coalitions.

The effective power of a military coalition is not merely the product of raw materials—it depends largely on strategic choices by its constituent states. This is true for individual states as well (Sullivan 2007), but coalitions add significant complications. The most important strategic consideration is the free-rider problem. Weaker members of a coalition have an incentive to contribute even less to collective military efforts than they would if they had to fight on their own, leaving the most powerful partners to take on the bulk of the effort (Olson and Zeckhauser 1966). The likelihood of free-riding within a coalition may have other strategic knock-on effects in crisis bargaining. Bringing in more coalition partners might intimidate an opponent into making concessions at the negotiating table, but only if the new partners' efforts are not expected to be offset by other members' free-riding. And if the crisis escalates into war, each side's choice of how much to spend trying to win will depend on how it expects the other side to behave, which is in part a function of free-riding incentives.

In this paper, we propose a new theoretical model of crisis bargaining between coalitions that accounts for all of these strategic dynamics, allowing anticipated free-riding by coalition partners to shape strategic decisions in both negotiating and fighting. We then estimate the structural parameters of the model using data on militarized disputes from 1816 to 2010. This approach allows us to simultaneously quantify the determinants of *individual* states' military capabilities and how these are *aggregated* into effective power in crises between coalitions. Situating the analysis within a game-theoretical model of crises also allows us to answer counterfactual questions about strategic behavior, such as how historical crises would have turned out if different coalition partners had joined or dropped out.

Using our theoretical framework, we decompose a coalition's effective military power into the product of two distinct factors. The first of these is the *effort* each state exerts—the resources used or the consumption foregone in order to increase their side's chance of success in the conflict. Effort is an endogenous, strategic choice. This is where free-riding enters our picture of effective power, when states decrease the effort they would have otherwise exerted in anticipation of their coalition partners' efforts. By contrast, the second determinant of effective power is a fixed, exogenous factor, which we call the *force multiplier*. This represents the "bang for the buck" states get from their military resources; the higher a state's force multiplier, the more its side benefits militarily from the same exertion of effort. This force multiplier may depend on military technology, political organization, economic strength, or a host of other factors. The endogenous equilibrium levels of effort are functions of participants' force multipliers, but we take the multipliers themselves to be exogenous.

Our game-theoretical model links the equilibrium chance of war and the probability of victory by each side (in case war occurs) to the *individual* force multipliers of the participants on each side. Our empirical analysis exploits this linkage, using the Militarized Interstate Dispute data to estimate the structural determinants of individual countries' force multipliers. This exercise allows us to characterize the strongest influences on the effectiveness of countries' individual efforts. In addition, by calculating the force multipliers for countries at particular points in time, we are able to answer counterfactual questions about conflict contributions and outcomes—such as how much less likely a coalition would be to win a war if one of its partners had dropped out. By assuming equilibrium choices of effort in these counterfactuals, we account for free-riding behavior and other strategic incentives in a way that would be impossible if we relied on traditional reduced-form statistical modeling.

In analyzing the determinants of individual countries' force multipliers, we find that demographic and economic factors loom largest. According to our estimates, states with large, dispersed populations and rich economies have the greatest effectiveness per unit of military resources. Political and geopolitical factors have smaller influences, though there is a discernible democratic advantage in the employment of military resources. For example, despite China's demographic advantage and rough economic parity, we estimate a larger force multiplier for the United States as of 2010 due to its more democratic political regime. Altogether, the most valuable coalition partners—and those whose inclusion are most likely to cause others to free-ride—are rich, populous democracies.

Once we have calculated force multipliers for countries at a particular point in time using our model estimates, we can use the unique equilibrium of the underlying bargaining model to simulate individual contributions and likely outcomes in modified or hypothetical coalitional disputes. We take this approach to analyze individual contributions to the most important conflicts between coalitions in our sample, the world wars. We find that earlier American intervention in the First World War would have raised the Allied Powers' chance of victory by about 14 percent. We also find that the Central Powers' chance of victory would have increased by about 10 percent if France had been knocked out of the conflict in 1914, partially validating the military philosophy of the Schlieffen Plan. For the Second World War, we find that earlier Soviet participation would have made a moderate difference in the European theater, but only if the United States had continued to stay out. Similarly, in the Pacific theater as of 1941, Soviet participation would have made almost no difference, being offset by British and Chinese free-riding.

By breaking power into the endogenous selection of effort and the exogenous effectiveness of that effort, we follow other scholars who have recognized the distinction between the raw resources a state possesses and its ability or willingness to use those resources (Sullivan 2007; Beckley 2010). We innovate on these earlier efforts in important ways. Most importantly, our analysis encompasses military coalitions as well as individual states. We show how the problem of translating potential power into real military effects is even more complicated in coalitions. In addition, we provide a general equilibrium model of the relationship between raw force multipliers and strategic choices of effort. This allows us to predict explicitly how the efforts of individual states—and thus their fortunes in crisis bargaining—depend on the military potentials of their friends and foes.

1 Coalitions, Power, and War: Existing Literature

The canonical bargaining model of war is purely dyadic, treating both crisis negotiations and war itself as two-player affairs (Fearon 1995). Our theoretical model fits into a growing body of work extending the bargaining model to allow coalition relationships among the players. Scholarship in this tradition has answered a number of important questions about how actions and outcomes in crisis bargaining change when coalitions are involved. Fang, Johnson and Leeds (2014) show how public recommendations by coalition partners can lead states to accept a wider range of demands in crisis bargaining. Wolford (2014*b*) demonstrates how the need to maintain coalition support can complicate costly signaling, thereby increasing the likelihood of war. Trager (2015) finds that coalitions open up possibilities for cheap talk signaling to adversaries that would otherwise lack credibility, while Smith (forthcoming) shows how free-riding incentives might complicate such signaling.

Our our innovation relative to this literature is to endogenize individual states' choices of military effort—and thus the effective power of military coalitions in times of war. In all of the models noted above, aggregation of power is treated in reduced form: the power of one side increases by a fixed amount when a new partner joins.[1] Consequently, these theories do not speak to *how* coalitions aggregate power, nor to which partners would most increase their side's effective power. Because military efforts are not modeled, the only way for a state to free-ride in these models is to stay out of the conflict altogether. Our model, by contrast,

[1] Wolford (2014*b*) allows endogenous military mobilization, but only by one state in the interaction (and before it knows whether its partner will participate in the conflict).

produces sharp empirical implications about how individual states' capabilities translate into collective power, depending on which states they are partnered with and fighting against. We also introduce a richer conception of free-riding, quantifying the difference one state's participation makes to its partners' efforts. The formal model closest to ours is by Garfinkel (2004*a*,*b*), who models conflict as an N-player contest with coalitions (see also Skaperdas 1998; Krainin 2014).[2] We simplify Garfinkel's model by treating the coalition structure as exogenous, while expanding on it by modeling pre-conflict negotiations.

We also contribute to the empirical literature on disputes and wars between coalitions of states. Despite the classic theoretical findings demonstrating incentives to free-ride in alliances (Olson and Zeckhauser 1966), empirical studies of coalitions typically measure the power of a coalition as the sum of its members' capabilities (e.g., Grant 2013; Fang, Johnson and Leeds 2014; Wolford 2014*a*, 2017; Morey 2016; Chiba and Johnson 2019). By contrast, our calculations of coalitions' effective power are driven by theory—namely, the equilibrium allocations of effort in our formal model of war fighting.

Our work is also part of a recent tradition breaking away from purely dyadic research designs (see Poast 2010; Dorff and Ward 2013). Cranmer and Desmarais (2016) critique the bias toward "dyadic theory" and "dyadic analysis" in studies of conflict. Theoretically, we bridge between dyadic and k-adic approaches by modeling crisis bargaining as occur between a pair of coalitions, each of which may consist of many individual states. We then carry this hybrid conceptualization into our empirical work by directly estimating the parameters of our formal model. We do not artificially treat a conflict between coalitions as a loose collection of dyadic conflicts; our unit of analysis is the dispute. Yet, by using a structural model, we are still able to estimate state-level parameters, such as how demographic and economic variables shape individual states' force multipliers.

By structurally estimating a model of negotiations and conflict, we also advance the burgeoning literature on the empirical study of the bargaining model of war (see Gartzke

[2]Krainin and Wiseman (2016) also model N-player coalitional war, though their dynamic modeling framework is otherwise quite distinct from ours.

and Poast 2017). Substantively, we build on prior work relating power and uncertainty to the likelihood and outcome of conflict (Reed 2000; Clark and Reed 2003; Reed et al. 2008). We innovate on this work substantively by introducing coalitions and by conceiving of effective power as the product of (exogenous) force multipliers and (endogenous) effort. We also innovate methodologically by structurally estimating a game-theoretical model and by using its equilibrium to perform counterfactual simulations. While structural estimation has long been a staple of industrial organization, it has seen increasing use in political science as scholars seek to answer counterfactual questions about complex strategic interactions (e.g., Carter 2010; Whang, McLean and Kuberski 2013; Crisman-Cox and Gibilisco 2018; Ascencio and Rueda 2019; Abramson and Montero forthcoming). In this sense, our work is part of a long tradition in international relations using formal models to reason through counterfactual outcomes in real-world conflicts (see Levy 2008). Whereas such work has traditionally analyzed counterfactuals through qualitiative case studies, structural estimation allows us to scale this up to the analysis of quantitative data.

Finally, our work has some notable methodological antecedents in political science and economics. Our estimation of force multipliers is similar to a Bradley–Terry model (Bradley and Terry 1952), and thus to the recent work by Renshon and Spirling (2015) using such models to estimate the determinants of military effectiveness. We extend this technique by allowing for competitions between coalitions. The war-fighting portion of our model is also similar to work by Kang (2016), who estimates a game-theoretic model of lobbying as a contest. Meanwhile, our estimation of optimal offers and the probability of bargaining breakdown in an ultimatum game is similar to the estimator by Ramsay and Signorino (2009). We extend this technique by allowing for coalitions and by endogenizing the reservation values through a model of war.

2 A Model of Crisis Bargaining between Coalitions

We model crisis bargaining in the shadow of conflict between two coalitions, each comprised of one or more distinct states. In the model, the military contribution of each state depends on how much effort it chooses to exert, as well as an exogenous force multiplier describing its technological effectiveness. Bargaining takes place between representatives of each coalition who anticipate the likely outcome of conflict and calibrate their negotiating stances accordingly. As in the workhorse model of conflict (Fearon 1995), bargaining breakdown occurs due to incomplete information.[3]

2.1 Setup

The coalitions are labeled A and B, consisting of I_A and I_B countries respectively. In the crisis, the sequence of play is as follows. First, a representative of side A makes an offer $x \in \mathbb{R}$. A representative of side B may accept or reject the offer. If it accepts the offer, side A receives x to divide among its constituent countries and side B receives $1 - x$. Each representative's objective is to maximize the total payoff to her side.[4]

If the offer is rejected, the representative for each side $K \in \{A, B\}$ pays a cost for bargaining breakdown, $\theta_K \geq 0$, and the game proceeds to war. Each of these bargaining breakdown costs is drawn according to a cumulative distribution function F_K; these distributions are common knowledge, but the realized values of θ_K are private information.[5] We assume each F_K has full support on $\mathbb{R}_+$ and is continuously differentiable, with an associated density

[3]This is in line with the focus on informational explanations in formal theories of war between coalitions (Fang, Johnson and Leeds 2014; Wolford 2014*b*; Trager 2015; Smith forthcoming). The development and estimation of a general model of war due to commitment problems between coalitions would be a considerable project in its own right—and thus one that we leave for future research.

[4]Given this, we do not model the distribution of spoils across states in case of peace, as it is immaterial to the equilibrium.

[5] You can think of A and B as the initiator and the target, with the other coalition members as being allies and the conflict joiners. This is a natural structure for the empirical data on war. It is also true that we need a stochastic component in order for war to occur probabilistically, as required for estimation. One might instead model the marginal costs of effort in the war contest as the stochastic component. However, contests with a continuous distribution of marginal costs generally lack explicit equilibrium solutions even with just two players (Fey 2008), which creates practical problems for estimation.

function f_K.

If war occurs, each country involved in the crisis selects a level of effort $e_i \geq 0$. Denote the vector of each country's effort by $e = (e_1, \ldots, e_I)$, where $I = I_A + I_B$, the total number of states involved in the crisis. The postwar distribution of spoils depends on how much effort each country exerts, as well as their force multiplier (or military effectiveness), an exogenous parameter $m_i > 0$. The probability that side A wins is given by the contest success function

$$p_A(e) = \frac{\sum_{i \in A} m_i e_i}{\sum_{i=1}^{I} m_i e_i},$$

with $p_B(e) = 1 - p_A(e)$. Spoils of size 1 are divided among the countries on the winning side in proportion to their effectiveness-weighted contribution. The expected payoff from the contest to country i on side K is

$$\begin{aligned} \pi_i(e) &= p_K(e) \frac{m_i e_i}{\sum_{j \in K} m_j e_j} - c_i e_i \\ &= \frac{m_i e_i}{\sum_{j=1}^{I} m_j e_j} - c_i e_i, \end{aligned}$$

where $c_i > 0$ is the country's marginal cost of effort. Each country's force multiplier and marginal cost of effort are common knowledge. As the equilibrium would be identical if we replaced each country's force multiplier with m_i/c_i and normalized their marginal cost of effort to 1, we will call the ratio m_i/c_i the *effective force multiplier* for country i. Without loss of generality, assume countries are labeled in order of their effective force multipliers: $m_1/c_1 \geq m_2/c_2 \geq \cdots \geq m_I/c_I$.

