Social Conflict and the Predatory State*

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Abstract

I model the political economy of predatory governance in an internally divided society. Two competing logics characterize the relationship between social conflict and the revenues that a predatory ruler can extract. Internal conflict may benefit the government by discouraging collective resistance against predation (the divide-and-rule logic), but it may reduce revenues by shrinking economic output (the productivity logic). The productivity logic is inoperative for states funded by natural resource wealth, so these states benefit from social conflict. However, I find a sharp opposite for predatory states funded by the output of the population's labor—in equilibrium, the economic benefits of greater productivity always outweigh the political benefits of a divided population. Consequently, social fractionalization benefits the predatory state if and only if its revenues derive from natural resources or a similar fixed source. However, changes in the labor costs of internal conflict have more ambiguous effects.

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A key imperative of the sovereign state is to provide for its subjects' security. Even predatory states—those that seek to profit from rule—have provided justice systems and protected property rights, as doing so raises productivity and thus increases revenues (North 1981). This line of thought, the *productivity logic*, would seem to imply that even a predatory state benefits from social order. Yet since at least Machiavelli, scholars of politics and rulers themselves have also thought that predatory rulers might benefit from rivalry and disorder among their subjects. Conflict within the population may distract from or otherwise discourage collective resistance against extractive state policies, according to this *divide-and-rule logic*. There is thus a tension in the political imperatives of the predatory state: to tamp down internal conflicts enough to boost economic productivity, but to stoke internal divisions enough to discourage resistance and maintain control.

In this paper, I develop a theory to explain how predatory states resolve these competing imperatives—and to understand when these states benefit from social conflict rather than internal order. I study the political economy of predatory governance in a divided society, modeling it as an interaction between a revenue-maximizing ruler and a set of subjects divided into distinct political factions. In the model, there is weak property rights protection, as each faction's economic holdings are subject to expropriation by both the government and by other factions. The government's revenues depend on how much labor the population devotes to collective resistance against predatory rule, as opposed to factional conflict that merely shifts the distribution of untaxed surplus. The population's division of labor in turn depends on the ruler's choice of tax rate, as well as structural conditions such as the extent of internal fractionalization and the labor cost of internal conflict. The central question is when the structural conditions that promote social order—low levels of expropriative conflict *among* factions—increase state revenues in equilibrium.

I find that the relationship between social order and the profitability of predatory rule depends critically on the nature of the ruler's revenue base. I distinguish between capital-financed states, whose rents derive primarily from some fixed source of wealth (e.g., natural resource reserves), and labor-financed states, who extract commodities produced by the population they govern (e.g., staple crops). For state that is capital-financed in this sense, the productivity of the subject labor force is essentially immaterial. It follows that the divide-and-rule logic prevails for capital-financed states, and these states benefit from internal strife rather than order.

Most significantly, I find a sharp converse for a labor-financed state: the productivity-enhancing logic *always* dominates, meaning a predatory ruler benefits from conditions that promote social order. In this case, seemingly in line with the divide-and-rule logic, the state is able to implement a greater tax rate without provoking costly resistance when baseline social conflict is greater. At the same time, however, more internal conflict means less labor devoted to productive activity, resulting in a contraction of the state's revenue base. I show that the latter effect is dominant—that the pie shrinks enough to offset the government getting a proportionally larger slice of it. This follows from two key features of the strategic environment. First, the incentive to engage in collective resistance depends on the government's overall revenue, not just the tax rate. Second, the extent of internal division does not influence a faction's incentive to divert labor from *production* to resistance. Combined, these mean that production is low enough and the threat of resistance is strong enough to hold down state revenues in settings with high baseline internal conflict.

The goal of the analysis is not merely to observe when social strife works to the benefit of a predatory state, but also to identify structural variables that might influence the baseline level of internal conflict in a divided society. To this end, I analyze the effects of two features of the political economy. The first is the extent of fractionalization in society, operationalized in the model as the number of distinct competing factions. The main finding for this variable is simple: in equilibrium, the extent of social conflict—the proportion of labor devoted to the internal competition, rather than collective resistance or economic production—increases with fractionalization. This result follows from the classic logic of collective action (Olson 1965). From the social factions' perspective, resistance against predatory taxation is a public good, as it increases consumption for all factions, even those that do not contribute to the collective effort. An individual faction always internalizes the opportunity cost of labor devoted to resistance, but internalizes a smaller share of the benefits as the number of factions increases. Meanwhile, internal conflict, whose costs and benefits are both fully internalized by each faction, becomes relatively more attractive.

The second factor I examine is the technological effectiveness, or labor cost, of the means of social conflict—how much labor it takes for one faction to increase its share of output by some fixed amount. I find countervailing effects on the baseline level of social conflict, meaning that improvements in the technology of competition could work to the benefit or detriment of either type of government. Improved technology allows a faction to produce the same level of effective force with less labor, thereby freeing up more for collective resistance or economic production. However, by increasing the benefit per labor unit of internal conflict, greater effectiveness also increases each faction's optimal level of effective force. The overall effect of a technological change on the equilibrium level of social conflict depends on whether the labor-saving effect or the incentive effect is dominant.

The findings on the technology of conflict are important for understanding governance choices by predatory states. Whereas social fractionalization would be difficult to alter through policy, the state (or another outside actor) could improve the technology of internal conflict by arming the population, or degrade it by providing protection against internal raiding and banditry. If providing justice would ultimately reduce the amount of labor to social conflict, then a capital-financed predatory state will prefer not to do so. However, it is profitable for a labor-financed state to protect social factions' claims to property against one another, as long as such protection would reduce internal conflict and is not too costly to provide.

If internal conflict only works to the benefit of predatory states under particular conditions, why is the divide-and-rule logic so widely accepted? The formal analysis indicates how casual observation might be misleading. First, as noted above, the equilibrium tax rate increases with fractionalization even for labor-financed governments. What decreases is total production and therefore overall revenue, which is even more difficult to measure.[1] Second, at the optimal tax rate for a labor-financed government, the level of internal conflict is relatively high. It just happens that the policy which minimizes resistance is the one that maximizes internal conflict. The government would benefit from a change in structural conditions that reduced the overall incentives for social conflict, but such changes would naturally be more difficult to observe. Finally, as many predatory governments have historically been the products of imperial or colonial conquest, it is important to distinguish the process of taking over a society from governing it. In an extension to the model, I show that the conquest process is analogous to capital-financed predatory governance—a contest with a fixed prize. Therefore, as in the famous case of Cortés in Mexico, internal strife might help a predatory outsider gain control the state, even when it then reduces their subsequent revenues from labor extraction.

1 Related Literature

Broadly speaking, this paper contributes to the long tradition of work in political science and political economy that conceives of the state as an extractive or predatory institution (North 1981; Tilly 1985; Levi 1989; Olson 1993). More specifically, the paper builds on the idea that conflict within a society shapes the policy choices of a predatory government, and vice versa (Bates, Greif and Singh 2002; North, Wallis and Weingast 2009). The key contributions my paper makes to this literature are, first, to develop a theoretical link between social conflict and the profits of predatory rule and, second, to show how this relationship is conditional on the nature of the state's revenue base.

In an early model of predatory governance, Moselle and Polak (2001) characterize how extractive states harm social welfare. Like them, I model predatory governance in a political economy where individuals might steal from each other rather than engaging in productive

[1] For example, see Frankema's (2010) analysis of revenue extraction in the British empire.

activity. Whereas they focus on the social welfare consequences of the predatory state versus other social arrangements, I take a predatory government as given and analyze how social and economic variables affect its revenues. Critically, I introduce the possibility of collective resistance against extractive rule, allowing me to directly assess arguments about how a predatory government might benefit from weaker incentives for collective action among the governed population.

More recent work has assessed the strategic underpinnings of extractive states. Gailmard (2017) examines institutional arrangements, arguing that the separation of powers may arise as a solution to agency problems in imperial governance. I abstract away from institutional specifics in order to allow for appropriation and conflict *among* the governed population, which Gailmard treats as unitary. Meanwhile, Acharya and Lee (2018) analyze collusion between predatory states vying for the loyalty of subjects. My analysis provides a possible microfoundation for their assumption of declining marginal benefits to acquiring additional subjects, at least under some conditions. If fractionalization increases with population size, then the resulting social conflict might decrease the profitability of rule—but only if the state's revenue base is dependent on labor output, not natural resources.

Perhaps the most closely related formal analysis is by Tyson (2020), who identifies axes of conflict that parallel the ones this paper is concerned with. He describes two security problems that the state must solve: the horizontal problem, whereby citizens might appropriate from each other, and the vertical problem, whereby the sovereign might appropriate from the citizenry. These security problems also form the core of the political economy that I study. But despite the similar underlying structure, the analyses have divergent approaches and aims. Whereas Tyson focuses on the conditions that allow for a social contract to form between the ruler and the ruled, I take the predatory nature of the government as given and analyze how its choices and payoffs are affected by underlying political and economic variables.

As I analyze when predatory rulers benefit from social conflict, my analysis is also closely related to theoretical studies of divide-and-rule politics. Acemoglu, Verdier and Robinson (2004) identify divide-and-rule strategies as an explanation for the persistence of kleptocracies. In their model, a kleptocrat facing a divided opposition can credibly threaten to divert resources to a favored group in case of a challenge, thereby deterring a challenge in the first place. Padró i Miquel (2007) presents a similar model, except in which rulers' leverage comes from fear of political succession turning over power to a different group. Debs (2007) introduces an informational mechanism, finding that it is in a government's interest to manipulate the media in a way that polarizes social preferences over policy. While I abstract away from the specific divide-and-rule strategies examined in these theoretical works, I innovate on them by allowing for direct conflict among the internal factions, drawing from the political economy literature on social conflict (see below). In the model, each faction's economic output is subject to appropriation by others, which affects the incentive to produce and thereby the profitability of predatory rule. The analysis here shows that the conventional wisdom about the profitability of divide-and-rule strategies holds only conditionally, namely in societies where the ruler's revenue comes from fixed resources rather than endogenous labor.

Finally, my model of social conflict as a contest draws heavily from the political economy literature on this subject. Earlier work examines variation in the extent and outcome of social conflict as a function of the participants' coercive capabilities (Hirshleifer 1991), the technology of conflict (Skaperdas 1992; Grossman and Kim 1995), the number of competing groups (Tornell and Lane 1999), the technology of production (Silve 2017), market conditions (Dal Bó and Dal Bó 2011), and access to external finance (Azam 2002). The main innovation in this paper is the introduction of a predatory state actor and the possibility of collective resistance against state predation. All else equal, predatory tax policies decrease the level of internal conflict compared to the baseline environment considered in previous models, as higher taxes increase groups' incentive to partake in collective resistance instead of internal conflict. A government that is financed by labor and therefore benefits from internal harmony may also prefer to enact policies, such as legal protection of claims to property, that further reduce the amount of internal conflict. The effect of predation by a capital-financed government is more ambiguous, however, as such states may seek to arm competing factions or otherwise increase social conflict, offsetting the direct effects of predatory taxation on internal conflict.

2 The Model

I model the political economy of predatory governance in a divided society. The players are the government, denoted G, and a set of $N \geq 2$ identical factions within the society, denoted $\mathcal{N} = \{1, \ldots, N\}$. Let $i \in \mathcal{N}$ denote a generic faction.