Our assumption that coalition membership is fixed from the outset is, of course, a simplification. It is also a departure from models that treat coalition wars as resulting from the expansion of bilateral crises (Wolford 2014*a*; Fang, Johnson and Leeds 2014, e.g.,). This assumption provides important tractability in estimating the model,[6] but we do not see

[6] Otherwise, for each dispute in our data, we would need to model hundreds of states' decisions whether to become involved.

it as a major limitation of the analysis. If states have rational expectations about who will ultimately enter the crisis, their behavior at the bargaining stage should reflect those expectations—even if those states are not literally present at the table when bargaining takes place. Moreover, a critical parameter in most theories of war expansion is the military difference made by each new coalition partner. By modeling and estimating how power is aggregated in military coalitions, our analysis provides a necessary first step toward a richer empirical model of how those coalitions form in the first place.

2.2 Equilibrium

We solve the model for a perfect Bayesian equilibrium. Proceeding backward, we first derive the equilibrium choices of effort in case of war. This subgame is essentially an n-player rent-seeking contest with asymmetries among players, so it has a unique equilibrium with a simple structure. Equilibrium effort is greatest among the players that have the highest effective force multipliers. Players with particularly low ratios, m_i/c_i, may opt out of exerting any effort at all. At least two players exert some effort, however, and all players exert effort in equilibrium if the effective force multipliers do not vary much among players. If the greatest multipliers are disproportionately concentrated on side A, it is possible that all the players on side B exert no effort, resulting in $p_A(e^*) = 1$. The following proposition formalizes the equilibrium of the war subgame.[7]

Proposition 1. *The war subgame has a unique Nash equilibrium e^*, in which the number of players who exert positive effort is*

$$J = \max\left\{ j \in A \cup B : \sum_{i=1}^{j} \frac{c_i}{m_i} > (j-1)\frac{c_j}{m_j} \right\}.$$

[7] See Stein (2002) for the proof.

Equilibrium effort allocations are

$$e_i^* = \begin{cases} \frac{1}{c_i} q_i(1-q_i) & i \leq J, \\ 0 & i > J, \end{cases}$$

where each q_i *is defined as*

$$q_i = \frac{(J-1)\frac{c_i}{m_i}}{\sum_{j=1}^{J} \frac{c_j}{m_j}}.$$

Figure 1 illustrates the equilibrium quantities defined in the proposition for a war subgame involving three players. Each player's effort decreases as their marginal cost of effort increases, all else equal. When one player's effective force multiplier is much lower than the others', that player drops out of the contest altogether. Short of that, however, a decrease in one player's effort is not always associated with lower efforts by the others. For example, in the leftmost panel of Figure 1, even as player 3's effort decreases with its marginal cost of effort, the other two players' efforts increase.

We now solve for equilibrium bargaining behavior, assuming strategies in case of war are given by the equilibrium defined in Proposition 1. As usual, each side's acceptance set depends on the value of its outside option. Let $\pi_A^* = \sum_{i \in A} \pi_i(e^*)$ denote side A's expected utility from fighting, and define π_B^* analogously. The war-fighting equilibrium depends only on the participants' effective force multipliers, which are common knowledge, which means π_A^* and π_B^* are common knowledge as well. However, each side must also consider its cost of bargaining breakdown, θ_K, which is private information. Let $w_K^*(\theta_K)$ denote the overall expected utility of bargaining breakdown for side K of type θ_K, so that $w_K^*(\theta_K) = \pi_K^* - \theta_K$. The following proposition characterizes the equilibrium offer strategy of side A and acceptance strategy of side B.

Proposition 2. *Assume the hazard function* $f_B/(1-F_B)$ *is nondecreasing. Let* $\mu^* = f_B(0)^{-1} - 1 + \pi_A^* + \pi_B^*$. *In any perfect Bayesian equilibrium of the bargaining game:*

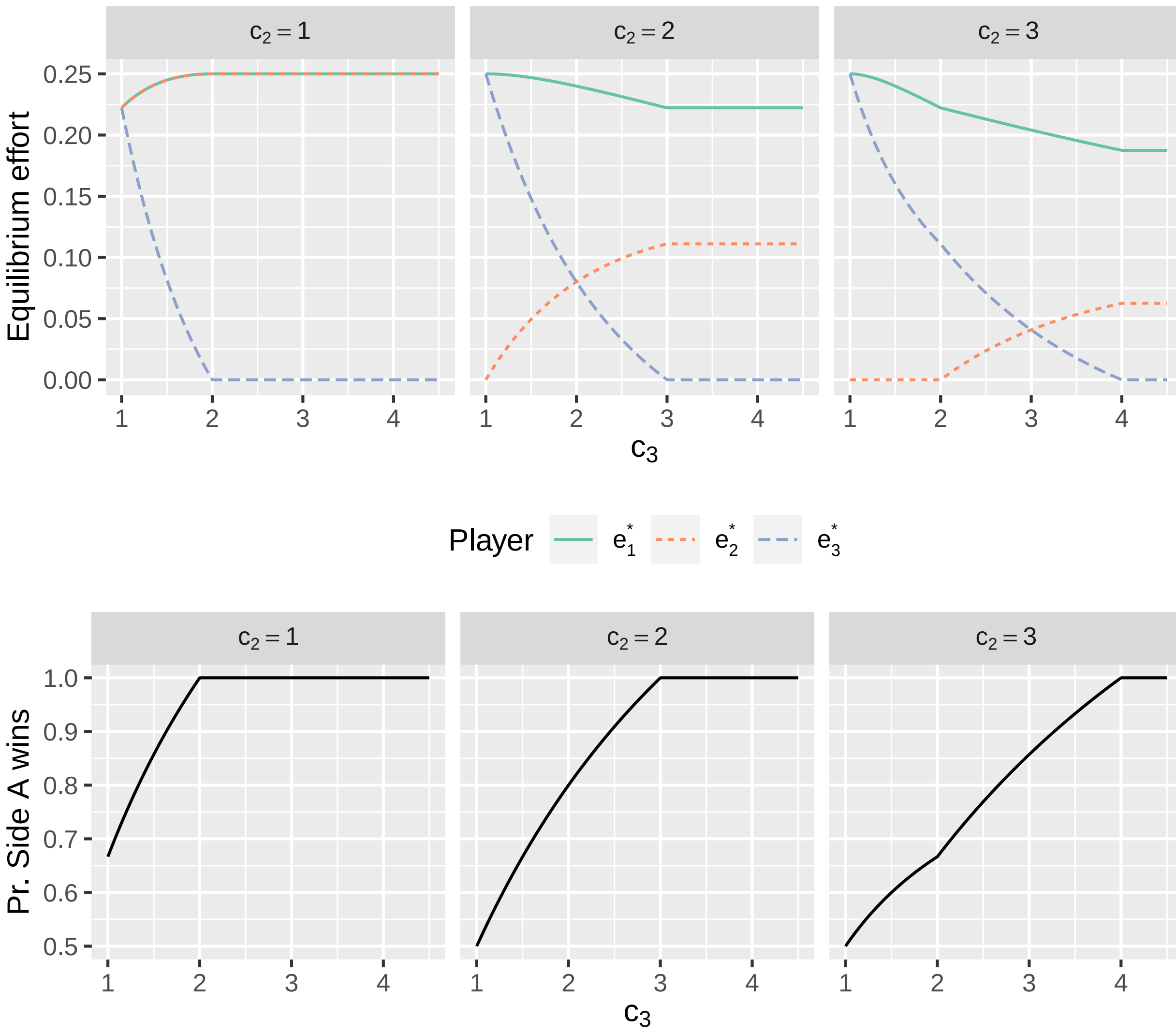

Figure 1. Equilibrium outcomes in the war subgame with three players, $A = \{1, 2\}$ and $B = \{3\}$. Each player's effectiveness is normalized to $m_i = 1$, and player 1's marginal cost is fixed at $c_1 = 1$ throughout.

(a) Types $\theta_A \leq \mu^*$ of side A make the offer that is the unique solution to

$$x^*(\theta_A) = w_A^*(\theta_A) + \frac{1 - F_B(x^*(\theta_A) - 1 + \pi_B^*)}{f_B(x^*(\theta_A) - 1 + \pi_B^*)}.$$

Types $\theta_A > \mu^*$ offer $x^*(\theta_A) = 1 - \pi_B^*$.

(b) Each type θ_B of side B accepts any offer $x < 1 - w_B^*(\theta_B)$ and rejects any offer $x > 1 - w_B^*(\theta_B)$.

(c) Strategies in all war subgames are given by Proposition 1.

Proof. Since payoffs in case of war do not depend on x or θ_K, statement (c) is immediate from Proposition 1. Statements (a) and (b) then follow from Fearon (1995, Claim 2). □

We observe the usual risk-reward tradeoff in the bargaining equilibrium. If side A's war payoff or private cost of bargaining breakdown is low enough, then it makes a risky offer with a chance of being rejected by side B. The probability of war is driven by side A's incentive to take risks to increase its settlement payoff, which depends on both A's cost of bargaining breakdown and A's uncertainty about B's willingness to accept offers.

3 Quantifying the Bargaining Model

Our goal is to quantify the major components of the bargaining model of war laid out in the previous section. First, we want to understand which real-world indicators contribute most to the main parameters of the model: countries' military effectiveness (m_i), their marginal costs of effort (c_i), and the extent of uncertainty about the costs of bargaining breakdown (the variance of θ). To do this, we write these parameters as functions of the relevant variables, then structurally estimate their weights from data on interstate crises. The structural estimation procedure identifies the parametric form of the model that best corresponds to what we observe empirically. Second, with our structural estimates in hand, we wish to quantify the features of crisis bargaining that would otherwise be difficult to observe (or at least to measure systematically) for particular countries, in particular crises, or as a function of directly observable variables. These include the individual effort contributions of coalition partners and the extent of free-riding in conflicts with more than two actors.

The formal description of our structural estimation procedure follows. We consider data consisting of a sequence of N crises. The n'th crisis involves I_n states, with $I_{nA} \geq 1$ on side A and $I_{nB} \geq 1$ on side B. The variable $Y_n^{\text{war}} \in \{0, 1\}$ denotes whether the n'th crisis resulted in war, and $Y_n^{\text{win}} \in \{A, B\}$ denotes its winner.[8] The other endogenous components of the game—offers and effort—are not observed. We estimate the parameters of the model as a function of observable characteristics:

- Dispute-level characteristics affecting the distribution of uncertainty about each side's cost of bargaining breakdown (θ_A and θ_B), collected in a vector W_n.[9]
- State-level characteristics affecting military effectiveness, collected for each crisis participant i in the vector X_{ni}.
- State-level characteristics affecting the marginal cost of effort, collected for each crisis participant i in the vector Z_{ni}.

Consider the i'th state in the n'th crisis. We assume a log-linear model for the relationships that military effectiveness and the marginal cost of effort have with their respective component variables:

$$m_{ni}(\beta) = \exp(X_{ni}^\top \beta);$$
$$c_{ni}(\gamma) = \exp(Z_{ni}^\top \gamma).$$

This gives the coefficients the natural interpretation of the effort multiplier effect. The vector β determines the relationship between state-level characteristics and military effectiveness,

[8]If the n'th crisis does not end in war, we write $Y_n^{\text{win}} = \emptyset$.
[9]We treat all vectors as column vectors.

and the vector γ does the same with the marginal cost of effort. We treat these as unknown parameters to be estimated. For the n'th observation, let $p^*_{nA}(\beta, \gamma)$ denote the equilibrium probability of victory by side A under the given parameters, and let p^*_{nB}, π^*_{nA}, and π^*_{nB} be defined analogously. In a bilateral crisis, with exactly one state on each side, the equilibrium probability of victory takes a simple form:[10]

$$p^*_{ni}(\beta, \gamma) = \frac{1}{1 + \exp\left((X_{nj} - X_{ni})^\top \beta + (Z_{ni} - Z_{nj})^\top \gamma\right)}. \tag{1}$$

Equation 1 has two important implications. First, in bilateral disputes, the probability of victory takes the form of a logistic regression. Second, there must be no overlap in the terms included in the cost and effectiveness equations. Otherwise, the model is not identified. When this restriction seems substantively insensible, we may consider the parameter estimates as net effects—effectiveness minus cost for X variables, and vice versa for Z variables.

To estimate how the distribution of prior beliefs about the costs of bargaining breakdown varies with dispute characteristics, we must impose additional structure on the model. Specifically, we assume that each side's type in the n'th crisis is drawn from an exponential distribution with parameter λ_n. The choice of a single-parameter family is also motivated by a data limitation. We only observe whether an offer is accepted or rejected (via whether war occurs) but not the value of the offer itself, leaving us with enough degrees of freedom only to estimate a single parameter of the prior distribution. The exponential distribution is a convenient choice among the set of single-parameter distributions with support on $\mathbb{R}_+$, as it yields simple expressions for the optimal offer and the resulting probability of war.

To model the relationship between the dispute-level variables and the distribution of prior beliefs, we again impose a log-linear model:

$$\lambda_n(\alpha) = \exp(W_n^\top \alpha).$$

[10]This follows from Proposition 3 in the Appendix.

The vector α is an unknown parameter to be estimated.

Under our distributional assumptions, the variance of the privately known cost of bargaining breakdown is $1/\lambda^2$. An increase in λ therefore reduces uncertainty, which in turn reduces the equilibrium probability of war.[11] The upshot is that we infer that a variable in W increases both uncertainty and the risk of war if its coefficient in α is negative, and vice versa if its coefficient is positive.

We estimate the parameters (α, β, γ) by maximum likelihood. The log-likelihood function is

$$\begin{aligned} \log \ell(\alpha, \beta, \gamma) = & \sum_{n: Y_n^{\text{war}}=0} \log(1 - r_n^*(\alpha, \beta, \gamma)) \\ & + \sum_{n: Y_n^{\text{war}}=1} \log r_n^*(\alpha, \beta, \gamma) \\ & + \sum_{k \in \{A, B\}} \left[\sum_{n: Y_n^{\text{win}}=k} \log p_{nk}^*(\beta, \gamma), \right] \end{aligned} \tag{2}$$

where $r_n^*(\alpha, \beta, \gamma)$ denote the probability of war for the n'th crisis as a function of the unknown parameters.[12] We calculate inferential statistics via the jackknife.[13]

4 Data and Specification

We use historical data on disputes to estimate the parameters of the formal model. The key assumption is that the data we observe is generated according to equilibrium play of the model. Because the model accommodates purely bilateral as well as coalitional disputes, our sample includes both types. We identify how political, economic, and technological characteristics shape states' military effectiveness, marginal costs of effort, and uncertainty

[11] See Proposition 4 in the Appendix for a proof.

[12] A formal definition of the probability of war appears in Proposition 4 in the Appendix. Because we may have $r_n^* = 0$, $p_{nA}^* = 0$, or $p_{nB}^* = 0$ for certain parameters, it is possible that $\log \ell(\alpha, \beta, \gamma) = -\infty$. In Proposition 5 in the Appendix, we verify that there always exist parameters at which the log-likelihood is finite.

[13] Kinks in the likelihood function preclude ordinary gradient-based estimation of the standard errors. We employ the jackknife rather than the nonparametric bootstrap to ensure reproducibility and to reduce computational issues.

about each other's willingness to risk war. Because the analysis is grounded in a formal model of crisis bargaining between coalitions, we can use the resulting estimates to address counterfactual questions such as how much less effort one state would have exerted in a conflict if another state had joined their coalition.

4.1 Sample and Outcomes

The unit of observation is the interstate dispute, and our sample consists of all Militarized Interstate Disputes (MIDs) between 1816 and 2010 (Gibler, Miller and Little 2016).[14] The sample size is $N = 2{,}101$. There are 284 disputes in which at least one side consists of more than one state, which we call *coalitional disputes.* The coalitional disputes involve about 4.7 disputants each, giving us a total of 4,962 distinct dispute participants in the data.[15]

We follow the MID data codings to determine which side is side A and which is side B. In other words, we code the side judged to have initiated the dispute as the one that makes the offer in our bargaining model. We code a dispute as having gone to war, $Y_n^{\text{war}} = 1$, if it resulted in more than 25 fatalities. In case of war, we code side A as the winner, $Y_n^{\text{win}} = A$, when the MID data records victory by side A or yield by side B as the outcome. In all other cases we code side B as the winner, $Y_n^{\text{win}} = B$.[16] Table 1 reports the distribution of dispute outcomes in the sample.

Outcome	Frequency
$Y_n^{\text{war}} = 0$	1851
$Y_n^{\text{war}} = 1, Y_n^{\text{win}} = A$	65
$Y_n^{\text{war}} = 1, Y_n^{\text{win}} = B$	185

Table 1. Distribution of dispute outcomes.

[14]Version 2.1.1 of the Gibler–Miller–Little MID data.

[15]Six countries are coded as being on different sides of World War II at different times. We include them each only on the side on which they spent the most time.

[16]This entails assuming that a stalemate favors side B, which we see as natural given that side A is the initiator of the dispute.

Variable	Units	Mean	Std. Dev.
State-Level: Demographic			
Total Population	Thousands of persons‡	86,494	192,649
Urban Population	Thousands of persons‡	18,250	44,268
State-Level: Economic			
Energy Consumption	Thousands of coal-tons‡	223,027	581,944
GDP	Millions USD, 2011†	654,983	1,848,001
Import Percentage	Percentage‡	22.09	17.33
Iron and Steel Production	Thousands of tons‡	13,556	39,598
State-Level: Political			
Democracy	Polity IV score	−0.21	7.22
State-Level: Geopolitical			
Distance to Dispute	Miles‡	1,098	1,871
Nuclear Weapons	Binary	0.14	0.35
Dispute-Level			
Contiguity	Binary	0.665	0.45
Democracy–Greater	Polity IV score	3.147	6.53
Democracy–Lower	Polity IV score	−4.846	5.02
Interest Similarity	S-score	0.550	0.40
Major Power–Both	Binary	0.095	0.29
Major Power–Either	Binary	0.434	0.50
Number of Participants	Count†	2.362	1.83
Peace Years	Years‡	11.754	23.36

Table 2. State- and dispute-level characteristics included in the empirical model. †: log-transformed in estimation. ‡: $\log(1 + x)$-transformed in estimation.

4.2 State Characteristics

Even though our dependent variables are the collective outcomes of whether war occurred and who won if so, our structural approach allows us to estimate the determinants of *individual* states' effective force multipliers. We model these as functions of individual state characteristics, dividing the underlying state-level variables into four groups: demographic, economic, political, and geopolitical characteristics. Table 2 summarizes all of the variables included in the model.

Demographics. Indices of national power, including the popular CINC score, typically include population measures. We therefore include **Total Population** and **Urban Population** in the equation for m_i, raw military effectiveness. We use the measures from the National Material Capabilities data from the Correlates of War project (Singer, Bremer and Stuckey 1972).[17] Due to missingness in the measure of Urban Population, as well as numerous other state-level variables, we use multiple imputation for estimation and inference (Honaker and King 2010).

In principle, these demographic characteristics could have either a positive or a negative effect on the effective force multiplier. On one hand, large states might enjoy greater military effectiveness because of their ability to field mass armies (Posen 1993). This is particularly relevant in our time period of study, which starts just after the Napoleonic Wars. Conversely, the costs of governing and extracting taxes from large populations might inhibit military effectiveness (Beckley 2018).

Economics. Economic development has long been considered a key component of military effectiveness. Richer states have more resources available for military use and may also have access to more effective coercive technologies. We include a variety of measures in the military effectiveness equation, m_i, to capture economic influences on the force multiplier. The first is **GDP**, which we measure with data from the Penn World Tables (Feenstra, Inklaar and Timmer 2015).[18] As additional proxies for economic development, we also include **Energy Consumption** and **Iron and Steel Production** from the National Material Capabilities data.[19]

The last economic component we include in the effective force multiplier is **Import**

[17] Version 5.0.

[18] Version 9.0. We use the $RGDP^0$ measure, the one best suited for comparisons of "productive capacity across countries and across years." To estimate missing values of GDP as accurately as possible, we also include GDP measures from the Maddison Project (Bolt et al. 2018) and the World Development Indicators dataset (The World Bank 2019) in the multiple imputation model. (Maddison: version 2018; WDI: accessed 2019-02-28.)

[19] This leaves Military Expenditures and Military Personnel as the only two CINC components not included in our equation for military effectiveness. We see these as endogenous state choices more so than structural components of the force multiplier, making them inappropriate for inclusion here.

Percentage, the ratio of the state's total imports to its GDP. We draw this measure from the World Development Indicators (The World Bank 2019). Unlike the other economic variables, we include Import Percentage in the equation for c_i, the marginal cost of effort. If states must acquire raw materials or other resources via international markets in times of war, it might be more costly to field the same level of military effort than if those resources were available domestically.

Political. We include a single political component in the model: **Democracy**, measured as the net score (Democracy minus Autocracy) from the Polity IV project (Marshall, Gurr and Jaggers 2014).[20] We place Democracy in the equation for c_i, the marginal cost of effort, as citizens of democracies can more easily resist extractive or economically inefficient military production than can the subjects of authoritarian regimes (Bennett and Stam 1996). Scholars have also observed, however, that the political and cultural features of democratic societies may lend themselves to battlefield success (Reiter and Stam 2002). Our statistical model does not allow us to separately identifying the force-multiplying and cost-increasing effects of a single variable. If the force-multiplying effect of democratic politics is stronger, that would manifest itself as a negative coefficient on Democracy in the cost equation.

Geopolitical. Finally, we include two geopolitical determinants of a state's effective force multiplier in the equation for m_i, raw military effectiveness. The first is a technological variable, **Nuclear Weapons**, an indicator for whether a state has a deliverable nuclear weapon (Gartzke and Kroenig 2009). Of course, we only observe a single dispute in which a state used nuclear weapons (World War II). However, even when nuclear weapons are not explicitly put to use, they may serve as a force multiplier by increasing the effectiveness of states' coercive threats (Kroenig 2013). Alternatively, nuclear weapons might act to reduce a state's force multiplier, as the increased risk of nuclear disaster impels states to show restraint in their deployment of conventional capabilities (Asal and Beardsley 2007).

[20] Version 2017.

The other geopolitical factor is the **Distance to Dispute** for each participant state. This variable measures the distance from each state's capital to the location of the dispute as coded in the MIDLOC project.[21] We expect distance to act as a negative force multiplier, as states' ability to project power declines over long distances (Boulding 1962).

State-Specific Multipliers. In an alternative specification reported in the Appendix, we include state identifiers in the equation for m_i alongside the substantive variables noted above. This entails the addition of hundreds of parameters to the model and a corresponding degradation in model fit, per the AIC and BIC statistics. Consequently, we use the baseline model without state identifiers for our substantive analysis and counterfactual simulations reported in the remainder of the text.

4.3 Dispute Characteristics

We include dispute-level variables in the equation for λ, which determines the precision of the disputants' prior beliefs about each other's costs of bargaining breakdown. We describe each characteristic below as if the dispute involved just one state on each side. In coalitional disputes, for most of the variables here, we take the average across all dyads within the dispute (i.e., all pairs of states consisting of one from side A and one from side B).[22]

The first characteristic we include in the prior belief equation is **Contiguity**, an indicator for whether the states have a land border or are separated by no more than 150 miles of water. We take this measure from the Correlates of War project's Direct Contiguity Data (Stinnett et al. 2002).[23] Contiguity is a well-known predictor of international conflict (Bremer 1992), corresponding to high uncertainty in our model. While it may seem counterintuitive to think of neighbors as having more uncertainty about each other in an abstract sense,

[21] MIDLOC: version 2.0. Capital locations from EUGene, version 3.212 (Bennett and Stam 2000). We recode Distance to Dispute as 0 for disputes occurring on or within a state's borders.

[22] The exceptions are the major power indicators, for which we take the maximum across dispute dyads, and the joint democracy variables, for which we take the minimum.

[23] Version 3.20.

remember that what matters is uncertainty about one's willingness to risk conflict to achieve gains at the negotiating table. The stakes are lower for distant countries, and therefore so are the potential incentives to take risks. Similarly, because the stakes are likely greater in relationships with major powers, we record whether either or both sides of the dispute are major powers in **Major Power–Either** and **Major Power–Both**, respectively. We take these indicators from the Correlates of War project's State System Membership data (Correlates of War Project 2016).

We also model the degree of uncertainty as a function of **Interest Similarity**, the extent to which the disputants have similar relationships in the international system. We measure this via the S-score of states' alliance relationships (Signorino and Ritter 1999), per the Alliance Treaty Obligations and Provisions project (Chiba, Johnson and Leeds 2015). By facilitating information transmission among members, alliance relationships might reduce uncertainty and thereby mitigate conflict in the international system (Bearce, Flanagan and Floros 2006).

A long line of research has found that joint democracy reduces conflict (Russett and Oneal 2001), perhaps because democracies are better able to transmit information to each other (Schultz 2001). To account for this possibility, we include regime type measures in the prior belief equation. **Democracy–Lower** and **Democracy–Higher** record the Polity IV regime type scores for the least and most democratic state in the dyad, respectively.

We also include **Peace Years**, a measure of the years since the last militarized interstate dispute between states.[24] States that have maintained peaceful relations over time may be more certain about each other's intentions and willingness to risk conflict.[25] Finally, to identify whether uncertainty is systematically greater in disputes with many participants, we include the **Number of Participants** as a variable in the prior belief equation.

[24]For dyads that existed in 1816, we use the corrected measure developed by Werner (2000).

[25]We include peace years to capture its substantive influence on uncertainty in disputes, not to control for unmodeled temporal dependence. Our data consists solely of disputes and thus does not have the common time-series cross-section structure. Consequently, we eschew the high-dimensional peace year specifications advocated by Beck, Katz and Tucker (1998) and Carter and Signorino (2010).

5 Structural Estimation Results

5.1 Force Multipliers

To examine the strongest sources of influence on individual countries' effective force multipliers (m_i/c_i), we break them into four components corresponding to the four categories of variable defined above: demographic, economic, political, and geopolitical. A dispute participant's effective force multiplier is the product of these components:

$$\text{Total Multiplier}_i = \text{Demographic}_i \times \text{Economic}_i \times \text{Political}_i \times \text{Geopolitical}_i.$$

For example, $\text{Economic}_i > 1$ would indicate that a country's economic variables enhance its effectiveness per unit of military force, whereas $\text{Economic}_i < 1$ would indicate that its economic position is a net drain on its martial effectiveness. The components of the multipliers are calculated using the raw model estimates (Table 6 below), with terms in the effectiveness equation m_i entering positively and those in the cost equation c_i entering negatively. For example, the demographic component is computed as

$$\text{Demographic}_i = \exp(0.141 \times \text{Total Population}_i - 0.108 \times \text{Urban Population}_i).$$

Overall, we find that demographic and economic characteristics have the strongest influence on countries' total effective force multipliers. As an illustration, Table 3 displays the ten highest and lowest estimated force multipliers for dispute participants in our data. Our estimates range from a maximum of 3.19 for India in its 1971 dispute with China, to a minimum of 0.69 for Jordan in its 1962 dispute with Egypt and Yemen. The top spots are occupied by large democracies involved in disputes on or within their own borders—India in the 1960s–70s and the United States in the 1910s. By contrast, the lowest force multipliers belong to Jordan in the 1950s–60s and Singapore in the 1980s; these are small autocracies involved (in most cases) some distance from their home territory. While the top and bottom

	Country	Year	MID#	Multiplier	Demographic	Economic	Political	Geopolitical
1	India	1971	2099	3.19	2.03	1.36	1.15	1.00
2	India	1969	2098	3.15	2.06	1.33	1.15	1.00
3	India	1969	2633	3.15	2.06	1.33	1.15	1.00
4	India	1969	2634	3.15	2.06	1.33	1.15	1.00
5	India	1969	2635	3.15	2.06	1.33	1.15	1.00
6	United States	1919	2185	3.07	1.71	1.54	1.17	1.00
7	United States	1911	1653	3.03	1.72	1.50	1.17	1.00
8	United States	1918	2184	3.02	1.71	1.51	1.17	1.00
9	United States	1916	321	2.99	1.72	1.49	1.17	1.00
10	India	1965	623	2.97	2.04	1.27	1.15	1.00
4953	Singapore	1988	2762	0.77	1.31	0.71	0.97	0.86
4954	Jordan	1957	607	0.77	1.63	0.54	0.87	1.00
4955	Jordan	1967	1067	0.74	1.52	0.60	0.87	0.93
4956	Jordan	1962	1018	0.73	1.57	0.58	0.87	0.93
4957	Jordan	1967	1035	0.73	1.52	0.60	0.87	0.92
4958	Jordan	1963	1019	0.73	1.56	0.58	0.87	0.93
4959	Jordan	1959	3231	0.72	1.61	0.56	0.87	0.93
4960	Jordan	1966	3412	0.72	1.52	0.59	0.87	0.92
4961	Jordan	1961	122	0.70	1.58	0.57	0.87	0.88
4962	Jordan	1962	1108	0.69	1.57	0.58	0.87	0.88

Table 3. Greatest and lowest estimated force multipliers for dispute participants in the analysis data.

countries differ along many dimensions, it is evident from the table that the quantitative difference in their estimated force multipliers is due largely to the demographic and economic components. For each dispute participant listed in the table, the economic component affects their total multiplier more than the political and geopolitical components combined.