The interaction proceeds in two stages. First, the government chooses a tax rate, $t \in [0, 1]$.[2] Second, after observing the tax rate, each faction simultaneously allocates its labor among activities that affect the level and distribution of economic output. These are production, denoted p_i; resistance against the government, r_i; and internal conflict or competition, c_i. Each faction has finite labor available, facing the budget constraint

$$\frac{p_i}{\pi^p} + \frac{r_i}{\pi^r} + \frac{c_i}{\pi^c} = \frac{L}{N}, \tag{1}$$

where $L > 0$ denotes the total size of the population and each $\pi^p, \pi^r, \pi^c > 0$ denotes the society's productivity in the respective activity.[3] For example, the greater π^r is, the less labor is required to produce the same amount of resistance; total resistance cannot exceed $\pi^r L$. After these choices are made, the game ends and each player receives her payoffs. Let $p = (p_1, \ldots, p_N)$ denote the vector of production choices, and define the vectors r and c similarly.

[2] The tax rate is the same for all factions. I relax this assumption in an extension.

[3] I write each faction's constraint as a fraction of L to take comparative statics on the number of factions while holding fixed the total size of the society. In the Appendix, I derive equilibrium existence and uniqueness results for the more general case in which factions may differ in their size and productivities.

The goal of each player, including the government, is to maximize their own consumption. The nature of this consumption depends on the state's revenue base. If the state is *capital-financed*, there is a fixed resource of value $X > 0$ that the players compete over.[4] To simplify the presentation of this case, I assume production by the factions is payoff-irrelevant when the state is capital-financed, which implies $p = 0$ in equilibrium. In formal terms, the production function is $f(p) = X$, a constant.[5] On the other hand, a *labor-financed* state depends solely on the productive output of the population. In this case, I assume a linear production function,

$$f(p) = \sum_{i=1}^{N} p_i, \tag{2}$$

where $p = (p_1, \ldots, p_N)$ denotes the vector of each faction's production choice.[6] Whether the state is capital- or labor-financed, each player's utility is ultimately a fraction of $f(p)$. The other choice variables—the tax rate, resistance, and internal conflict—determine what these fractions are.

Resistance, the second way the factions can expend their labor, determines how much the government can actually collect in taxes. Given the nominal tax rate t and the vector of resistance allocations r, let $\tau(t, r)$ denote the proportion of economic output that goes to the government, or the effective tax rate, and let $\bar{\tau} = 1 - \tau$ denote the share that remains to be divided among the factions. Resistance determines what proportion of the nominal tax

[4]Different actors might value the same resources differently. For example, control of oil fields might be more valuable in an absolute sense to the government than to rebel groups (Le Billon 2013, 29–30). The results of the analysis would not change if each actor valued the pie at a potentially different level $X_i > 0$, as this would simply entail the multiplication of each actor's utility function by a positive constant.

[5]The main results for capital-financed states would be the same if production affected an individual faction's payoffs, but competition among the factions and the government only concerned the exogenous resource X. The main results would also continue to hold if we let $f(p) = X + \sum_i p_i$, provided that X were large enough. The setup here allows for the simplest presentation of the baseline finding that capital-financed states benefit from internal strife.

[6]The linear production technology rules out complementarity between different factions' economic activities (see Silve 2017). This is a reasonable assumption for historically common forms of labor extraction by predatory states, such as staple crop production and mining of raw materials or precious metals, but would be less applicable to more developed economies.

rate the government can collect:

$$\tau(t, r) = t \times g\left(\sum_{i=1}^{N} r_i\right), \tag{3}$$

where $g : [0, \pi^r L] \to [0, 1]$ is a strictly decreasing function. Given total resistance $R = \sum_{i=1}^{N} r_i$, the function $g(R)$ may represent either the proportion of t that the government can collect or the probability that it collects t as opposed to nothing. To ensure the existence of an equilibrium and ease its characterization, I assume g is twice continuously differentiable, convex, and log-concave.[7] Convexity implies that resistance has diminishing returns, so the factions face a collective action problem: the more each faction expects the others to contribute to resistance, the less it prefers to contribute itself. To simplify the exposition, I assume the government fully collects the announced tax rate if there is no resistance: $g(0) = 1$. The linear function $g(R) = 1 - \frac{R}{\pi^r L}$ is one example of the many functional forms that meet these conditions.

Internal conflict, the third and final outlet for the factions' labor, determines the share each group receives of what is left over after the government takes its cut. I model the internal conflict over output as a contest, in which each faction expends costly effort to increase its share of the pie. Given the conflict allocations $c = (c_1, \ldots, c_N)$, a faction's share of post-tax output is given by the contest success function

$$\omega_i(c) = \frac{\phi(c_i)}{\sum_{j=1}^{N} \phi(c_j)}, \tag{4}$$

where $\phi : [0, \pi^c L/N] \to \mathbb{R}_+$ is strictly increasing. Factions that devote more effort to internal conflict end up with larger shares of the output. If every faction spends the same amount, or they all spend nothing, they all end up with equal shares, $\omega_i(c) = 1/N$.[8] Again, as regularity conditions to ensure equilibrium existence, I assume ϕ is twice continuously differentiable

[7]Given the first condition, the latter two are equivalent to $0 \leq g''(R) \leq g'(R)^2/g(R)$ for all $R \in [0, \pi^r L]$.
[8]If $\phi(0) = 0$, in which case (4) is not well-defined at $c = 0$, let each $\omega_i(0) = 1/N$.

and log-concave. Both of the most popular contest success functions satisfy these criteria: the ratio form, with $\phi(c_i) = c_i$, and the difference form, with $\phi(c_i) = e^{c_i}$ (Hirshleifer 1989).

Each faction's utility is simply the amount of economic output it receives. This is a function of how much is produced, how much the government extracts through taxation, and the faction's standing in the internal contest. Together, these yield faction i's utility function,

$$u_i(t, p, r, c) = \omega_i(c) \times \bar{\tau}(t, r) \times f(p). \tag{5}$$

The multiplicative payoff structure is similar to that of Hirshleifer (1991) and Skaperdas (1992). It is a strategic environment with weak protection of property rights, in which possession is determined through appropriation (Skaperdas 1992).[9]

The government in the model is predatory insofar as its motivation is to increase revenues for its own consumption (Levi 1989). Its utility is how much of the economic output it receives, accounting for reductions in the effective tax rate due to resistance:

$$u_G(t, p, r, c) = \tau(t, r) \times f(p). \tag{6}$$

The government does not use tax revenues to provide public goods or redistribute wealth within society. In addition, the government does not have preference over the distribution of post-tax consumption among the factions. Internal conflict does not directly enter the state's utility, though it matters indirectly insofar as it reduces production or resistance.

2.1 Notes on the Model Setup

The model sets up stark strategic tradeoffs for both the factions and the government. Each faction must choose between increasing the total size of the post-tax pie, namely through production or resistance, and securing its own share of that pie through internal conflict. They face a collective action problem, as production and resistance are collectively beneficial

[9]For a model of conflict with (endogenously) partial property rights, see Grossman and Kim (1995).

while one's share of the internal contest only has private benefits. For the government, the key tradeoff comes in how it calibrates its extractive demand. Holding fixed the behavior of the factions, the government would always prefer a higher tax rate. But a high rate is counterproductive if it diverts social effort away from production and into resistance.

By separating resistance and internal conflict into separate choices, the model assumes that effort spent resisting government predation does not help a faction in resource competition with other groups, and vice versa. In reality, some activities, such as building fortifications, may serve both purposes. However, it is analytically useful to focus on the case in which there is a stark separation between the two activities. Most importantly, the negative effect of fractionalization on incentives for collective action is strongest in an environment without spillovers between collective resistance and internal conflict. The less complementarity there is between these two activities, the more that fractionalization ought to facilitate government extraction. Therefore, the assumption of no complementarity makes it all the more surprising that I find that labor-financed states always benefit from internal conflict. This result would only become stronger in the presence of spillovers.

It may seem natural to interpret the internal conflict in the model as the eruption of violent conflict between factions. However, this might raise the question of why the factions do not simply determine the distribution of post-tax surplus through peaceful negotiation (Fearon 1995). For the model, what matters is not whether violence occurs, but the extent to which competition between factions results in them spending labor that neither increases the level of economic output nor contributes to collective resistance against predatory rule. The mere threat of violence might result in this kind of labor allocation, even when that threat remains latent, as groups would need to spend time and resources acquiring the coercive capability to maintain their leverage at the bargaining table (Coe 2011; Fearon 2018).

3 Capital-Financed Governments Prefer Strife

I begin the model analysis by focusing on the case of a capital-financed government, in which the object of both predatory taxation and internal conflict is to control a resource of fixed value. In the absence of concerns over labor productivity, the divide-and-rule logic is naturally dominant: the government's equilibrium revenue increases with the extent of internal conflict. I use this baseline case to identify important comparative statics on the structural determinants of internal conflict. Social fractionalization, here conceived of as the number of distinct factions, raises the level of internal conflict and thereby works to the benefit of a capital-financed predatory state. Efficiency improvements in the technology of internal conflict have countervailing effects, however.

The clearest example of a capital-financed government is one funded by natural resources, particularly oil. Oil exploitation is capital-intensive, often takes place offshore, and can be conducted by workers imported from abroad (Le Billon 2013, 28–30). In the absence of mass resistance, a government can extract value from oil deposits with minimal regard for local labor contributions. Similarly, an empire that seeks land for settlement—rather than for agricultural or mineral exploitation by a local labor force—is capital-financed in this sense. For example, the English colonial empire in North America, which sought sparsely populated territories and fought to expel American Indians where they settled, was capital-financed by this definition (Elliott 2007, 36–38). By contrast, the Spanish colonial empire, with its notorious *encomienda* system of native labor exploitation, was labor-financed.

3.1 Equilibrium

I solve the model via backward induction, first identifying the factions' equilibrium labor allocations given the government's choice of tax rate. When the state is capital-financed, the factions essentially face a tradeoff between two dimensions of competition, both over the fixed prize X. There is a vertical dimension, consisting of competition between the government and the factions collectively over how much the government is to expropriate. Resistance determines whether, or to what extent, this competition is resolved in favor of the governed society rather than the government. The other dimension of competition is horizontal, consisting of the conflict among factions over their division of whatever the state fails to expropriate. By diverting labor from collective resistance to internal conflict, an individual faction reduces overall post-tax output (by allowing the government to collect a greater fraction of t), but increases its own share of what is left.

As the government's tax demand increases, the vertical dimension of competition weighs more heavily in each individual faction's decision calculus. Formally, the marginal benefit of resistance increases with t, increasing each faction's incentive to spend labor on resistance rather than internal competition. The equilibrium among factions, considered as a function of the government's choice of tax rate, therefore has a simple structure. If taxes are low enough, all labor is devoted to internal conflict, none to resistance. At a certain point, resistance begins to increase at the expense of internal competition. When taxes are sufficiently high, there may be no internal conflict at all, depending on the parameters of the model.[10] In all cases, because they are *ex ante* symmetric, all factions exert the same amount of effort in the internal contest, leaving each with a share $1/N$ of post-tax output. The following proposition summarizes the equilibrium, and Figure 1 illustrates.[11]

Proposition 1. *If the government is capital-financed, every labor allocation subgame has a unique equilibrium. There exists a tax rate $\hat{t}_0^X \in (0,1)$ such that each $r_i = 0$ in equilibrium if and only if $t \leq \hat{t}_0^X$. There exists $\hat{t}_1^X > \hat{t}_0^X$ such that each $c_i = 0$ in equilibrium if and only if $t \geq \hat{t}_1^X$. For $t \in (\hat{t}_0^X, \hat{t}_1^X)$, in equilibrium each $r_i = \tilde{R}_X(t)/N > 0$ (strictly increasing in t) and each $c_i = \tilde{c}_X(t) > 0$ (strictly decreasing).*

A high tax rate galvanizes the population, giving the factions an incentive to act in concert

[10] This is possible if the labor productivity of resistance, π^r, is great enough relative to that of internal conflict, π^c. See Equation 24 in the Appendix.