The importance of demographic and economic factors also comes through in a more systematic analysis of all dispute participants, summarized in Table 4. Across all disputants in the data, we find by far the most variation in the demographic force multipliers. Moreover, if we were to remove the demographic component from the equation, the remaining variation in total force multipliers would decrease by 71%. The economic component has the next-highest variation across dispute participants, and removing either the economic or the political component would lead to a roughly 20% decrease in the variation in total force multipliers. Interestingly, the geopolitical component tends to suppress variation in the total force multiplier across countries. This is because the countries in the strongest demographic, economic, and political positions tend to have relatively unfavorable geopolitical multipliers—for one thing, these are the countries most likely to be involved in disputes far from their homeland, reducing their ability to project power.

Component	SD	%Explained	Correlation
Total	0.39	100.0	1.00
Demographic	0.30	71.5	0.43
Economic	0.22	21.5	0.66
Political	0.11	18.3	0.42
Geopolitical	0.10	−62.8	−0.11

Table 4. Determinants of force multipliers in the empirical model. SD = standard deviation. %Explained = $100 \times \frac{V[\text{total}]-V[\text{total/component}]}{V[\text{total}]}$. (Values do not sum to 100 due to correlations between components.) Correlation = Spearman correlation between total and component.

We see a similar pattern—demographic and economic characteristics taking on the most importance—when we look exclusively at major powers. Figure 2 plots the estimated force multiplier for China, Japan, Russia, the United Kingdom, and the United States throughout our sample period. Unlike in the tables above, we calculate these for every year, not just when these countries were involved in disputes.[26] We are thus extrapolating out of sample, which in theory entails some risk of bias since our sample (dispute-years) is not a random sample of all country-years. However, as most of the countries depicted in Figure 2 are involved in a dispute in most years,[27] we do not expect selection bias to affect these estimates much.[28]

Both within each country's time series and when comparing different countries at the same point in time, we see the most variation in the demographic and economic components.[29] But that is not to say politics are unimportant. For example, in 2010, we estimate a larger total force multiplier for the United States than for China, despite their economic parity

[26]We assume zero distance to dispute in this figure, which raises the overall force multiplier and slightly decreases variation in the geopolitical component.

[27]The sole exception is Japan, which is in a dispute in 39% of years.

[28]In subsection 6.2 below, we perform a validity test to further study extrapolation performance.

[29]The negative trend in the demographic multiplier time series is because urban population growth has outpaced total population growth, and we find a moderate negative coefficient on urban population in the equation for m_i.

and the latter's demographic advantage. The American advantage comes from its greater political multiplier, a product of its relatively democratic political system.

On the whole, we observe a downward secular trend in the major powers' force multipliers. In terms of the question of who would be likely to win in case of a war, a common trend like this has little effect, as equilibrium probabilities of victory are a function of the relative force multipliers at any point in time. Still, the downward trends imply that it takes more effort for a major power to achieve a given military objective now than at their mid–20th century peaks. For example, we observe this in how the United States spent similar amounts in real terms on World War II and on the Iraq War (Stiglitz and Blimes 2008), despite accomplishing far less militarily in the latter.

Table 5 breaks down each component of the total multiplier into its constituent variables, displaying the dispute participants with the highest and lowest values of each component. The constituent variables are given as their contribution to the force multiplier, so that each component multiplier equals the product of the constituent variable multipliers.

We see the highest demographic multipliers for countries with moderately sized, totally rural populations. This reflects the countervailing effects of Total Population and Urban Population in the structural model. Total Population has one of the largest positive effects among the variables we consider, with a 1% population increase being associated with a 0.14% increase in military effectiveness. As our sample covers the post-Napoleonic world, this finding is consistent with the idea that large national states possess a military advantage through their ability to field mass armies (Posen 1993). However, this effect appears to be limited mostly to increases in a country's non-urban population. We find a partially offsetting force multiplication effect of Urban Population, with a 1% increase in urbanites associated with a 0.1% decline in the effectiveness parameter.

Turning to the economic multiplier, we find that indicators of economic development generally lead to greater military effectiveness. Energy Consumption and Iron and Steel Production, the development proxies from the National Material Capabilities data, both

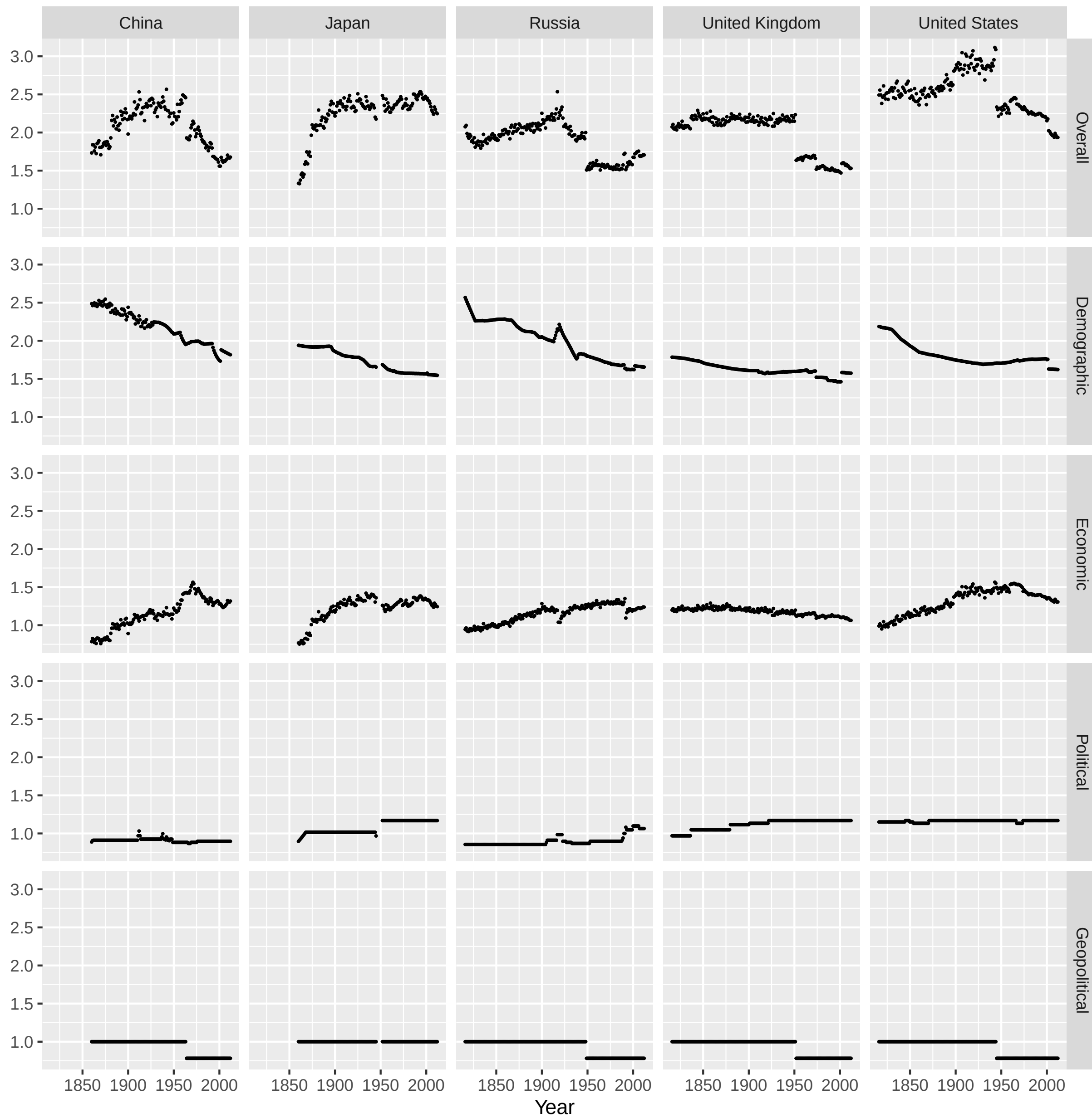

Figure 2. Estimated force multipliers for major powers over time.

	Country	Year	MID#	Demographic	Total Pop. $\beta = 0.14$	Urban Pop. $\beta = -0.11$
1	Ethiopia	1931	407	3.77	3.77	1.00
2	Ethiopia	1923	1669	3.67	3.67	1.00
3	Afghanistan	1934	3159	3.52	3.52	1.00
4	Afghanistan	1925	1781	3.47	3.47	1.00
5	Papua New Guinea	2008	4589	3.46	3.46	1.00
4958	Singapore	1987	2785	1.30	3.03	0.43
4959	Singapore	1987	2789	1.30	3.03	0.43
4960	Qatar	1986	2572	1.30	2.32	0.56
4961	Qatar	1984	3617	1.28	2.27	0.56
4962	Bahamas	1984	3038	1.25	2.15	0.58

(a) Greatest and lowest demographic force multipliers.

	Country	Year	MID#	Economic	GDP $\beta = -0.01$	Energy $\beta = 0.05$	Iron and Steel $\beta = 0.01$	Imports $\gamma = 0.15$
1	China	1971	2099	1.56	0.88	1.89	1.11	0.84
2	China	1971	2947	1.56	0.88	1.89	1.11	0.84
3	United States	1964	611	1.55	0.87	2.00	1.13	0.79
4	United States	1964	1213	1.55	0.87	2.00	1.13	0.79
5	United States	1964	2220	1.55	0.87	2.00	1.13	0.79
4958	Panama	1937	2306	0.53	0.94	1.00	1.00	0.57
4959	Jordan	1948	1793	0.53	0.93	1.00	1.00	0.56
4960	Jordan	1948	3229	0.53	0.93	1.00	1.00	0.56
4961	Jordan	1950	1006	0.53	0.94	1.00	1.00	0.56
4962	Jordan	1949	3161	0.51	0.93	1.00	1.00	0.55

(b) Greatest and lowest economic force multipliers.

	Country	Year	MID#	Geopolitical	Distance $\beta = -0.02$	Nuclear $\beta = -0.25$
4958	United States	1975	356	0.659	0.843	0.782
4959	Russia	1979	2224	0.659	0.843	0.782
4960	United States	1968	3300	0.659	0.843	0.782
4961	Russia	1977	3122	0.659	0.842	0.782
4962	United States	2003	4460	0.658	0.841	0.782

(c) Lowest geopolitical force multipliers.

Table 5. Estimated force multiplier components by category for dispute participants in the analysis data.

increase m_i according to our estimates. GDP enters the equation negatively, but the effect is small in magnitude. This may reflect the limitations of GDP as a measure of economic development as it pertains to war (Beckley 2018). It may also simply indicate that the other proxies are capturing development more precisely—particularly early in our sample period, when GDP is often imputed due to missingness. Meanwhile, we find that import dependence increases the marginal cost of exerting military effort, thereby decreasing a country's effective force multiplier. In particular, each percentage point increase in Import Percentage is associated with a 0.15% increase in the marginal cost of effort. Altogether, we find the greatest economic multipliers for large economies with relatively low import dependence, such as the United States in the 1960s and China in the 1970s. Small, open economies like Panama and Jordan have the lowest economic multipliers. Notice that the dispute participants with the highest and lowest economic multipliers correspond most closely to those with the most extreme overall multipliers, again highlighting the importance of economic variables in the total equation.

The geopolitical multiplier is a function of a dispute participant's nuclear status and distance to the location of the dispute.[30] Perhaps surprisingly, we find that possession of nuclear weapons reduces a country's effective force multiplier: the military effectiveness of a country that possesses nuclear weapons is only about 78% that of an otherwise equivalent country without nuclear weapons. This provides at least suggestive evidence in favor of the idea that the possession of nuclear weapons inhibits the deployment of conventional force (Asal and Beardsley 2007). In the shadow of nuclear weapons, states may have extra incentives to limit their efforts so as to forestall devastating nuclear conflict (Schelling 1966). In line with usual expectations (Boulding 1962), we find that distance from a dispute erodes military effectiveness. But the effect is small, with a 1% increase in distance corresponding to just a 0.02% reduction in the effective force multiplier. Combining these findings, we find the lowest geopolitical force multipliers for nuclear powers involved in disputes far from their

[30] Table 5 does not display the greatest geopolitical force multipliers, as all 1,896 non-nuclear states with zero distance to dispute attain the maximum geopolitical multiplier of 1.0.

homeland—the United States and Russia during the Cold War, as well as the United States in the 2003 Iraq War.

The political component is omitted from Table 5, as it depends solely on a country's Polity IV score. We find that democracies have lower marginal costs of effort ($\gamma = -0.02$), and thus a greater effective force multiplier. Specifically, the marginal cost of effort for a fully autocratic country (Democracy $= -10$) is about 38% greater than that for an otherwise identical fully democratic country (Democracy $= 10$). This apparently runs contrary to theoretical expectations that political accountability makes war-fighting more costly for democratic leaders (Bueno de Mesquita et al. 1999; Jackson and Morelli 2007). However, recall that because we cannot separately identify the influence of the same variable on m_i and c_i, our estimates of γ represent the *net* effect of each variable on the overall cost-to-effectiveness ratio. Consequently, it is possible that political accountability increases the costs of effort, but that these costs are outweighed by democratic advantages in raw military effectiveness. (see Reiter and Stam 1998*a*,*b*, 2002).

5.2 Precision of Prior Beliefs

We now turn to the dispute-level influences on the distribution of prior beliefs, collected in the parameter α. Remember that an increase in α corresponds to a decrease in the variance of the prior, and therefore in the probability of war. Table 6 reports the full estimated model.

We first consider a geographical determinant of uncertainty, namely Contiguity. As with Distance to Dispute in the military effectiveness equation, our point estimate for Contiguity is in the expected direction—it increases uncertainty, and thus the chances of war—but is substantively small and statistically insignificant. On the whole, we see little evidence for geography as primary determinants of states' negotiating behavior or bargaining power.

By contrast, we again see strong effects of traditional measures of military capability. Uncertainty is greatest in disputes with a major power on exactly one side and lowest in disputes with major powers on both sides, with those lacking major powers somewhere in

Term	Estimate	Std. Err.	Z	p
State-Level: Demographic				
m_i: Total Population	$0.141^{\dagger}$	0.076	1.865	0.062
m_i: Urban Population	$-0.108^{\dagger}$	0.056	-1.925	0.054
State-Level: Economic				
m_i: Energy Consumption	0.048	0.040	1.217	0.224
m_i: GDP	-0.009	0.083	-0.110	0.912
m_i: Iron and Steel Production	0.011	0.033	0.324	0.746
c_i: Import Percentage	0.148	0.162	0.913	0.361
State-Level: Political				
c_i: Democracy	-0.016^{*}	0.008	-1.985	0.047
State-Level: Geopolitical				
m_i: Distance to Dispute	-0.019	0.021	-0.887	0.375
m_i: Nuclear Weapons	-0.246	0.177	-1.392	0.164
Dispute-Level				
λ: Contiguity	-0.062	0.131	-0.473	0.636
λ: Democracy–Higher	0.020^{*}	0.010	2.077	0.038
λ: Democracy–Lower	0.002	0.010	0.193	0.847
λ: Interest Similarity	-0.168	0.131	-1.287	0.198
λ: Major Power–Both	0.550^{*}	0.269	2.042	0.041
λ: Major Power–Either	-0.264^{*}	0.132	-1.995	0.046
λ: Number of Participants	-6.246^{*}	2.471	-2.528	0.011
λ: Peace Years	0.150^{*}	0.028	5.273	<0.001
λ: Intercept	4.288	1.737	2.469	0.014
Disputes	2101			
Participants	4962			
Log-likelihood	-859.830			
AIC	1755			
BIC	1857			

Table 6. Structural estimation results. †: $p < 0.1$. *: $p < 0.05$.

between. Both differences are statistically significant. Compared to a baseline of no major power involvement, introducing a major power to one side increases the variance of the prior beliefs by about 70%. Compared to that same baseline, however, introducing major powers to both sides reduces the variance of prior beliefs by about 44%. This may reflect a reduction in uncertainty arising from the frequency of interaction among major power states.

We also observe some evidence for political determinants of uncertainty in crisis bargaining. Surprisingly, we find greater variance in prior beliefs in disputes between states with similar alliance portfolios, though our estimate is not statistically significant. To interpret this finding, remember that our sample consists of international disputes, and that we are modeling the escalation to war given that crisis bargaining has begun. It might be that states with similar preferences are less likely to have a dispute in the first place—our structural model is agnostic as to the origins of the crisis bargaining situation itself. In fact, if states with similar preferences end up in crises relatively infrequently, this unfamiliarity could be a source of the uncertainty we identify in the rare cases where they do. On the other hand, more in line with conventional expectations (e.g., Schultz 2001), we find that interactions among democracies are characterized by less uncertainty.

We identify two final determinants of uncertainty in international disputes. First, the coefficient on Peace Years is positive and statistically significant, indicating greater uncertainty between countries that have recently clashed. Second, the coefficient on Number of Participants is negative and statistically significant, meaning there is more uncertainty in disputes involving more than one state on each side. This is consistent with what we observe descriptively, with war occurring in 32% of coalitional disputes, compared to just 9% of those involving only two states. Our structural model thus directly recovers what many previous studies of coalitions and conflict have posited: the participation of more countries in a crisis raises the degree of uncertainty and thus the probability of war (Wolford 2014*a*; Bas and Schub 2016; Vasquez and Rundlett 2016; Smith forthcoming).

6 Counterfactual Analyses

One of the primary virtues of the structural estimation approach is that it allows us to calculate counterfactual quantities of substantive interest. We are particularly interested in counterfactuals involving military coalitions, such as how much difference a country made to its side's military fortunes, or how much more effort its partners would have exerted had it not been involved. Additionally, for dyads as well as coalitions, we can estimate optimal bargaining behavior even for countries that did not experience a dispute with each other, based on their observable characteristics.

We use the following procedure to calculate counterfactual estimates:

1. Given two (perhaps hypothetical) coalitions of countries at a particular time, use the estimates from Table 6 to calculate the military effectiveness of each participant (m_i), the marginal cost of effort of each participant (c_i), and the degree of prior uncertainty in the dispute (λ).

2. Use Proposition 1 to calculate each individual country's effort and each side's probability of winning in case of war, according to the equilibrium of the war subgame.

3. Use Proposition 2 to calculate the equilibrium offer and the probability of war.

6.1 Contributions to Coalitions

Perhaps the most novel feature of our estimation approach is that it allows us to estimate the contributions of individual countries or groups of countries to the outcomes of coalitional conflicts. To illustrate this capability, in this section we analyze counterfactual outcomes as a function of the set of participants in a few prominent historical conflicts.

We begin by analyzing the contributions of three participants on the Allied side of World War I—Russia, the United Kingdom, and the United States.[31] Specifically, we model each

[31]Given our assumption that informational problems cause war, one might worry about the validity of these estimates if the war were instead caused by commitment problems. However, the bulk of the identifying

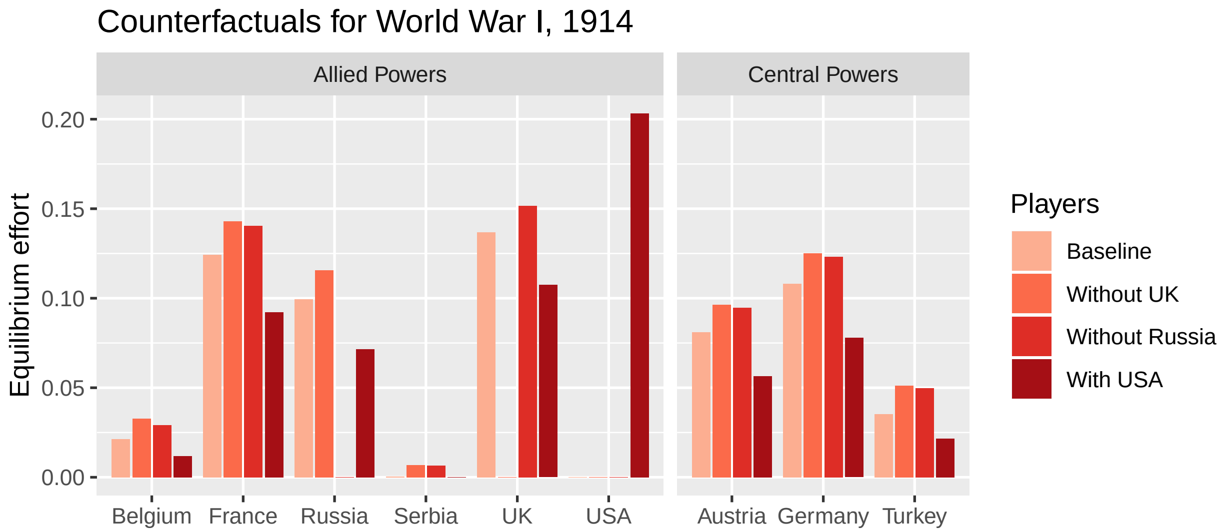

Figure 3. Equilibrium choices of effort in World War I, 1914, as a function of Allied Power participants. Estimates are from Model 1.

country's equilibrium effort, the probability of Allied victory, and the *ex ante* probability of war as of 1914. We vary the set of participants on the Allied side (adding the United States, or subtracting Russia or the United Kingdom) while holding fixed participation among the Central Powers. Equilibrium choices of war effort under each scenario are plotted in Figure 3, while Table 7 reports the dispute-level equilibrium quantities.

Counterfactual	Pr(Allied Powers Win)
Baseline	0.62
Without Russia	0.52
Without UK	0.51
With USA	0.76

Table 7. Counterfactual equilibrium quantities in World War I, 1914. Estimates are from Model 1.

Our results suggest that Russia and the United Kingdom both made meaningful contributions to the war effort, but that these contributions were at least somewhat offset by free-riding. In the baseline equilibrium, Russia contributes 31% of the Allied Powers' effective

power for our force multiplier estimates—which drive these counterfactuals—comes from the contest subgame after bargaining has broken down.

effort (effort weighted by military effectiveness, $m_i e_i$), and the United Kingdom contributes about 34%. Yet most of each country's contribution would have been made up by other Allied Powers had it not participated in the conflict, meaning their observed efforts far outweigh their effects. Meanwhile, according to our estimates, American entry at the outset of the dispute would have made an even bigger difference than British entry did. We estimate that the Allied Powers' chance of winning would increased 14 percentage points to 76% had the United States participated in 1914, with the United States contributing almost half of this side's effective efforts. This is because the United States has a much larger effective force multiplier than the UK, with most of the difference owing to the United States' larger population and lower import dependence.

Still in the context of World War I, we can also evaluate the fortunes of the Central Powers as of 1915 if the "Schlieffen Plan" had succeeded.[32] In particular, we can calculate equilibrium military outcomes as of 1915 if France, Belgium, or both had been knocked out of the war in the prior year, while holding fixed the rest of the participants on each side. Would the French and Belgian military efforts just have been made up by Russia and the United Kingdom?

Counterfactual	Pr(Allied Powers Win)
Baseline	0.61
Without Belgium	0.60
Without France	0.52
Without both	0.50

Table 8. Counterfactual equilibrium quantities in World War I, 1915, if the "Schlieffen Plan" had succeeded. Estimates are from Model 1.

Table 8 presents the results of the counterfactual analysis of the "Schlieffen Plan" scenario. Our model suggests that the success of the plan would indeed have made a substantial difference in the likely war outcome. In the baseline scenario, our model projects a 61% chance of victory by the Allied Powers as of 1915. Knocking out Belgium alone would have

[32]See Zuber (1999) for some historical skepticism of the mythology around the Schlieffen Plan.

made a small difference of about 1 percentage point, while eliminating France would have reduced the Allied Powers' chances of victory by 9 percentage points. The effect of knocking out both would be 11 percentage points, greater than the combined effect of eliminating either alone—this is because Belgium would have made up some of the lost effort had only France been eliminated, and vice versa. Overall, the success of the Schlieffen Plan would have changed the likely outcome as of 1915 from favoring the Allied Powers to being a complete toss-up.

We analyze similar counterfactuals for World War II. First, we consider the European theater. We analyze equilibrium war efforts and the probability of victory if the Soviet Union, the United States, or both had contributed to the Allies from the war's outset in 1939. Figure 4 plots the predicted contributions, and Table 9 reports the dispute-level equilibrium quantities for each scenario.

Counterfactual	Pr(Allies Win)
Baseline	0.80
With USSR	0.86
With USA	0.94
With both	0.96

Table 9. Counterfactual equilibrium quantities in the European theater of World War II, 1939. Estimates are from Model 1.

According to our model, the Soviet contribution to the Allies in 1939 would have had a moderate effect. We estimate that Soviet involvement would have raised the Allied probability of victory by 6 percentage points, from 80% to 86%. On the other hand, we find that the United States would have made a substantial difference by joining the war effort two years earlier. Its involvement would have raised the Allies' probability of victory 14 percentage points, to 94%. Interestingly, conditional on the United States joining, the participation of the Soviet Union would have been almost irrelevant, making only a 2 percentage point difference in the Allies' chances of winning. In other words, American participation would have created a strong incentive for the Soviet Union to free-ride—as well as the other pow-

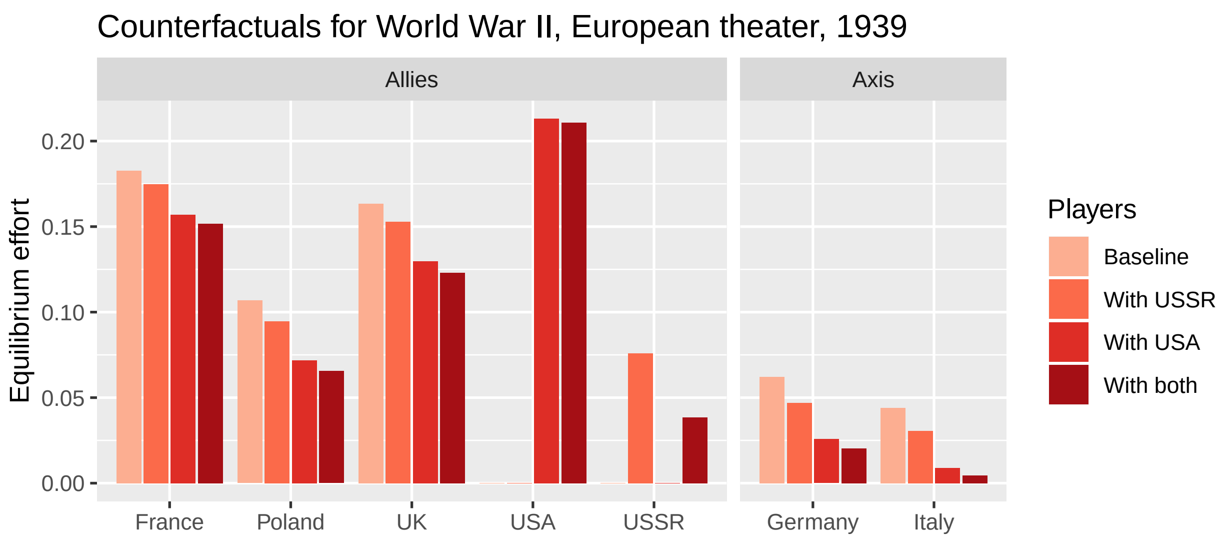

Figure 4. Equilibrium choices of effort in the European theater of World War II, 1939, as a function of Allied participants. Estimates are from Model 1.

ers, as seen in Figure 4. As before, the United States has a larger effective force multiplier than its prospective allies. It compares favorably to the UK and France in terms of total population and import dependence, while its advantage over the Soviet Union is largely due to its democratic political system.