[11] All proofs are in the Appendix.

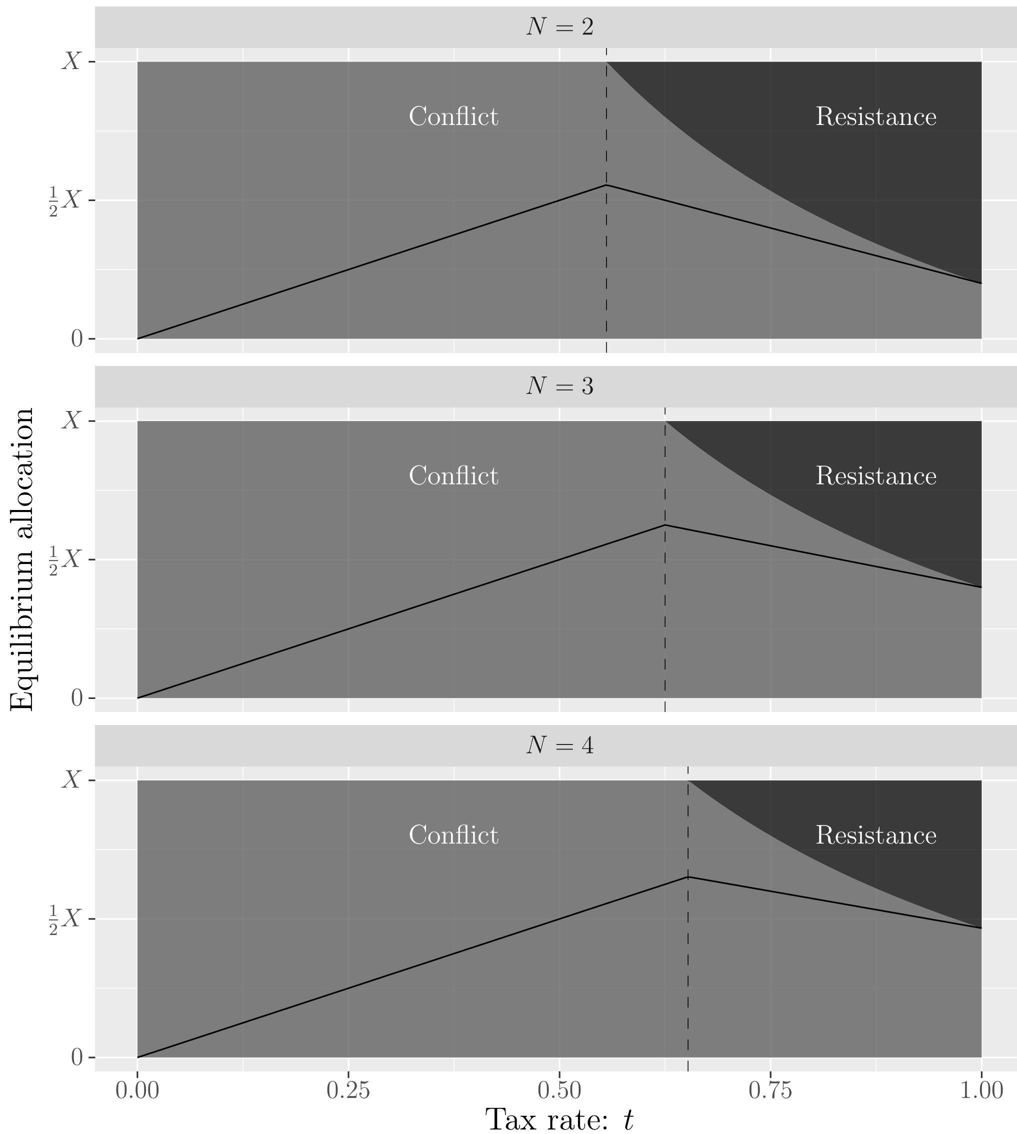

Figure 1. Equilibrium labor allocations with a capital-financed government as a function of the tax rate and the number of factions (Proposition 1). The solid curve is the government's payoff as a function of the tax rate, and the dashed line is the equilibrium tax rate. This and subsequent figures use the parameters $\pi^p = \pi^r = \pi^c = 1$ and $L = X = 2.5$ and the functional forms $g(R) = 1 - \frac{R}{\pi^r L}$ and $\phi(c_i) = \exp(c_i)$.

to reduce government expropriation. In other words, predatory state behavior endogenously reduces the structural impediments to collective action within a divided society. As an example of this dynamic, excessive extraction by colonial empires led to unified resistance both among the American colonies during the Stamp Act crisis of 1765 and between creoles and Indians during a contemporaneous tax revolt in Quito (Elliott 2007, 310–314).

Moving up the game tree, the government selects a tax rate in anticipation of the factions' responses. Assuming that the factions employ equilibrium responses, the government's induced utility function is

$$Eu_G(t) = \overbrace{t}^{\text{increasing}} \times \underbrace{g(R^*(t))}_{\text{decreasing}} \times \overbrace{X}^{\text{constant}}.$$

where R^* is total resistance in the corresponding equilibrium. Here we see countervailing effects of the tax rate on the government's revenues. All else equal, greater t means greater revenues. But all else is not equal: equilibrium resistance increases with the tax rate, partially or even fully offsetting whatever gains the government might reap. While there is no general expression for a capital-financed government's optimal choice of tax rate, some results are evident.[12] First, because total resistance is continuous as a function of t, an optimal rate exists.[13] Second, no $t < \hat{t}_0^X$ is an equilibrium, as the government could raise the rate to the cutpoint without provoking resistance. Similarly, no $t \in (\hat{t}_1^X, 1)$ may be an equilibrium either, as resistance is at its maximum and thus cannot further increase with taxes in this range.

3.2 Comparative Statics

I now consider the structural determinants of social order and their effects on a capital-financed government's equilibrium revenue. The first is the number of factions, N, which I

[12] In Figure 1 the equilibrium tax rate is always $\hat{t}_0^X$, but this is an artifact of the particular functional forms used.

[13] See Lemma 6 in the Appendix.

take to represent the extent of social fractionalization. I find that greater fractionalization results in a greater portion of the factions' labor spent on internal conflict. In the capital-financed case, this must come at the expense of collective resistance, thereby benefiting the government. To see why resistance decreases with fractionalization, remember that in equilibrium, each faction receives $1/N$ of post-tax wealth, and thus only internalizes $1/N$ of its efforts toward collective resistance. As this fraction grows smaller, so does the individual incentive to engage in collective resistance. Therefore, by the classic logic of group size and collective action (Olson 1965), high fractionalization discourages social mobilization against predatory rule. Because a capital-financed predatory state is indifferent to internal conflict except insofar as it affects resistance, this in turn means that such states gain from governing fractionalized populations—the divide-and-rule logic holds here.

Proposition 2. *A capital-financed government's equilibrium payoff is increasing in the number of factions, N.*

To see formally why this result holds, observe that the first-order condition for an equilibrium with total resistance $R > 0$ and nonzero internal conflict is

$$\pi^r \frac{\partial u_i}{\partial r_i} = \pi^c \frac{\partial u_i}{\partial c_i} \quad \Leftrightarrow \quad \pi^r \times \underbrace{\frac{t(-g'(R))}{1 - tg(R)}}_{\text{decreases with } R} = \pi^c \times \overbrace{\frac{N-1}{N}}^{\text{increases with } N} \times \underbrace{\frac{\phi'(C)}{\phi(C)}}_{\text{decreases with } C},$$

where $C = \frac{\pi^c}{N}(L - \frac{R}{\pi^r})$ per the budget constraint. An exogenous increase in N essentially increases the relative marginal utility of internal conflict, and thus must result in lower total resistance. More precisely, an exogenous increase in N causes the right-hand side of the above expression to increase. To maintain the equality, C must increase and R must decrease.

The effects of improvements in the technology of internal conflict, π^c, on the equilibrium level of resistance—and thus on government revenues—are more complicated. An increase in π^c has two countervailing effects. The first is a *labor-saving effect*: greater π^c means that a faction can achieve the same result in the internal conflict with less labor devoted to it. This promotes resistance by allowing each faction to maintain its current position in the conflict at a strictly lower labor cost. The second is an *incentive effect*: effectively, the marginal utility of internal conflict increases with π^c. This increases the incentives for fierce competition in the contest among factions, discouraging resistance.

Neither the labor-saving effect nor the incentive effect is guaranteed to dominate. Their relative influence depends on the equilibrium level of internal conflict and the functional form of the contest success function. To see why, remember that $r_i = \pi^r(\frac{L}{N} - \frac{c_i}{\pi^c})$, so the effect of a marginal change in conflict effectiveness on equilibrium resistance is

$$\frac{dr_i^*}{d\pi^c} = \frac{\pi^r}{\pi^c}\left[\overbrace{\frac{c_i^*}{\pi^c}}^{\text{labor-saving effect}} - \underbrace{\frac{dc_i^*}{d\pi^c}}_{\text{incentive effect}}\right].$$

If there is minimal conflict in equilibrium ($c_i^* \approx 0$), then the incentive effect dominates, and an increase in π^c results in less resistance. More generally, the incentive effect dominates if

$$\frac{d\log\phi(c_i^*)}{dc_i^*} + c_i^*\frac{d^2\log\phi(c_i^*)}{d(c_i^*)^2} > 0, \tag{7}$$

which may hold at some tax rates but not others.[14]

Proposition 3. *If there is a unique equilibrium tax rate t^*, the government's equilibrium payoff is locally increasing in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect (i.e., Equation 7 holds) at the corresponding equilibrium level of internal conflict.*

This result implies that a capital-financed predatory state might choose not to protect

[14]For the derivation of this expression, see Lemma 9 in the Appendix. Additionally, in Lemma 10 in the Appendix, I verify that the incentive effect always outweighs the labor-saving effect for the "difference" contest success function ($\phi(C) = \theta e^{\lambda C}$) and that the effects are exactly offsetting for the "ratio" contest success function ($\phi(C) = \theta C^{\lambda}$).

factions' property rights claims against each other, even if it were costless to do so. In fact, if the baseline level of social conflict were low enough, the state might try to increase it—e.g., by dismantling extant justice-providing institutions, or by arming the population—if the cost were low enough.

These results for capital-financed governments rely on the assumption that internal conflict has no *direct* externality on the state's utility. Conflict among factions only matters to the government insofar as it affects resistance. If instead there were negative externalities from internal conflict, then fractionalization and conflict effectiveness would have countervailing effects on the state's equilibrium utility. For example, if civil conflict makes it harder for an occupying power to maintain control over a territory, then even a capital-financed occupier might benefit from unity rather than fractionalization.