Turning to the Pacific theater, we find that the Allies might have been best off without the help of the Soviet Union as of 1941. As shown in Table 10, Soviet involvement would not have made a discernible difference in the Allies' chances of victory. We predict that the Soviet Union would have contributed relatively little effort and that even this small contribution would have been largely offset by decreases in British and Chinese contributions.

Counterfactual	Pr(Allies Win)
Baseline	0.69
With USSR	0.69

Table 10. Counterfactual equilibrium quantities in the Pacific theater of World War II, 1941. Estimates are from Model 1.

6.2 Crisis Bargaining Behavior

The structural model also allows us to identify equilibrium bargaining behavior—optimal offers and the probability of war—as a function of countries' observable characteristics. As long as the variables included in our structural model are observed, we may perform this counterfactual exercise even for countries or coalitions that did not experience a dispute. For the sake of simplicity, here we will focus on optimal bargaining behavior in purely dyadic relationships.

Figure 5 plots the results of this type of counterfactual exercise for the United States and Russia throughout our sample period. In each year, even those in which there was not actually a United States–Russia MID, we plug in both countries' covariates to estimate equilibrium bargaining behavior in case they were to experience a dispute. For each year we plot the United States' baseline optimal offer (greater values indicate more demands by the United States),[33] the *ex ante* probability of war, and the United States' probability of victory in case of war.

Two patterns are evident in the results of the counterfactual bargaining analysis. First, we see a clear historical trend in the optimal demand for the United States. The optimal American bargaining position became more accommodating in the century prior to World War I, spiked during the Russian Revolution, then grew again in the Cold War period before declining afterward. Besides 1919–1921, the high point for American aggressiveness occurs in the 1980s. Second, the increasing riskiness of the United States' bargaining posture during the Cold War period is due more to greater uncertainty than to greater American power. In fact, the United States' probability of winning declines throughout and after the Cold War period.

We also conduct a more formal evaluation of the face validity of our counterfactual equilibrium predictions. We begin by observing that the optimal demand, x^*, should be

[33]Per Proposition 2, the optimal offer varies with the country's type, which is unobserved. In Figure 5, as well as in the broader counterfactual exercise described below, we obtain the optimal offer for the lowest-cost type, $x^*(0)$, as a baseline.

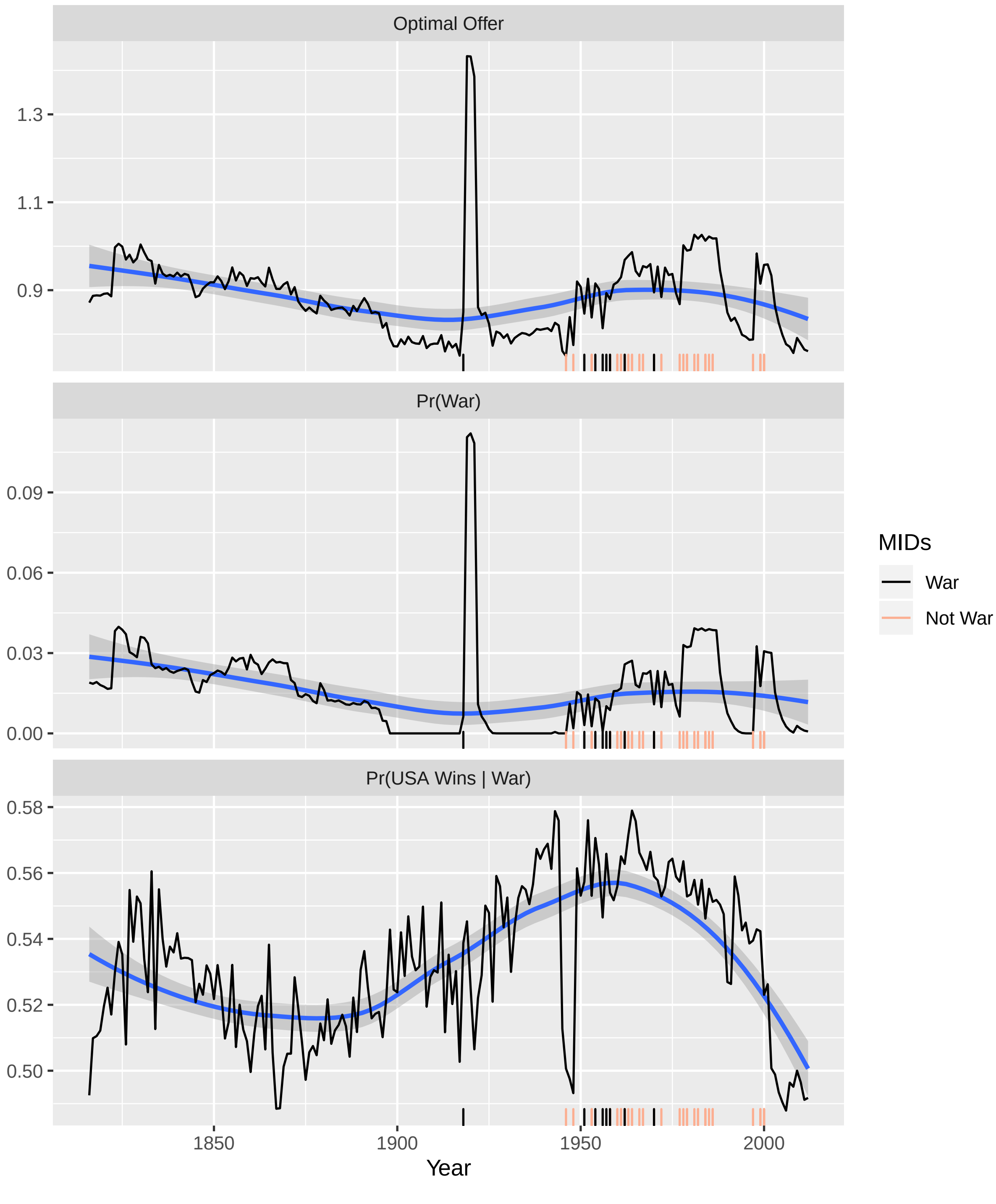

Figure 5. Counterfactual analysis of a hypothetical United States–Russia dispute, 1816–2010. Estimates are from Model 1. The rug plot at the bottom of each panel indicates years in which a MID occurred between them.

related to a country's latent satisfaction with the dyadic status quo. The more a country would demand at the bargaining table, the less likely it is that the prevailing status quo remains acceptable to that country (Powell 1996). Therefore, all else equal, we should expect crisis bargaining to ensue when a country's latent demand grows stronger.

To evaluate this claim empirically, we examine the relationship between optimal demands and the initiation of MIDs. As in our structural model, we take the outbreak of a MID as evidence that crisis bargaining is occurring; the difference here is that we are now analyzing selection into a crisis, whereas our structural estimator models behavior once in a crisis. For each directed dyad–year in the international system during our sample period (1816–2010), we calculate the baseline optimal offer x^* using the structural estimates in Model 1. Using a linear probability model, we regress MID initiation in each directed dyad–year on the optimal offer. If greater demands are indeed associated with greater dissatisfaction with the status quo, as we would expect if our structural model is picking up important components of states' utility functions, then we would expect a positive relationship in this regression.[34] We employ random and fixed effects specifications so as to capture only variation within each directed dyad, and we cluster all standard errors at the directed dyad level.

Table 11 displays the results from regressing MID initiation on the estimated optimal offer. Across all three specifications, we see a statistically significant and positive relationship between the optimal offer and the likelihood of a dispute, as expected. The fixed effects estimates are the most reliable, as we reject the null hypothesis that the random effects model is consistent via a Hausman test ($\chi^2 = 489$, $p < 0.001$).

These results also give us some confidence in our model's ability to extrapolate to state-years not used in estimation. If selection bias were very high, then we would expect our estimates of x^* to be rife with measurement error, making it difficult to identify a significant correlation with dispute onset. Table 11 shows that this is not the case.

[34] The probability of war may also depend on the location of the status quo (Reed et al. 2008), which we do not measure. Even so, we ought to expect a positive correlation with the latent optimal offer, particularly when making within-dyad comparisons.

	1	2	3
Optimal offer x^*	0.0101^* (0.00069)	0.0056^* (0.00043)	0.0035^* (0.00043)
Intercept	-0.0086 (0.00065)	-0.0046 (0.00041)	
Random effects	N	Y	N
Fixed effects	N	N	Y
Observations	1685370	1685370	1685370
Clusters	40864	40864	40864
R^2	0.002	0.055	0.073

Table 11. Regression of MID initiation on the estimated optimal offer, x^*, in directed dyad–years, 1816–2010. x^* is estimated from Model 1. Standard errors are clustered at the directed dyad level. *: $p < 0.05$.

7 Conclusion

We have developed a new model of crisis bargaining between coalitions and structurally estimated its parameters using data on interstate disputes. The model allows us to estimate the determinants of individual states' effective force multipliers and of the precision of prior beliefs in crises. We find that states' force multipliers are primarily functions of demographic and economic characteristics, with political and geopolitical features playing a less important role. We have also shown how the model allows us to estimate optimal contributions by individual states and the extent of free-riding in actual and counterfactual conflicts.

An important lesson of both our formal model and our empirical results is that we cannot simply look to how much a country contributed to a war effort to determine the importance of its contribution to the outcome. The key counterfactual question is how much of the country's contribution would have been made up by its allies if that country had not been involved. Our theory-driven approach allows us to develop at least provisional answers to this difficult counterfactual question.

On a broader methodological note, our approach and findings illustrate the utility of the structural estimation of game-theoretical models. Debates about these techniques in political science have largely revolved around whether structural estimation is necessary to test comparative statics predictions without bias (Carrubba, Yuen and Zorn 2007; Signorino 2007). But we see the marriage of formal theory and statistical estimation as more than just a tool for bias reduction. For one thing, structuring our statistical analysis around a formal model led us to develop a theory-driven alternative to problematic dyadic research designs (Cranmer and Desmarais 2016). More importantly, it directed our attention toward latent variables (the optimal offer in crisis bargaining) and counterfactual questions (changes in contributions with different coalition structures) that would have been much harder to uncover in a traditional analysis. We encourage political scientists to think of structural estimation as a tool to expand the range of substantive questions that we can answer with data, not merely as a methodological fix for particular statistical problems.

References

Abramson, Scott F and Sergio Montero. forthcoming. "Learning about Growth and Democracy." *American Political Science Review* .

Asal, Victor and Kyle Beardsley. 2007. "Proliferation and International Crisis Behavior." *Journal of Peace Research* 44(2):139–155.

Ascencio, Sergio J. and Miguel R. Rueda. 2019. "Partisan Poll Watchers and Electoral Manipulation." *American Political Science Review* 113(3):727–742.

Bas, Muhammet and Robert Schub. 2016. "Mutual Optimism as a Cause of Conflict: Secret Alliances and Conflict Onset." *International Studies Quarterly* 60(3):552–564.

Bearce, David H., Kristen M. Flanagan and Katharine M. Floros. 2006. "Alliances, Internal Information, and Military Conflict among Member-States." *International Organization* 60(3):595–625.

Beck, Nathaniel, Jonathan N. Katz and Richard Tucker. 1998. "Taking Time Seriously:

Time-Series-Cross-Section Analysis with a Binary Dependent Variable." *American Journal of Political Science* 42:1260–1288.

Beckley, Michael. 2010. "Economic Development and Military Effectiveness." *The Journal of Strategic Studies* 33(1):43–79.

Beckley, Michael. 2018. "The Power of Nations: Measuring What Matters." *International Security* 43(2):7–44.

Bennett, D Scott and Allan C Stam. 1996. "The Duration of Interstate Wars, 1816–1985." *American Political Science Review* 90(2):239–257.

Bennett, D. Scott and Allan C. Stam. 2000. "EUGene: A Conceptual Manual." *International Interactions* 26(2):179–204.

Bolt, Jutta, Robert Inklaar, Herman de Jong and Jan Luiten. 2018. "Rebasing 'Maddison': New Income Comparisons and the Shape of Long-Run Economic Development.". Maddison Project Working Paper 10.
URL: https://www.rug.nl/ggdc/html_publications/memorandum/gd174.pdf

Boulding, Kenneth. 1962. *Conflict and Defense: A General Theory.* Harper and Row.

Bradley, Ralph Allan and Milton E Terry. 1952. "Rank Analysis of Incomplete Block Designs: I. The Method of Paired Comparisons." *Biometrika* 39(3/4):324.

Bremer, Stuart A. 1992. "Dangerous Dyads: Conditions Affecting the Likelihood of Interstate War, 1816-1965." *Journal of Conflict Resolution* 36(2):309–341.

Bueno de Mesquita, Bruce, James D. Morrow, Randolph M. Siverson and Alastair Smith. 1999. "An Institutional Explanation of the Democratic Peace." *American Political Science Review* 93(4):791–807.

Carrubba, Clifford J., Amy Yuen and Christopher Zorn. 2007. "In Defense of Comparative Statics: Specifying Empirical Tests of Models of Strategic Interaction." *Political Analysis* pp. 465–482.

Carter, David B. 2010. "The Strategy of Territorial Conflict." *American Journal of Political Science* 54(4):969–987.

Carter, David B. and Curtis S. Signorino. 2010. "Back to the Future: Modeling Time Dependence in Binary Data." *Political Analysis* 18(3):271–292.

Chiba, Daina and Jesse C. Johnson. 2019. "Military Coalitions and Crisis Duration." *The Journal of Politics* 81(4):1466–1479.

Chiba, Daina, Jesse C. Johnson and Brett Ashley Leeds. 2015. "Careful Commitments: Democratic States and Alliance Design." *Journal of Politics* 77(4):968–982.

Clark, David H. and William Reed. 2003. "A Unified Model of War Onset and Outcome." *Journal of Politics* 65(1):69–91.

Correlates of War Project. 2016. "State System Membership List, v2016." Online dataset. **URL:** http://correlatesofwar.org

Cranmer, Skyler J. and Bruce A. Desmarais. 2016. "A Critique of Dyadic Design." *International Studies Quarterly* 60(2):355–362.

Crisman-Cox, Casey and Michael Gibilisco. 2018. "Audience Costs and the Dynamics of War and Peace." *American Journal of Political Science* 62(3):566–580.

Dorff, Cassy and Michael D. Ward. 2013. "Networks, Dyads, and the Social Relations Model." *Political Science Research and Methods* 1(2):159.

Fang, Songying, Jesse C. Johnson and Brett Ashley Leeds. 2014. "To Concede or to Resist? The Restraining Effect of Military Alliances." *International Organization* 68(4):775–809.

Fearon, James D. 1995. "Rationalist Explanations for War." *International Organization* 49(3):379–414.

Feenstra, Robert C., Robert Inklaar and Marcel P. Timmer. 2015. "The Next Generation of the Penn World Table." *The American Economic Review* 105(10):3150–82.

Fey, Mark. 2008. "Rent-seeking contests with incomplete information." *Public Choice* 135(3-4):225–236.

Garfinkel, Michelle R. 2004*a*. "On the Stability of Group Formation: Managing the Conflict Within." *Conflict Management and Peace Science* 21(1):43–68.

Garfinkel, Michelle R. 2004*b*. "Stable Alliance Formation in Distributional Conflict." *European Journal of Political Economy* 20(4):829–852.

Gartzke, Erik A and Paul Poast. 2017. Empirically Assessing the Bargaining Theory of War: Potential and Challenges. In *Oxford Research Encyclopedia of Politics.*

Gartzke, Erik and Matthew Kroenig. 2009. "A Strategic Approach to Nuclear Proliferation." *Journal of Conflict Resolution* 53(2):151–160.

Gibler, Douglas M., Steven V. Miller and Erin K. Little. 2016. "An Analysis of the Militarized Interstate Dispute (MID) Dataset, 1816–2001." *International Studies Quarterly* 60(4):719–730.

Grant, Keith A. 2013. "Outsourcing Security: Alliance Portfolio Size, Capability, and Reliability." *International Studies Quarterly* 57(2):418–429.

Honaker, James and Gary King. 2010. "What to Do about Missing Values in Time-Series Cross-Section Data." *American Journal of Political Science* 54(2):561–581.

Jackson, Matthew O. and Massimo Morelli. 2007. "Political Bias and War." *American Economic Review* 97(4):1353–1373.

Kang, Karam. 2016. "Policy Influence and Private Returns from Lobbying in the Energy Sector." *The Review of Economic Studies* 83(1):269–305.

Krainin, Colin. 2014. "Multilateral War Bargaining." *Quarterly Journal of Political Science* 9(4):407–439.

Krainin, Colin and Thomas Wiseman. 2016. "War and Stability in Dynamic International Systems." *Journal of Politics* 78(4):1139–1152.

Kroenig, Matthew. 2013. "Nuclear Superiority and the Balance of Resolve: Explaining Nuclear Crisis Outcomes." *International Organization* 67(1):141–171.

Levy, Jack. 2008. Counterfactuals and Case Studies. In *The Oxford Handbook of Political Methodology*. Oxford University Press.

Marshall, Monty G, Ted Robert Gurr and Keith Jaggers. 2014. Polity IV Project: Political Regime Characteristics and Transitions, 1800–2013. Technical report.

Morey, Daniel S. 2016. "Military Coalitions and the Outcome of Interstate Wars." *Foreign Policy Analysis* 12(4):533–551.

Olson, Mancur and Richard Zeckhauser. 1966. "An Economic Theory of Alliances." *The Review of Economics and Statistics* pp. 266–279.

Poast, Paul. 2010. "(Mis)Using Dyadic Data to Analyze Multilateral Events." *Political Analysis* 18(4):403–425.

Posen, Barry R. 1993. "Nationalism, the Mass Army, and Military Power." *International Security* 18(2):80–124.

Powell, Robert. 1996. "Stability and the Distribution of Power." *World Politics* 48(2):239–267.

Powell, Robert. 1999. *In the Shadow of Power*. Princeton University Press.

Ramsay, Kristopher W. and Curtis S. Signorino. 2009. "A Statistical Model of the Ultimatum Game.". Typescript, Princeton University.
URL: https://scholar.princeton.edu/sites/default/files/logitbarg_0.pdf

Reed, William. 2000. "A Unified Statistical Model of Conflict Onset and Escalation." *American Journal of Political Science* pp. 84–93.

Reed, William, David H. Clark, Timothy Nordstrom and Wonjae Hwang. 2008. "War, Power, and Bargaining." *Journal of Politics* 70(4):1203–1216.

Reiter, Dan. 2003. "Exploring the Bargaining Model of War." 1(1).
URL: http://dx.doi.org/10.1017/S1537592703000033

Reiter, Dan and Allan C. Stam. 1998*a*. "Democracy and Battlefield Military Effectiveness." *Journal of Conflict Resolution* 42(3):259–277.

Reiter, Dan and Allan C. Stam. 1998*b*. "Democracy, War Initiation, and Victory." *The American Political Science Review* 92(2):377.

Reiter, Dan and Allan C. Stam. 2002. *Democracies at War*. Princeton, N.J.: Princeton University Press.

Renshon, Jonathan and Arthur Spirling. 2015. "Modeling 'Effectiveness' in International Relations." *Journal of Conflict Resolution* 59(2):207–238.

Russett, Bruce M and John R Oneal. 2001. *Triangulating peace: democracy, interdependence, and international organizations*. New York: Norton.

Schelling, Thomas C. 1966. *Arms and Influence*. Yale University Press.

Schultz, Kenneth A. 2001. *Democracy and Coercive Diplomacy*. Cambridge University Press.

Signorino, Curtis S. 2007. "On Formal Theory and Statistical Methods: A Response to Carrubba, Yuen, and Zorn." *Political Analysis* pp. 483–501.

Signorino, Curtis S and Jeffrey M Ritter. 1999. "Tau-b or not tau-b: measuring the similarity of foreign policy positions." *International Studies Quarterly* 43(1):115–144.

Singer, David J, Stuart A Bremer and John Stuckey. 1972. Capability Distribution, Uncertainty, and Major Power War, 1820–1965. In *Peace, War, and Numbers*, ed. Bruce M Russett. Beverly Hills: pp. 19–48.

Skaperdas, Stergios. 1998. "On the Formation of Alliances in Conflict and Contests." *Public Choice* 96(1-2):25–42.

Smith, Bradley C. forthcoming. "Military Coalitions and the Politics of Information." *Journal of Politics* .

Stein, William E. 2002. "Asymmetric Rent-Seeking with More than Two Contestants." *Public Choice* 113(3/4):325–336.

Stiglitz, Joseph E. and Linda J. Blimes. 2008. *The Three Trillion Dollar War*. Norton.

Stinnett, Douglas M., Jaroslav Tir, Philip Schafer, Paul F. Diehl and Charles Gochman. 2002. "The Correlates of War Project Direct Contiguity Data, Version 3." *Conflict Management and Peace Science* 19:58–66.

Sullivan, Patricia L. 2007. "War Aims and War Outcomes: Why Powerful States Lose Limited Wars." *Journal of Conflict Resolution* 51(3):496–524.

The World Bank. 2019. "World Development Indicators." Online dataset.
URL: https://datacatalog.worldbank.org/dataset/world-development-indicators

Trager, Robert F. 2015. "Diplomatic Signaling among Multiple States." *The Journal of Politics* 77(3):635–647.

Vasquez, John A. and Ashlea Rundlett. 2016. "Alliances as a Necessary Condition of Multiparty Wars." *Journal of Conflict Resolution* 60(8):1395–1418.

Werner, Suzanne. 2000. "The effects of political similarity on the onset of militarized disputes, 1816-1985." *Political Research Quarterly* 53(2):343–374.

Whang, Taehee, Elena V. McLean and Douglas W. Kuberski. 2013. "Coercion, Information, and the Success of Sanction Threats." *American Journal of Political Science* 57(1):65–81.

Wolford, Scott. 2014*a*. "Power, Preferences, and Balancing: The Durability of Coalitions and the Expansion of Conflict." *International Studies Quarterly* 58(1):146–157.

Wolford, Scott. 2014*b*. "Showing Restraint, Signaling Resolve: Coalitions, Cooperation, and Crisis Bargaining." *American Journal of Political Science* 58(1):144–156.

Wolford, Scott. 2017. "The Problem of Shared Victory: War-Winning Coalitions and Postwar Peace." *The Journal of Politics* 79(2):702–716.

Zuber, Terence. 1999. "The Schlieffen Plan Reconsidered." *War in History* 6(3):262–305.

A Appendix

A.1 Additional Formal Results

In our first additional result, we derive the equilibrium shares of the prize and expected utilities in the war subgame.

Proposition 3. *In the unique equilibrium of the war subgame, country i's expected share of the prize is*

$$p_i^* = \begin{cases} 1 - q_i & i \leq J, \\ 0 & i > J, \end{cases}$$

and its expected utility is $\pi_i^* = \pi_i(e^*) = (p_i^*)^2$.

Proof. For $i > J$, we have $e_i^* = 0$ and therefore $p_i^* = \pi_i^* = 0$, as claimed. For $i \leq J$, we have

$$p_i^* = \frac{m_i e_i^*}{\sum_{j=1}^{I} m_j e_j^*} = 1 - q_i$$

and

$$\pi_i^* = p_i^* - c_i e_i^* = (1 - q_i)^2 = (p_i^*)^2,$$

as claimed. □

In our second additional result, we derive expressions for the optimal offer and the equilibrium probability of war under the assumption that both sides' cost of bargaining breakdown have an exponential distribution.

Proposition 4. *If F_A and F_B are exponential with parameter λ,*

(a) the optimal offer is

$$x^*(\theta_A) = \max\left\{1 - \pi_B^*, w_A^*(\theta_A) + \frac{1}{\lambda}\right\}; \tag{3}$$

(b) the ex ante *probability of war is*

$$\Pr(x^*(\theta_A) > 1 - w_B^*(\theta_B)) = \begin{cases} 0 & \mu^* \leq 0, \\ 1 - \exp(-\lambda\mu^*) - \lambda\mu^* \exp(-\lambda\mu^*) & \mu^* > 0, \end{cases} \tag{4}$$

which is nonincreasing in λ*.*

Proof. By hypothesis, $f_B(0) = \lambda$ and thus $\mu^* = \frac{1}{\lambda} - 1 + \pi_A^* + \pi_B^*$. If $\theta_A \leq \mu^*$, then the proposed offer solves (3), as required:

$$\begin{aligned} & w_A^*(\theta_A) + \frac{1 - F_B(x^*(\theta_A) - 1 + \pi_B^*)}{f_B(x^*(\theta_A) - 1 + \pi_B^*)} \\ &= w_A^*(\theta_A) + \frac{\exp(-\lambda(x^*(\theta_A) - 1 + \pi_B^*))}{\lambda \exp(-\lambda(x^*(\theta_A) - 1 + \pi_B^*))} \\ &= w_A^*(\theta_A) + \frac{1}{\lambda} \\ &= x^*(\theta_A). \end{aligned}$$

If $\theta_A > \mu^*$, then $w_A^*(\theta_A) + \frac{1}{\lambda} < 1 - \pi_B^*$ and thus $x^*(\theta_A) = 1 - \pi_B^*$, as required. This proves statement (a). The *ex ante* probability of war is

$$\begin{aligned} & \Pr(x^*(\theta_A) > 1 - w_B^*(\theta_B)) \\ &= \int F_B(x^*(\theta_A) - 1 + \pi_B^*)\, dF_A(\theta_A) \\ &= \int_0^{\max\{\mu^*, 0\}} F_B(\mu^* - \theta_A) f_A(\theta_A)\, d\theta_A \\ &= \int_0^{\max\{\mu^*, 0\}} (1 - \exp(-\lambda(\mu^* - \theta_A)))\lambda \exp(-\lambda\theta_A)\, d\theta_A \\ &= \begin{cases} 0 & \mu^* \leq 0, \\ 1 - \exp(-\lambda\mu^*) - \lambda\mu^* \exp(-\lambda\mu^*) & \mu^* > 0, \end{cases} \end{aligned}$$

as stated. When $\mu^* > 0$, the probability of war as a function of λ is

$$r(\lambda) = 1 - (2 - \lambda(1 - \pi_A^* - \pi_B^*)) \exp(\lambda(1 - \pi_A^* - \pi_B^*) - 1),$$

the derivative of which is

$$\frac{\partial r(\lambda)}{\partial \lambda} = \underbrace{(1 - \pi_A^* - \pi_B^*)}_{\geq 0} \underbrace{(\lambda(1 - \pi_A^* - \pi_B^*) - 1)}_{= -\lambda\mu^* < 0} \underbrace{\exp(\lambda(1 - \pi_A^* - \pi_B^*) - 1)}_{> 0} \leq 0,$$

which concludes the proof of statement (b). □

In our third additional result, we verify that there is always a positive-measure set of parameters at which the log-likelihood is finite.