4 Labor-Financed Governments Prefer Order

I now examine labor-financed states, in which the payoffs of all players depend on endogenous production choices by the factions. In this environment, both the productivity logic and the divide-and-rule logic ought to carry at least some weight. As with a capital-financed state, greater internal conflict may come at the cost of collective resistance, thereby increasing the state's ability to expropriate. But conflict among factions may now also detract from production, having the opposite effect on state revenues. In fact, in equilibrium both effects are evident: a greater baseline level of social conflict is associated with lower production, but allows the government to impose a greater tax rate without provoking resistance. Despite these countervailing effects, I find a sharp converse of the comparative statics from the previous case: structural conditions favoring social order work to the *benefit* of a labor-financed predatory state.

In order to focus on the most substantively relevant applications of the model, I impose the following assumption throughout this section:

Assumption 1. $\frac{N-1}{N} > \frac{\phi(0)}{\phi'(0)} \cdot \frac{1}{\pi^c L}$.

If Assumption 1 does not hold, then for every tax rate $t \in [0, 1]$, the equilibrium of the subsequent labor allocation subgame entails each faction spending nothing on internal conflict. This case, in which internal conflict never happens regardless of the tax rate, is of relatively little substantive interest for the analysis of the interplay between government predation and social order.

4.1 Equilibrium

As before, I begin by solving for the equilibrium labor allocation after each possible tax rate. The structure of the equilibrium is similar to that in the capital-financed case (Proposition 1), with one important exception—some labor is now devoted to economic production. In fact, total production must be nonzero following any tax rate, even $t = 1$, as otherwise every player would receive their lowest possible payoff.

When taxes are low enough, there is no resistance in equilibrium, as in the case of a capital-financed government. Assumption 1 guarantees positive internal conflict in this low-tax case. There is a threshold at which the incentive to resist becomes too great, and total resistance strictly increases with t after this point. The increase in resistance comes at the expense of both internal conflict and economic production. If taxes are sufficiently high, then internal conflict may dissipate altogether, leaving the factions to divide their labor solely between production and resistance. Proposition 4 summarizes the form of the labor allocation equilibrium.

Proposition 4. *Assume the government is labor-financed. There exist tax rates $\hat{t}_0 \in (0, 1)$ and $\hat{t}_1 > \hat{t}_0$ such that in every equilibrium of the labor allocation with tax rate t:*

- *If $t \leq \hat{t}_0$, then each $p_i = \bar{P}_0/N > 0$, each $r_i = 0$, and each $c_i = \bar{c}_0 > 0$.*
- *If $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = \tilde{P}_1(t) > 0$ (weakly decreasing in t), $\sum_i r_i = \tilde{R}_1(t) > 0$*

(strictly increasing), and each $c_i = \tilde{c}_1(t) > 0$ *(strictly decreasing).*

- *If* $t \geq \hat{t}_1$*, then* $\sum_i p_i = \tilde{P}_2(t) > 0$ *(strictly decreasing in* t*),* $\sum_i r_i = \tilde{R}_2(t) > 0$ *(strictly increasing), and each* $c_i = 0$*.*

Unlike in the capital-financed case, there is not necessarily a unique equilibrium of the labor allocation subgame. However, the payoff-relevant quantities—total production and total resistance—are identical across equilibria. Moreover, the equilibrium in the low-tax case ($t \leq \hat{t}_0$) is unique, with each individual faction exerting the same amount of effort on both production and internal conflict.

Even when $t = 0$, the outcome is social welfare inefficient *and* Pareto inefficient from the perspective of the factions. Every faction would receive the same share, $1/N$, of a larger pie if they devoted all their labor to production. A kind of prisoner's dilemma logic explains why this Pareto efficient allocation of labor is not sustainable: if no faction planned to spend on the internal conflict, then any single faction could obtain a large share by spending relatively little. Under Assumption 1, the temptation is large enough that every faction has an incentive to deviate from a strategy profile with no conflict.

Having solved for labor allocations as a function of the tax rate, I now solve for the government's optimal choice of t. One basic tradeoff from the capital-financed case carries over here: more taxes (above the cutpoint $\hat{t}_0$) lead to more resistance, at least partially offsetting the intended increase in the government's share of output. But now there is an additional downside to predatory taxation. Collective resistance no longer merely siphons away from internal conflict—it also reduces economic production, thereby shrinking the pool of output from which government revenues are drawn. The combination of these effects allows me to pin down the equilibrium tax rate for a labor-financed government more clearly than in the capital-financed case. A labor-financed government prefers $t = \hat{t}_0$, the greatest rate that engenders no resistance. Figure 2 illustrates equilibrium labor allocations as a function of the chosen tax rate, showing how government revenues are maximized at $\hat{t}_0$.

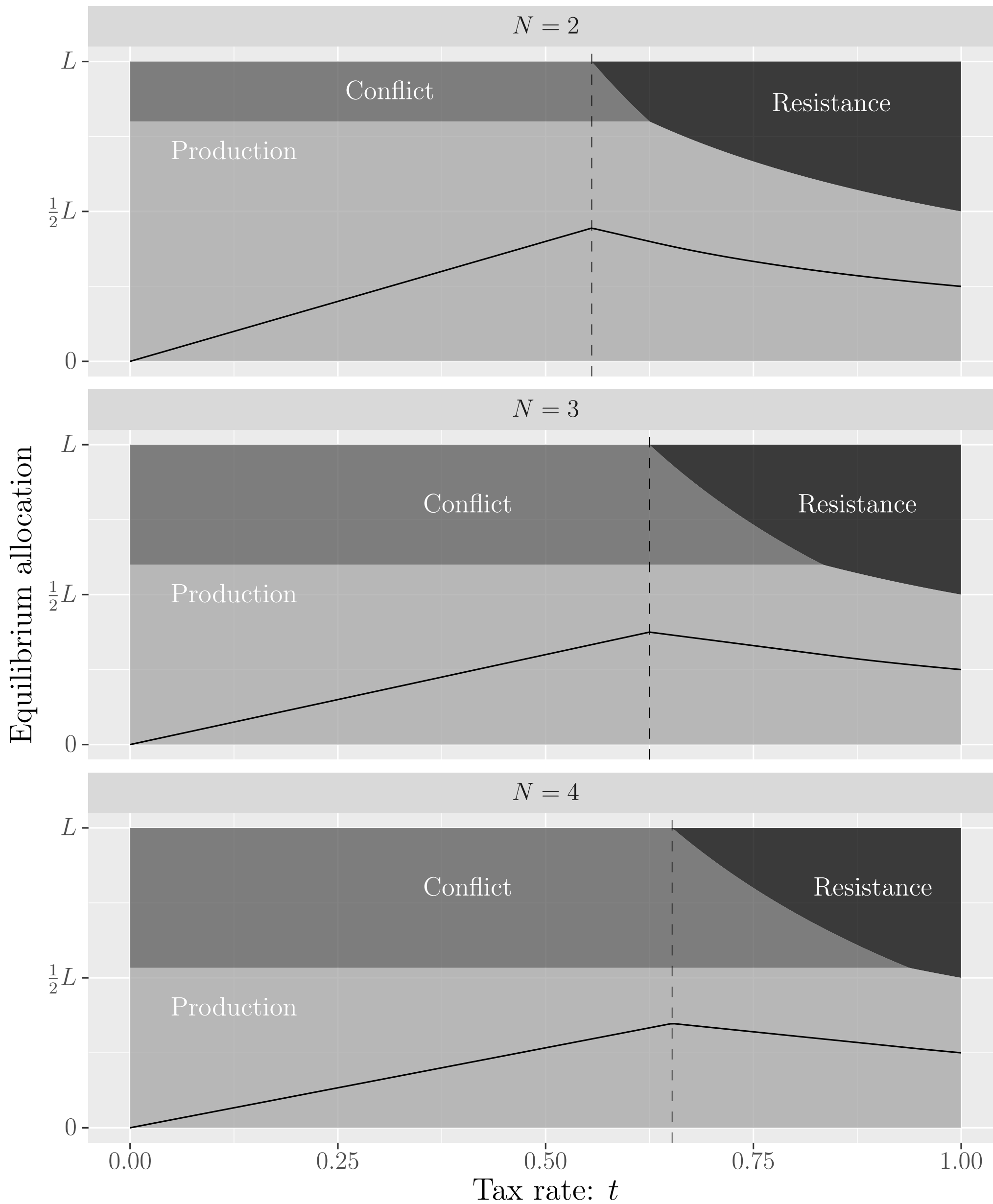

Figure 2. Equilibrium labor allocations under a labor-financed government as a function of the tax rate and the number of factions (Proposition 4). The solid curve is the government's payoff as a function of the tax rate, and the dashed line is the equilibrium tax rate.

Proposition 5. *If the government is labor-financed, there is an equilibrium in which the government chooses the greatest tax rate that engenders no resistance, $t = \hat{t}_0$. If g or ϕ is strictly log-concave, this is the unique equilibrium tax rate.*

To see why it is optimal for the government to avoid engendering resistance, consider a tax rate $t' > \hat{t}_0$ that does lead to resistance. This will result in an effective tax rate of $\tau(t', r) < t$, with some of the increase in resistance coming at the expense of production. It would be better for the government to make the announced tax rate equal the effective one, $t'' = \tau(t', r)$, and reap the gains of the additional production.[15]

At a glance, this result might appear to imply that the government benefits from internal disorder. It is evident from Figure 2 that the equilibrium tax rate corresponds to a high point for internal conflict. But this correlation is not causal. Where internal conflict is at its maximum, production is also at its maximum, and resistance is at its minimum. To analyze whether a labor-financed government benefits from social order or disorder, it is more apt to examine how its equilibrium utility varies with an exogenous shock to the structural conditions favoring social conflict.

4.2 Comparative Statics

Per Proposition 5, a labor-financed government will choose the highest tax rate at which there is no resistance, $\hat{t}_0$. This implies that equilibrium production will equal its baseline value, $\bar{P}_0$, defined in Proposition 4. The government's equilibrium payoff is thus $\hat{t}_0 \bar{P}_0$. I now examine the effects of fractionalization and internal conflict technology on taxation, production, and overall state revenue in equilibrium. Even though the divide-and-rule logic remains operative in the labor-financed case, insofar as social conflict reduces resistance, I now find that the predatory state now benefits from conditions that favor social order.

On the equilibrium path, all the factions' labor is divided between social conflict and

[15] Of course, if $t'' > \hat{t}_0$, there will still be positive resistance at t'' and the government will not recoup the full share. Nonetheless, as I show in the proof of this proposition in the Appendix, the increase in production by moving to a lower rate is great enough to be profitable for the government.

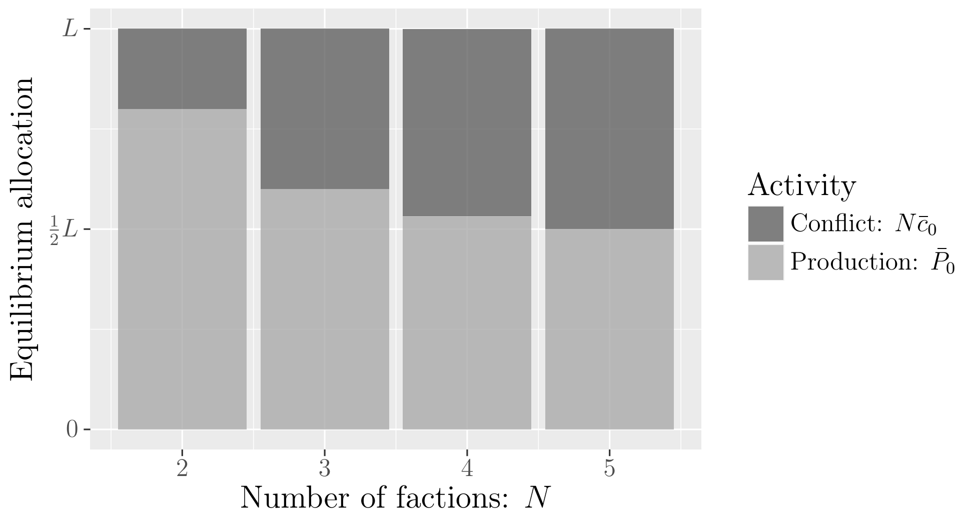

Figure 3. Division of labor on the equilibrium path under a labor-financed government (Proposition 4) as a function of the number of factions.

economic production. Anything that raises the labor devoted to internal conflict therefore reduces production. Following the same logic as in the capital-financed case, I find that greater fractionalization (higher N) increases equilibrium internal conflict. Figure 3 illustrates this result. Meanwhile, the effect of improvements in conflict technology again depend on the relative strengths of the incentive effect and the labor-saving effect. These in turn depend on the specific contest success function and the equilibrium level of internal conflict, as described by Equation 7 above.

Proposition 6. *Equilibrium production with a labor-financed government, $\bar{P}_0$, is strictly decreasing in the number of factions, N. It is locally decreasing in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

This result highlights the productivity logic—that even a predatory state benefits from social order, as this creates the conditions for the economic productivity that forms the state's revenue base. If the tax rate were exogenously fixed, this result would directly imply that a labor-financed government benefits from conditions that discourage internal conflict.

The picture looks different when we look at the equilibrium tax rate. Even though there is zero resistance on the equilibrium path, the threat of resistance remains important here, as it determines how highly the government can set taxes without engendering a backlash. And just as in the case of a capital-financed government, the threat to resist grows weaker as the marginal utility of internal conflict increases. The equilibrium tax rate, $\hat{t}_0$, is the point at which the marginal benefit of resistance equals that of internal conflict. Consequently, as structural conditions increasingly favor internal conflict, the state is able to extract a greater share of output in equilibrium.

Proposition 7. *The equilibrium tax rate of a labor-financed government,*

$$\hat{t}_0 = \frac{\pi^p}{\pi^p - \pi^r \bar{P}_0 g'(0)},$$

is strictly increasing in fractionalization, N. It strictly increases with a marginal increase in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.

Here we see the divide-and-rule logic in action. By discouraging collective resistance, greater internal conflict allows the government to extract a greater proportion of economic production. If the size of the pie were exogenous, as in the capital-financed case, this would imply the government benefits from internal division and disorder.

Proposition 6 and Proposition 7 identify countervailing forces on the predatory state's revenues: the productivity logic and the divide-and-rule logic are both operative for a labor-financed government. Structural conditions that promote internal conflict, such as fractionalization, result in the state extracting a larger share of a smaller pie. What is the net effect on government revenues? I find that the productivity logic is dominant: on balance, a revenue-maximizing ruler would benefit from structural conditions that favor internal peace rather than social conflict. Even for an essentially kleptocratic state, it is worse to rule a divided society than a unified one, as long as the state's revenue is based on the economic output of its subjects. This is evident from inspecting the government's revenue curve in Figure 2: even though the equilibrium tax rate increases with N, overall revenue decreases.

Proposition 8. *A labor-financed government's equilibrium payoff is strictly decreasing in the number of factions, N. It strictly decreases with a marginal increase in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

This result arises from two key features of the factions' incentives in the labor allocation subgame. The first is that the marginal benefit of production decreases with the tax rate. Letting (p^*, r^*, c^*) be an equilibrium of the subgame with $t = \hat{t}_0$, the marginal utility of production is

$$\pi^p \frac{\partial u_i(\hat{t}_0, p^*, r^*, c^*)}{\partial p_i} = \frac{\pi^p}{N}(1 - \hat{t}_0).$$

The second key feature of the factions' incentives is that the marginal benefit of resistance increases with the amount the government can extract through taxation. Specifically, the marginal benefit of resistance in equilibrium is

$$\pi^r \frac{\partial u_i(\hat{t}_0, p^*, r^*, c^*)}{\partial r_i} = \frac{-\pi^r g'(0)}{N} \hat{t}_0 \bar{P}_0,$$

where $g'(0) < 0$ represents how quickly the effective tax rate shrinks with a marginal increase in resistance.

Because the equilibrium tax rate pushes the factions just to the point where resistance would become profitable, in equilibrium the marginal benefits of production and resistance must be equal even though no resistance takes place. Importantly, while N and π^c reduce the incentive to divert labor from internal conflict into collective resistance, they do not directly affect the incentive to divert labor from economic production. The formal condition for production and resistance to have equal marginal benefits in equilibrium is

$$\pi^p(1 - \hat{t}_0) = -\pi^r g'(0) \hat{t}_0 \bar{P}_0.$$

Proposition 7 shows that fractionalization increases $\hat{t}_0$ and thereby decreases the marginal return to production (the left-hand side of the above expression). Therefore, in order to maintain the equality of marginal benefits, production $\bar{P}_0$ must decrease enough with fractionalization that the government's payoff, $\hat{t}_0\bar{P}_0$, also decreases. Fractionalization directly reduces the benefits of resistance, but it also reduces the benefits of production. Therefore, even as fractionalization increases, the factions' collective ability to threaten resistance remains strong enough to prevent the government from profiting.

The upshot of Proposition 8 is that social order increases the profitability of extraction from the population's labor. A labor-financed government is better off governing a society where structural conditions are favorable to social order. If we think of these conditions as at least partially endogenous, this implies that it is in the government's interest to promote social order if the costs of doing so are low enough. For example, the government might seek to divert labor away from looting and into productive economic activity by enforcing subjects' claims to property against appropriation by other factions.

4.3 Discussion

The analysis so far shows that the profitability of social conflict depends critically on the nature of the predatory state's revenue base. If the main source of value is the product of the population's labor, then fractionalization and internal disorder decrease the incentive to produce, ultimately reducing the profits of expropriation. But if the source of contention between the government and the population is some fixed pool of goods or resources, the opposite logic prevails. In this case, internal conflict does not reduce the value of the pie, but it does keep the population distracted from resistance against the government.

The major empirical implication of these results is that the relationship between social order and the policies and profitability of extractive governance is conditional on the nature of what is being extracted. All else equal, when a predatory state is financed by the population's labor, we should expect it to impose policies that reduce conflict and promote internal order, at least at the margins. By reducing appropriation among various ethnic groups or political factions, these policies raise the overall productivity of the governed population, which is profitable for the government. Moreover, we should expect extractive governance to be more profitable and stable in polities where structural conditions favor internal order. For example, labor-financed imperial regimes should be less willing to expend resources to gain or maintain control over internally divided societies.

The 17th-century experience of the Dutch East India Company in the Sulawesi region of present-day Indonesia is a clear example of a labor-financed predatory government benefiting from social order rather than division. When the Company arrived in Sulawesi in the 17th century, the region was beset with raiding and other violence, largely between neighboring rival villages (Schouten 1998). Instead of encouraging these conflicts, the Dutch sought to reduce looting and protect property rights. Warring parties regularly called on the Dutch to arbitrate their disputes, making the Company a kind of "stranger king" in Sulawesi society. And the Dutch found it in their interest to do so, as "any conflict quickly tended to interfere with the production and supply of the Minahasan rice which ... formed the Company's main economic interest in the area" (Henley 2004, 105). In other words, with its revenues in Sulawesi funded by agricultural output, the ruling class did not profit from social strife, and in fact sought to reduce it.

Similarly, on the frontiers of Latin Christendom prior to Frankish conquest in the late Middle Ages, "direct predation ... was not the occasional excess of the lawless but the prime activity of the free adult male population" (Bartlett 1993, 303). By establishing free villages and improving the protection of property rights, the immigrant Frankish nobility was able to profit by directing labor into productive activity rather than banditry. Later in Europe, the Ottoman empire maintained an advantage in trade and revenue in part by maintaining peace among the diverse religious and cultural groups that constituted its subjects (Burbank and Cooper 2010, 132–133). In yet another example, the Mughal empire that preceded British rule on the Indian subcontinent "defined their task as to keep an ordered balance between the different forces which constituted Indian society" (Wilson 2016, 17).

When the object of government extraction is a fixed source, such as a natural resource, we should expect these relationships to go the other direction. A capital-financed government will, at the margin, prefer policies that increase internal conflict and thereby reduce anti-government resistance, such as lax enforcement of competing groups' property rights. Colonies or occupations whose main objective is to secure control of some existing resource will be more successful when the population is more divided, or local conditions otherwise favor internal conflict.

5 Endogenous Inequality

The preceding analyses of government extraction have assumed that taxation affects all factions identically. In addition, in the model, all factions are *ex ante* identical in terms of their productivity and incentives. In this section, I briefly consider whether a labor-financed government might profit by creating inequality between groups, namely by taxing them at different rates. This extension is similar to the model of divide-and-rule politics in Acemoglu, Verdier and Robinson (2004); the most important difference is that the factions can engage in costly conflict to appropriate from each other. Because of the paradox of power identified by Hirshleifer (1991), unequal taxation ends up unprofitable—state revenues in equilibrium are the same as in the baseline game.

In the model with *asymmetric taxation*, the government is labor-financed and taxes each faction's production separately. To keep the analysis simple, I assume throughout the extension that $N = 2$. The government chooses a pair of tax rates, t_1 and t_2, where each $t_i \in [0, 1]$ as before. The factions then respond as before, by allocating their labor among production, resistance, and internal conflict, (p_i, r_i, c_i), subject to the budget constraint, Equation 1. I consider tax schemes such that $t_1 \geq t_2$; as the factions remain identical *ex ante*, this restriction is without loss of generality. The utility functions for the government and the factions are now

$$u_G(t,p,r,c) = \tau(t_1,r)p_1 + \tau(t_2,r)p_2,$$

$$u_i(t,p,r,c) = \omega_i(c)\left[\bar{\tau}(t_1,r)p_1 + \bar{\tau}(t_2,r)p_2\right].$$

If the government chooses the same tax rate for both groups, $t_1 = t_2$, then each player's utility is the same as in the original model with that rate. Throughout the analysis of this extension, I impose an additional technical condition on the function that translates c_i into effective strength in the internal conflict: I assume ϕ'/ϕ is convex.[16]

To analyze the extension, I first consider how the factions would respond to the choice of unequal tax rates. Naturally, as taxation reduces the marginal benefit of production, the faction that is taxed more produces less in equilibrium. The more highly taxed faction then shifts some of the labor it would have spent on economic production into resistance and internal conflict. This has the counterintuitive implication that the equilibrium payoff for the more-taxed faction is no less than that of the less-taxed faction. By reducing a group's incentive to produce, the government increases its incentive to appropriate from the other group, resulting in it taking home a disproportionate share of the total post-tax output. This result is reminiscent of the "paradox of power" characterized by Hirshleifer (1991), wherein seemingly weaker groups expend disproportionate effort on appropriation. The following proposition summarizes the equilibrium responses to unequal taxation.