Proposition 5. *The log-likelihood is finite in a neighborhood of* $(\alpha, \beta, \gamma) = (0, 0, 0)$.

Proof. Consider the n'th crisis. By Proposition 3, $p_{ni}^*(0,0) = 1/I_n$ for each country i. Consequently, $p_{nA}^*(0,0) = I_{nA}/I_n$, so each side has a positive probability of winning in case of war. Additionally, $\lambda_n(0) = 1$ implies $\mu_n^*(0,0,0) > 0$, so there is a positive probability of war. The chance of peace is always positive. Therefore, $\log \ell(0,0,0) > -\infty$. Since the equilibrium is continuous in the parameters at any point where all players exert positive effort in equilibrium, this holds in a neighborhood of zero. □

A.2 Fixed Effects

As noted in the text, we estimate a model that includes a dummy variable for each individual country in the military effectiveness equation, m_i, alongside the time-varying covariates noted in the text.[35] Estimates are reported in Table 12.

[35] The fixed effects for a few states are not identified because these states always appear together on the same side of the disputes they are in. Consequently, we estimate a common fixed effect for these groups of states: (1) Slovakia and Slovenia; (2) Antigua & Barbuda, Barbados, Dominica, Jamaica, St. Lucia, and St. Vincent and the Grenadines.

Term	Estimate	Std. Err.	Z	p
State-Level: Demographic				
m_i: Total Population	1.485	2.392	0.621	0.535
m_i: Urban Population	−0.558	0.819	−0.681	0.496
State-Level: Economic				
m_i: Energy Consumption	−0.106	1.144	−0.092	0.926
m_i: GDP	−0.069	1.075	−0.064	0.949
m_i: Iron and Steel Production	−0.155	0.204	−0.760	0.447
c_i: Import Percentage	−0.049	1.186	−0.042	0.967
State-Level: Political				
c_i: Democracy	0.013	0.058	0.220	0.826
State-Level: Geopolitical				
m_i: Distance to Dispute	−0.059	0.146	−0.407	0.684
m_i: Nuclear Weapons	−0.471	2.128	−0.221	0.825
Dispute-Level				
λ: Contiguity	−0.747	1.664	−0.449	0.654
λ: Democracy–Higher	0.041	0.059	0.699	0.485
λ: Democracy–Lower	0.131	0.193	0.680	0.496
λ: Interest Similarity	−1.221	1.858	−0.657	0.511
λ: Major Power–Both	−1.276	3.037	−0.420	0.674
λ: Major Power–Either	−1.519	1.730	−0.878	0.380
λ: Number of Participants	−12.130	12.663	−0.958	0.338
λ: Peace Years	−0.215	0.861	−0.249	0.803
λ: Intercept	2.362	3.554	0.665	0.506
Disputes	2101			
Participants	4962			
Log-likelihood	−834.312			
AIC	2074			
BIC	3221			

Table 12. Structural estimation results for the model with fixed effects. Estimates of 208 country fixed effects omitted.

This model nests the baseline model, so it has a greater log-likelihood. However, this appears to come at the cost of substantial overfitting in the fixed effects specification. By estimating a separate base force multiplier for each country in the same, we use up an additional 208 degrees of freedom. The baseline model has a substantially lower AIC and BIC than the fixed effects model, indicating that the improvement in fit does not make up for the additional degrees of freedom used. This is why we report only the baseline model in the main text and rely on it for our substantive analysis and counterfactual simulations.

We plot the estimated fixed effects in Figure 6. To reduce clutter, we only report the estimates for countries that are involved in at least five wars in our sample. Because the fixed effects are included in the military effectiveness equation, the estimates essentially represent the logarithm of the baseline force multiplier for each country in the sample. At any given time, this baseline will be modified depending on the country's values on the substantive variables at that time.

The results of the fixed effects analysis broadly conform with intuitions about states' military prowess. We see above-average force multipliers for countries that have historically been regarded as militarily powerful, such as Germany, Japan, Israel, the United Kingdom, and the United States. Looking at the extremes, we estimate the greatest force multipliers for the Democratic Republic of the Congo and New Zealand. It is not surprising that we estimate the largest fixed effects for smaller countries, as these capture variation not accounted for by the substantive variables. New Zealand's high force multiplier is accounted for by its battlefield success: it was on the winning side of all six wars it participated in, two of which did not include the United States as a coalition partner. Meanwhile, the DRC's large force multiplier appears to be due to its successful push for independence (without coalition partners) against Belgium, which itself is estimated as having a fairly high military effectiveness.

At the other pole, we estimate the lowest force multipliers for Canada and El Salvador. The Canadian case is interesting, as Canada is on the winning side of all five wars in which

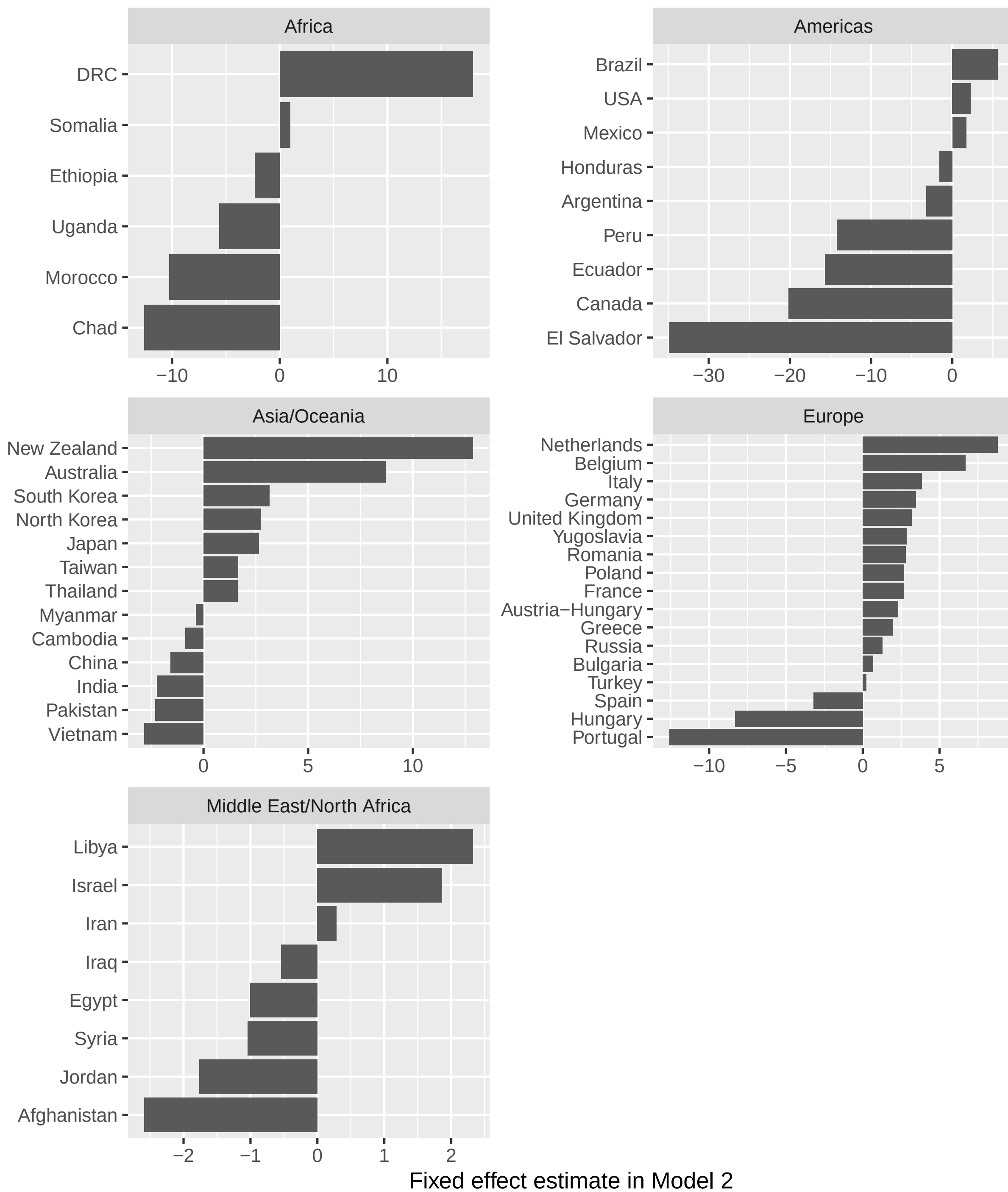

Figure 6. Estimated country fixed effects from Model 2.

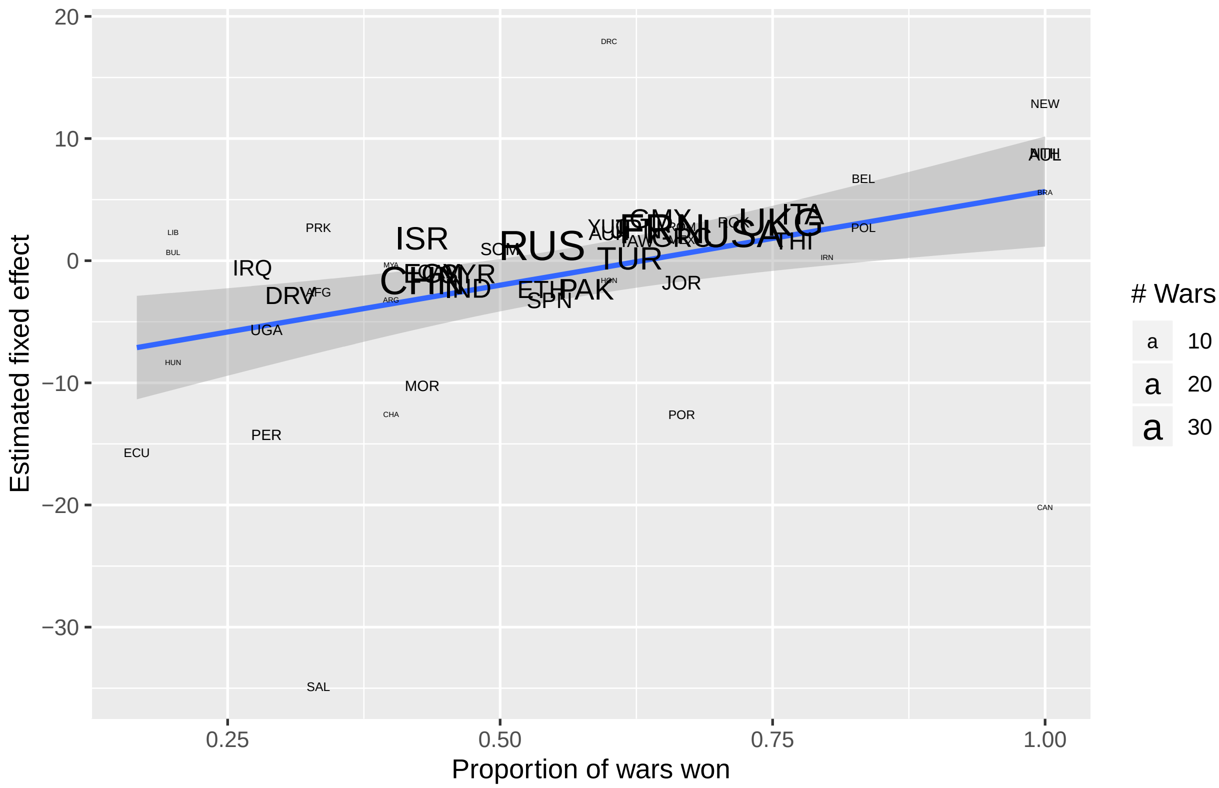

Figure 7. Relationship between raw win rates in war and estimated fixed effects from Model 2.

it was a participant. However, Canada appears to have been a free rider in these conflicts, as it had more than a dozen coalition partners in each. Moreover, two of these conflicts were against opponents with below-average military effectiveness and no coalition partners (Iraq in 1990 and Afghanistan in 2001). Meanwhile, the low force multiplier for El Salvador is due more to poor outcomes than to free riding. El Salvador is on the winning side of just two of its five wars; in one of these (the Second Central American War) it has powerful partners including the United States and Mexico, and its sole opponent is Guatemala. Meanwhile, El Salvador's three losses are all to relatively weak states that lack coalition partners in their respective wars: Nicaragua, Honduras, and Guatemala.

The Canadian example suggests that our force multiplier fixed effects are picking up more than just how often countries win their wars. For a broader look at the relationship at the relationship between the raw data and our estimates of strength, Figure 7 plots the fixed effects as a function of the proportion of wars won by each country. We see that countries with better raw win rates are indeed estimated to be stronger on average, but that the relationship is hardly one-to-one. The Spearman correlation between win rates and the estimated fixed effects is 0.59. Similar to a Bradley-Terry model (see Renshon and Spirling 2015), our estimates account for the force multipliers of one's opponents, so a country that wins many wars may still have a low force multiplier if it is taking on weak opponents. In addition, the structure imposed by our formal model allows us to account for free-riding on one's coalition partners in coalitional wars. Of course, these estimates might be further improved if we could directly measure the effort exerted by each country in each conflict, supplementing the estimates about effort that we derive from the Nash equilibrium conditions.