Proposition 9. *In the game with asymmetric taxation, if the government chooses $t_1 > t_2$, then $p_1 \leq p_2$, $r_1 \geq r_2$, and $c_1 \geq c_2$ in any equilibrium of the subsequent labor allocation subgame.*

The factions' responses show why asymmetric taxation is ultimately unprofitable for a labor-financed government. There is obviously no profit to be made from the more highly

[16] The baseline assumptions imply that ϕ'/ϕ is positive and decreasing, so convexity is a natural restriction. The difference and ratio functional forms described above in footnote 14 both satisfy this condition.

taxed faction, as it reduces its production in response to the greater taxation. But as the more-taxed faction increases its appropriative efforts, the less-taxed faction also loses some of its incentive to engage in productive activity. The decrease in the less-taxed faction's incentive to resist does not make up the difference, as the marginal benefit of resistance remains a function of the government's overall payoff, just as in the baseline model. Ultimately, then, the government is no better off having the ability to set unequal tax rates across groups.

Proposition 10. *Asymmetric taxation does not raise the equilibrium payoff of a labor-financed government.*

This brief extension demonstrates that the earlier results for labor-financed extraction do not depend on the assumption of equal tax rates across factions. All else equal, a labor-financed government benefits from social order and has no incentive to create inequality where none exists before. What this extension does not answer is how asymmetric tax rates would interact with *ex ante* asymmetries in productivity or size among factions, a topic that is beyond the scope of the present analysis.

6 Conquest

The preceding analysis takes the identity of the ruler as fixed. I have shown that the relationship between social conflict and the profitability of rule depends on the type of economic product from which the ruler's rents derive. I now briefly consider the process of taking control, prior to the selection of the tax rate and subsequent division of society's labor. When an outside force seeks to usurp authority, is it more likely to succeed when the population is more divided?

Whereas internal fractionalization is only conditionally beneficial for predatory governance, namely when the government is capital-financed, it unconditionally increases the prospects of an outsider seeking to gain control in the first place. The intuition behind this result mirrors the logic of the finding above that fractionalization increases the tax rate the government can impose without engendering resistance (Proposition 7). Resistance against an outsider's attempt to take control is effectively a public good. As the number of factions increases, the incentive to provide this public good rather than to fend for oneself decreases (Olson 1965). Therefore, an outsider can more easily take control of a divided society than a unified one.

In the *conquest model*, a set of N factions compete with each other and with an outsider, denoted O, for the chance to be the government in the future. The incremental value of being the government is $v(N) > 0$, which may increase with N (when capital is the main source of revenue) or decrease (when labor is the main source of revenue). Each faction has L/N units of labor, which it may divide between two activities: $s_i \geq 0$, to prevent the outsider from taking over; and $d_i \geq 0$, to influence its own chance of becoming the government if the outsider fails. Each faction's budget constraint is[17]

$$s_i + d_i = \frac{L}{N}. \tag{8}$$

The success of the attempted takeover depends on how much the factions spend to combat the outsider. I assume the outsider's military strength is a fixed value, $\bar{s}_O > 0$, so the outsider is not a strategic player here. The assumption that the outsider's strength is exogenous is of course a simplification, but it is plausible in situations where the outsider marshals its forces before fully understanding the internal political situation—such as in Cortés's incursion into the Mexican mainland, and other early maritime colonial ventures. The probability that the outsider becomes the government is

$$\frac{\bar{s}_O}{\bar{s}_O + \chi(\sum_{i=1}^{N} s_i)}, \tag{9}$$

[17]The assumption of unit productivity for each activity is without loss of generality. The model here with functional forms $\chi(S) = \tilde{\chi}(\pi^s S)$ and $\psi(D) = \tilde{\psi}(\pi^d D)$ is isomorphic to a model with the common budget constraint $s_i/\pi^s + d_i/\pi^d = L/N$ and functional forms $\tilde{\chi}$ and $\tilde{\psi}$.

where $\chi : [0, L] \to \mathbb{R}_+$ represents the translation of society's labor into its strength against the outsider. In case the outsider fails, the probability that faction i becomes the government is

$$\frac{\psi(d_i)}{\sum_{j=1}^{N} \psi(d_j)}, \tag{10}$$

where $\psi : [0, L/N] \to \mathbb{R}_+$ represents the translation of an individual faction's labor into its proportional chance of success against other factions. As with the function ϕ in the original model, I assume χ and ψ are strictly increasing and log-concave.

The factions simultaneously choose how to allocate their labor, subject to the budget constraint (8). A faction's utility function is

$$u_i(s, d) = \frac{\psi(d_i)}{\sum_{j=1}^{N} \psi(d_j)} \times \frac{\chi(\sum_{j=1}^{N} s_j)}{\bar{s}_O + \chi(\sum_{j=1}^{N} s_j)} \times v(N), \tag{11}$$

where $s = (s_1, \ldots, s_N)$ and $d = (d_1, \ldots, d_N)$.

The strategic tradeoff for the factions here is analogous to the tradeoff between resistance and internal conflict in the baseline model. Critically, the relative marginal benefit of fighting the outsider declines as the number of factions increases. When the number of factions is large, any individual faction's chance of becoming the government if the outsider loses is small, which in turn reduces its incentive to contribute to the collective effort against the outsider. Consequently, as the following result states, the outsider is more likely to win the more divided the society is.

Proposition 11. *In the conquest model, the probability that the outsider wins is increasing in the number of factions, N.*

To be clear, unlike some of the earlier results, Proposition 11 does not address how fractionalization affects the outsider's overall welfare in equilibrium. In particular, if the outsider's ultimate objective is to extract the population's labor, there is no guarantee that the increase in the chance of winning due to greater fractionalization would offset the decrease in $v(N)$. Proposition 11, in combination with the earlier results, implies that a society that is easy to conquer may nevertheless be difficult to govern. Specifically, fractionalization benefits a labor-financed government in the conquest stage, but not in the governance stage. Only for a capital-financed government, which benefits from internal chaos even while governing, does fractionalization have the same effect on ease of conquest and the profitability of rule.

The conquest model captures how Cortés benefited from internal divisions in Aztec society. He was able to conquer with significantly less military support than would have been necessary otherwise, because the incumbent regime also had to contend with its internal enemies (Elliott 2007; Burkholder and Johnson 2015). The Dutch East India Company similarly exploited internal divisions when initially establishing its foothold in present-day Indonesia (Scammell 1989, 20), despite its later efforts to bring about more peaceful internal relations (see the discussion in subsection 4.3). In pure military competition, the political economy issues that arise in extraction of labor output—namely, the tradeoff between internal conflict and economic productivity—are sidelined. Only after establishing control does internal fractionalization become a potential problem for the predatory state.

7 Conclusion

I have characterized the political economy of predatory rule in a divided society. The main result is that the profitability of fractionalization and social division depends on the nature of the economic product the ruler wishes to extract. When it is the output of the population's labor, the ruler is better off when society is less deeply divided. The opposite is true for a capital-financed government that seeks to expropriate from an exogenously fixed source of value.

In this paper, I have assumed the shape of the society being governed is exogenously fixed. A natural direction for future research would be to examine how social conflict of the type modeled here interacts with territorial competition and the drawing of political boundaries.

The analysis here highlights how the *economic* value of territory cannot be disentangled from its *social* structure. According to the model, natural resource wealth becomes all the more valuable—and thus more likely to be the source of costly conflict—when the local society is politically fractionalized. The opposite would be true, however, for territory that is primarily valued for its agricultural productivity, as was the case in various colonial conflicts. In addition to their implications for the study of territorial conflict, these are important issues for political economy models of state size and endogenous border formation (e.g., Alesina and Spolaore 2005; Acharya and Lee 2018).

To focus on how fractionalization and the technology of internal conflict affect a predatory state's payoffs, I have assumed away any *ex ante* inequality in size or productivity between social groups. Of course, inequality itself might increase social conflict (Esteban and Ray 2011). In light of the analysis here, we thus might expect capital-financed predatory states to prefer inequality and labor-financed ones to prefer equality. However, if productivity inequalities between factions lead to specialization in particular activities, then different strategic tradeoffs might emerge than in the symmetric case. For example, in a labor-financed state, if one group specializes in resistance while the other specializes in production, increased conflict between them may benefit a predatory government on net. An important question for future research is how these inequalities might interact with the political-economic incentives studied here, and to what extent they alter the baseline relationship between social conflict and the profits of predatory rule.

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A Appendix to "Social Conflict and the Predatory State"

Contents

A.1 Additional Notation

Throughout the appendix, let $\Phi(c) = \sum_i \phi(c_i)$. We have $\log \omega_i(c) = \log \phi(c_i) - \log \Phi(c)$ and thus

$$\begin{aligned}
\frac{\partial \log \omega_i(c)}{\partial c_i} &= \frac{\phi'(c_i)}{\phi(c_i)} - \frac{\phi'(c_i)}{\Phi(c)} \\
&= \frac{\phi'(c_i)}{\phi(c_i)} \left(1 - \frac{\phi(c_i)}{\Phi(c)}\right) \\
&= \hat{\phi}'(c_i)(1 - \omega_i(c)),
\end{aligned}$$

where $\hat{\phi} = \log \phi$. Because ϕ is strictly increasing and log-concave, $\hat{\phi}' > 0$ and $\hat{\phi}'' \leq 0$.

A.2 Equilibrium Existence and Uniqueness

For the existence and uniqueness results, I consider a more general version of the model presented in the text. I allow groups to be asymmetric in their size and productivities, which entails generalizing each faction i's budget constraint (1) to

$$\frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} = L_i, \tag{12}$$

where $\pi_i^p, \pi_i^r, \pi_i^c, L_i > 0$. I assume a labor-financed government throughout the existence and uniqueness results, as this is the more difficult case; all claims here also apply to a capital-financed state in which $f(p) = X$. In addition, the results here do not depend on Assumption 1.

Let $\Gamma(t)$ denote the subgame that follows the government's selection of t, in which the factions simultaneously decide how to allocate their labor. Let $\sigma_i = (p_i, r_i, c_i)$ be a strategy for faction i in the subgame, and let

$$\Sigma_i = \left\{ (p_i, r_i, c_i) \;\middle|\; \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} = L_i \right\}$$

denote the strategy space. Let $\sigma = (\sigma_1, \ldots, \sigma_N)$ and $\Sigma = \bigtimes_{i=1}^N \Sigma_i$.

I begin by proving that a Nash equilibrium exists in each subgame. The task is complicated by the potential discontinuity of the factions' payoffs, namely at $c = 0$ when $\phi(0) = 0$. I rely on Reny's (1999) conditions for the existence of pure strategy equilibria in a discontinuous game. The key condition is *better-reply security*—informally, that at least one player can assure a strict benefit by deviating from any non-equilibrium strategy profile, even if the other players make slight deviations.

Lemma 1. $\Gamma(t)$ *is better-reply secure.*

Proof. Let $U^t : \Sigma \to \mathbb{R}_+^N$ be the vector payoff function for the factions in $\Gamma(t)$, so that $U^t(\sigma) = (u_1(t, \sigma), \ldots, u_N(t, \sigma))$. Take any convergent sequence in the graph of U^t, call it $(\sigma^k, U^t(\sigma^k)) \to (\sigma^*, U^*)$, such that σ^* is not an equilibrium of $\Gamma(t)$. Because production and the effective tax rate are continuous in (p, r), we have

$$U_i^* = w_i^* \times \bar{\tau}(t, r^*) \times f(p^*)$$

for each i, where $w_i^* \geq 0$ and $\sum_{i=1}^N w_i^* = 1$. I must show there is a player i who can secure a payoff $\bar{U}_i > U_i^*$ at σ^*; i.e., there exists $\bar{\sigma}_i \in \Sigma_i$ such that $u_i(t, \bar{\sigma}_i, \sigma'_{-i}) \geq \bar{U}_i$ for all σ'_{-i} in a neighborhood of σ^*_{-i} (Reny 1999, 1032).

If $N = 1$ or $\Phi(c^*) > 0$, then U^t is continuous in a neighborhood of σ^*, so the conclusion is immediate. If $\bar{\tau}(t, r^*) \times f(p^*) = 0$, then each $U_i^* = 0$ and each faction can assure a strictly greater payoff by deviating to a strategy with positive production, resistance, and conflict. For the remaining cases, suppose $N > 1$, $\bar{\tau}(t, r^*) \times f(p^*) > 0$, and $\Phi(c^*) = 0$, the latter of which implies $c^* = 0$ and $\phi(0) = 0$. Since $N > 1$, there is a faction i such that $w_i^* < 1$. Take any $\epsilon \in (0, (1 - w_i^*)/2)$ and any $\delta_1 > 0$ such that

$$\bar{\tau}(t, r') \times f(p') \geq (w_i^* + 2\epsilon) \times \bar{\tau}(t, r^*) \times f(p^*)$$

for all σ' in a δ_1-neighborhood of σ^*. Since $w_i^* + 2\epsilon < 1$ and $\bar{\tau}(t, r) \times f(p)$ is continuous in (p, r), such a δ_1 exists. Then let $\bar{\sigma}_i = (\bar{p}_i, \bar{r}_i, \bar{c}_i)$ be any strategy in a δ_1-neighborhood of σ_i^* such that $\bar{c}_i > 0$. Because $c^*_{-i} = 0$ and ϕ is continuous, there exists $\delta_2 > 0$ such that

$$\omega_i(\bar{c}_i, c'_{-i}) = \frac{\phi(\bar{c}_i)}{\phi(\bar{c}_i) + \sum_{j \in \mathcal{N} \setminus \{i\}} \phi(c'_j)} \geq \frac{w_i^* + \epsilon}{w_i^* + 2\epsilon}$$

for all σ'_{-i} in a δ_2-neighborhood of σ^*_{-i}. Therefore, for all σ'_{-i} in a $\min\{\delta_1, \delta_2\}$-neighborhood of σ^*_{-i}, we have

$$u_i(t, \bar{\sigma}_i, \sigma'_{-i}) \geq (w_i^* + \epsilon) \times \bar{\tau}(t, r^*) \times f(p^*) > U_i^*,$$

establishing the claim. $\square$

The other main condition for equilibrium existence is that each faction's utility function be quasiconcave in its own actions. I prove this by showing that the logarithm of a faction's utility function is concave in its actions.

Lemma 2. $\Gamma(t)$ *is log-concave.*

Proof. Take any (p, r, c) such that $u_i(t, p, r, c) > 0$, and let $P = \sum_j p_j$ and $R = \sum_j r_j$. First, assume $\sum_{j \neq i} \phi(c_j) > 0$, so that u_i is continuously differentiable in (p_i, r_i, c_i). We have

$$\begin{aligned}
\frac{\partial \log u_i(t,p,r,c)}{\partial p_i} &= \frac{1}{P}, \\
\frac{\partial \log u_i(t,p,r,c)}{\partial r_i} &= \frac{-tg'(R)}{1 - tg(R)}, \\
\frac{\partial \log u_i(t,p,r,c)}{\partial c_i} &= \hat{\phi}'(c_i)(1 - \omega_i(c)),
\end{aligned}$$

and therefore

$$\begin{aligned}
\frac{\partial^2 \log u_i(t,p,r,c)}{\partial p_i^2} &= \frac{-1}{P^2} < 0, \\
\frac{\partial^2 \log u_i(t,p,r,c)}{\partial r_i^2} &= \frac{-tg''(R)(1 - tg(R)) - (tg'(R))^2}{(1 - tg(R))^2} \leq 0, \\
\frac{\partial^2 \log u_i(t,p,r,c)}{\partial c_i^2} &= \hat{\phi}''(c_i)(1 - \omega_i(c)) - \hat{\phi}'(c_i)\frac{\partial \omega_i(c)}{\partial c_i} \leq 0, \\
\frac{\partial^2 \log u_i(t,p,r,c)}{\partial p_i \partial r_i} &= \frac{\partial^2 \log u_i(t,p,r,c)}{\partial p_i \partial c_i} = \frac{\partial^2 \log u_i(t,p,r,c)}{\partial r_i \partial c_i} = 0,
\end{aligned}$$

so $\log u_i$ is concave in (p_i, r_i, c_i). By the same token, $\bar{\tau}(t, r) \times f(p)$ is log-concave in (p, r).

Now assume $\sum_{j \neq i} \phi(c_j) = 0$. Take any (p_i', r_i', c_i') such that $u_i(t, p', r', c') > 0$, where $(p', r', c') = ((p_i', p_{-i}), (r_i', r_{-i}), (c_i', c_{-i}))$. Take any $\alpha \in [0, 1]$, and let $(p^\alpha, r^\alpha, c^\alpha) = \alpha(p, r, c) + (1 - \alpha)(p', r', c')$. If $c_i = c_i' = 0$, then $\omega_i(c^\alpha) = \omega_i(c) = \omega_i(c') = 1/N$ and thus

$$\begin{aligned}
\log u_i(t, p^\alpha, r^\alpha, c^\alpha) &= \log \frac{1}{N} + \log \bar{\tau}(t, r^\alpha) + \log f(p^\alpha) \\
&\geq \log \frac{1}{N} + \alpha \left(\log \bar{\tau}(t, r) + \log f(p)\right) + (1 - \alpha)\left(\log \bar{\tau}(t, r') + \log f(p')\right) \\
&= \alpha \log u_i(t, p, r, c) + (1 - \alpha) \log u_i(t, p', r', c'),
\end{aligned}$$

where the inequality follows from the log-concavity of $\bar{\tau}(t, r) \times f(p)$ in (p, r). If $c_i > 0$ and $c_i' = 0$, then $\omega_i(c^\alpha) = \omega_i(c) = 1$, $\omega_i(c') = 1/N$, and thus

$$\begin{aligned}
\log u_i(t, p^\alpha, r^\alpha, c^\alpha) &= \log \bar{\tau}(t, r^\alpha) + \log f(p^\alpha) \\
&\geq \alpha \left(\log \bar{\tau}(t, r) + \log f(p)\right) + (1 - \alpha)\left(\log \frac{1}{N} + \log \bar{\tau}(t, r') + \log f(p')\right)
\end{aligned}$$

$$= \alpha \log u_i(t, p, r, c) + (1 - \alpha) \log u_i(t, p', r', c').$$

The same argument holds in case $c_i = 0$ and $c_i' > 0$. It is easy to see that the same conclusion holds if $c_i > 0$ and $c_i' > 0$, in which case $\omega_i(c^\alpha) = \omega_i(c) = \omega_i(c') = 1$. Therefore, $\log u_i$ is concave in (p_i, r_i, c_i). □

Equilibrium existence follows immediately from the two preceding lemmas.

Proposition 12. $\Gamma(t)$ *has a pure strategy equilibrium.*

Proof. The strategy space Σ is compact, each payoff function u_i is bounded on Σ, and $\Gamma(t)$ is better-reply secure (Lemma 1) and quasiconcave (Lemma 2). Therefore, a pure strategy equilibrium exists (Reny 1999, Theorem 3.1). □

I now turn to the question of uniqueness. I show that although $\Gamma(t)$ may have multiple equilibria, these equilibria are identical in terms of three essential characteristics: total production, $\sum_i p_i$; total resistance, $\sum_i r_i$; and the vector of individual expenditures on internal conflict, c.

To prove essential uniqueness, I must characterize the equilibrium more fully than I have up to this point. The following result rules out equilibria in which (1) a faction's share in the internal competition is zero or (2) a faction could raise its share to one by an infinitesimal change in strategy.

Lemma 3. *If* $N > 1$, *then each* $\phi(c_i) > 0$ *in any equilibrium of* $\Gamma(t)$.

Proof. Assume $N > 1$, and let (p, r, c) be a strategy profile of $\Gamma(t)$ in which $c_i = 0$ for some $i \in \mathcal{N}$. The claim holds trivially if $\phi(0) > 0$, so assume $\phi(0) = 0$. If $\Phi(c) > 0$ or $\bar{\tau}(t, r) \times f(p) = 0$, then $u_i(t, p, r, c) = 0$. But i could ensure a strictly positive payoff with any strategy that allocated nonzero labor to production, resistance, and conflict, so (p, r, c) is not an equilibrium. Conversely, suppose $\Phi(c) = 0$, which implies $c_j = 0$ for all $j \in \mathcal{N}$, and $\bar{\tau}(t, r) \times f(p) > 0$. Then $u_i(t, p, r, c) = (\bar{\tau}(t, r) \times f(p))/N$. But i could obtain a payoff arbitrarily close to $\bar{\tau}(t, r) \times f(p)$ by diverting an infinitesimal amount of labor away from production or resistance and into internal conflict, so (p, r, c) is not an equilibrium. □

This result is important because it implies utility functions are continuously differentiable in the neighborhood of any equilibrium. Equilibria can therefore be characterized in terms of first-order conditions.

Lemma 4. (p', r', c') *is an equilibrium of* $\Gamma(t)$ *if and only if, for each* $i \in \mathcal{N}$,

$$p_i' \left(\pi_i^p \frac{\partial \log f(p')}{\partial p_i} - \mu_i \right) = 0, \tag{13}$$

$$r_i' \left(\pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} - \mu_i \right) = 0, \tag{14}$$

$$c_i' \left(\pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} - \mu_i \right) = 0, \tag{15}$$

$$\frac{p_i'}{\pi_i^p} + \frac{r_i'}{\pi_i^r} + \frac{c_i'}{\pi_i^c} - L_i = 0, \tag{16}$$

where

$$\mu_i = \max \left\{ \pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i}, \pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} \right\}.$$

Proof. In equilibrium, each faction's strategy must solve the constrained maximization problem

$$\begin{aligned} \max_{p_i, r_i, c_i} \quad & \log u_i(t, p, r, c) \\ \text{s.t.} \quad & \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} - L_i = 0, \\ & p_i \geq 0, r_i \geq 0, c_i \geq 0. \end{aligned}$$

It follows from Lemma 3 that each u_i is C^1 in (p_i, r_i, c_i) in a neighborhood of any equilibrium. This allows use of the Karush–Kuhn–Tucker conditions to characterize solutions of the above problem. The "only if" direction holds because (13)–(16) are the first-order conditions for the problem and the linearity constraint qualification holds. The "if" direction holds because $\log u_i$ is concave in (p_i, r_i, c_i), per Lemma 2. □

A weak welfare optimality result follows almost immediately from this equilibrium characterization. If (p', r', c') is an equilibrium of $\Gamma(t)$, then there is no other equilibrium (p'', r'', c'') such that $c'' = c'$ and $\bar{\tau}(t, r'') \times f(p'') > \bar{\tau}(t, r') \times f(p')$. In other words, taking as fixed the factions' allocations toward internal conflict, there is no inefficient misallocation of labor between production and resistance.

Corollary 1. *If (p', r', c') is an equilibrium of $\Gamma(t)$, then (p', r') solves*

$$\begin{aligned} \max_{p,r} \quad & \log \bar{\tau}(t, r) + \log f(p) & \\ s.t. \quad & \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} = L_i - \frac{c_i'}{\pi_i^c}, & i = 1, \ldots, N, \\ & p_i \geq 0, r_i \geq 0, & i = 1, \ldots, N. \end{aligned}$$

Proof. This is a C^1 concave maximization problem with linear constraints, so the Karush–Kuhn–Tucker first-order conditions are necessary and sufficient for a solution. The result then follows from Lemma 4. □

I next prove that if post-tax output is weakly greater in one equilibrium of $\Gamma(t)$ than another, then each of the two individual components (production and the factions' total share) is weakly greater. The proof relies on the fact that if $c_i' \leq c_i''$ and $\omega_i(c') \leq \omega_i(c'')$, then

$$\frac{\partial \log \omega_i(c')}{\partial c_i} = \hat{\phi}'(c_i')(1 - \omega_i(c')) \geq \hat{\phi}'(c_i'')(1 - \omega_i(c'')) = \frac{\partial \log \omega_i(c'')}{\partial c_i}.$$

If in addition $\omega_i(c') < \omega_i(c'')$, the inequality is strict.

Lemma 5. *If (p', r', c') and (p'', r'', c'') are equilibria of $\Gamma(t)$ such that $\bar{\tau}(t, r') \times f(p') \geq \bar{\tau}(t, r'') \times f(p'')$, then $\bar{\tau}(t, r') \geq \bar{\tau}(t, r'')$ and $f(p') \geq f(p'')$.*

Proof. Suppose the claim of the lemma does not hold, so there exist equilibria such that $\bar{\tau}(t, r') \times f(p') \geq \bar{\tau}(t, r'') \times f(p'')$ but $\bar{\tau}(t, r') < \bar{\tau}(t, r'')$. Together, these inequalities imply $f(p') > f(p'')$. (The proof in case $\bar{\tau}(t, r') > \bar{\tau}(t, r'')$ and $f(p') < f(p'')$ is analogous.)

I will first establish that $p_i' > 0$ implies $r_i'' = 0$. Per Lemma 4 and the log-concavity of f and $\bar{\tau}$, $p_i' > 0$ implies

$$\pi_i^p \frac{\partial \log f(p'')}{\partial p_i} > \pi_i^p \frac{\partial \log f(p')}{\partial p_i} \geq \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} > \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i}.$$

Therefore, again by Lemma 4, $r_i'' = 0$.

Next, I establish that $\Phi(c'') > \Phi(c')$. Since $f(p') > f(p'')$, there is a faction $i \in \mathcal{N}$ such that $p_i' > p_i''$. As this implies $r_i'' = 0$, the budget constraint gives $c_i'' > c_i'$. If $\Phi(c'') \leq \Phi(c')$, then $\omega_i(c'') > \omega_i(c')$ and thus by Lemma 4

$$\pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} > \pi_i^c \frac{\partial \log \omega_i(c'')}{\partial c_i} \geq \pi_i^p \frac{\partial \log f(p'')}{\partial p_i} > \pi_i^p \frac{\partial \log f(p')}{\partial p_i}.$$

But this implies $p_i' = 0$, a contradiction. Therefore, $\Phi(c'') > \Phi(c')$.

Using these intermediate results, I can now establish the main claim by contradiction. Since $\bar{\tau}(t, r'') > \bar{\tau}(t, r')$, there is a faction $j \in \mathcal{N}$ such that $r_j'' > r_j'$. This implies $p_j' = 0$, so the budget constraint gives $c_j'' < c_j'$. Since $\Phi(c'') > \Phi(c')$, this in turn gives $\omega_j(c'') < \omega_j(c')$ and thus

$$\pi_j^c \frac{\partial \log \omega_j(c'')}{\partial c_j} > \pi_j^c \frac{\partial \log \omega_j(c')}{\partial c_j} \geq \pi_j^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j} > \pi_j^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j}.$$

But this implies $r_j'' = 0$, a contradiction. $\square$

I can now state and prove the essential uniqueness of the equilibrium of each labor allocation subgame.

Proposition 13. *If (p', r', c') and (p'', r'', c'') are equilibria of $\Gamma(t)$, then $f(p') = f(p'')$, $\bar{\tau}(t, r') = \bar{\tau}(t, r'')$, and $c' = c''$.*

Proof. First I prove that $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$. Suppose not, so that, without loss of generality, $\bar{\tau}(t, r') \times f(p') > \bar{\tau}(t, r'') \times f(p'')$. Then Lemma 5 implies $\bar{\tau}(t, r') \geq \bar{\tau}(t, r'')$ and $f(p') \geq f(p'')$, at least one strictly so, and thus

$$\max \left\{ \pi_i^p \frac{\partial \log f(p'')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i} \right\} \geq \max \left\{ \pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} \right\}$$

for all $i \in \mathcal{N}$, strictly so for some $j \in \mathcal{N}$. Since $\bar{\tau}(t, r'') \times f(p'') < \bar{\tau}(t, r') \times f(p')$, it follows from Corollary 1 that the set $\mathcal{N}^+ = \{i \in \mathcal{N} \mid c_i'' > c_i'\}$ is nonempty. For any $i \in \mathcal{N}^+$ such that $\omega_i(c'') > \omega_i(c')$,

$$\begin{aligned}
\pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} &> \pi_i^c \frac{\partial \log \omega_i(c'')}{\partial c_i} \\
&\geq \max\left\{\pi_i^p \frac{\partial \log f(p'')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i}\right\} \\
&\geq \max\left\{\pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i}\right\}.
\end{aligned}$$

But this implies $p_i' = r_i' = 0$, contradicting $c_i'' > c_i'$. So $\omega_i(c'') \leq \omega_i(c')$ for all $i \in \mathcal{N}^+$. Since $\mathcal{N}^+$ is nonempty and the conflict shares are increasing in c_i and sum to one, this can hold only if $\mathcal{N}^+ = \mathcal{N}$ and $\omega_i(c'') = \omega_i(c')$ for all $i \in \mathcal{N}$. This implies

$$\begin{aligned}
\pi_j^c \frac{\partial \log \omega_j(c')}{\partial c_j} &\geq \pi_j^c \frac{\partial \log \omega_j(c'')}{\partial c_j} \\
&\geq \max\left\{\pi_j^p \frac{\partial \log f(p'')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j}\right\} \\
&> \max\left\{\pi_j^p \frac{\partial \log f(p')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j}\right\},
\end{aligned}$$

which in turn implies $p_j' = r_j' = 0$, contradicting $c_j'' > c_j'$. I conclude that $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$ and thus, by Lemma 5, $\bar{\tau}(t, r') = \bar{\tau}(t, r'')$ and $f(p') = f(p'')$.

Next, I prove that $c' = c''$. Suppose not, so $c' \neq c''$. Without loss of generality, suppose $\Phi(c') \geq \Phi(c'')$. Since $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$ yet $c' \neq c''$, by Corollary 1 there exists $i \in \mathcal{N}$ such that $c_i' > c_i''$ and $j \in \mathcal{N}$ such that $c_j' < c_j''$. It follows from $\Phi(c') \geq \Phi(c'')$ that $\omega_j(c') < \omega_j(c'')$ and therefore

$$\begin{aligned}
\pi_j^c \frac{\partial \log \omega_j(c')}{\partial c_j} &> \pi_j^c \frac{\partial \log \omega_j(c'')}{\partial c_j} \\
&\geq \max\left\{\pi_j^p \frac{\partial \log f(p'')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j}\right\} \\
&= \max\left\{\pi_j^p \frac{\partial \log f(p')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j}\right\}.
\end{aligned}$$

But this implies $p_j' = r_j' = 0$, contradicting $c_j'' > c_j'$. □

Proposition 13 allows me to write the equilibrium values of total production, total resistance, and individual conflict allocations as functions of the tax rate. For each $t \in [0, 1]$, let $P^*(t) = P$ if and only if there is an equilibrium (p, r, c) of $\Gamma(t)$ such that $\sum_i p_i = P$. Let the functions $R^*(t)$ and $c^*(t)$, the latter of which is vector-valued, be defined analogously.

The only remaining step to prove the existence of an equilibrium in the full game is to show that an optimal tax rate exists. An important consequence of Proposition 13 is that the optimal tax rate (if one exists) does not depend on the equilibrium that is selected in each labor allocation subgame, since the government's payoff depends only on total production and resistance. The main step toward proving the existence of an optimal tax rate is to show that total production and resistance are continuous in t.

Lemma 6. *P^*, R^*, and c^* are continuous.*

Proof. Define the equilibrium correspondence $E : [0,1] \rightrightarrows \Sigma$ by

$$E(t) = \{(p, r, c) \mid (p, r, c) \text{ is an equilibrium of } \Gamma(t)\}.$$

Standard arguments (e.g., Fudenberg and Tirole 1991, 30–32) imply that E has a closed graph.[18] This in turn implies E is upper hemicontinuous, as its codomain, Σ, is compact. Let $F : \Sigma \to \mathbb{R}_+^{N+2}$ be the function defined by $F(p, r, c) = (\sum_i p_i, \sum_i r_i, c)$. Since F is continuous as a function, it is upper hemicontinuous as a correspondence. Then we can write the functions in the lemma as the composition of F and E:

$$(P^*(t), R^*(t), c^*(t)) = \{F(p, r, c) \mid (p, r, c) \in E(t)\} = (F \circ E)(t).$$

As the composition of upper hemicontinuous correspondences, (P^*, R^*, c^*) is upper hemicontinuous (Aliprantis and Border 2006, Theorem 17.23). Then, as an upper hemicontinuous correspondence that is single-valued (per Proposition 13), (P^*, R^*, c^*) is continuous as a function. □

Continuity of total production and resistance in the tax rate imply that the government's payoff is continuous in the tax rate, so an equilibrium exists.

Proposition 14. *There is a pure strategy equilibrium.*

Proof. For each labor allocation subgame $\Gamma(t)$, let $\sigma^*(t)$ be a pure strategy equilibrium of $\Gamma(t)$. Proposition 12 guarantees the existence of these equilibria. By Proposition 13, the government's payoff from any $t \in [0,1]$ is

$$u_G(t, \sigma^*(t)) = t \times g(R^*(t)) \times P^*(t).$$

This expression is continuous in t, per Lemma 6, and therefore attains its maximum on the compact interval $[0, 1]$. A maximizer t^* exists, and the pure strategy profile $(t^*, (\sigma^*(t))_{t\in[0,1]})$ is an equilibrium. □

A.3 Symmetric Game Properties

In the remainder of the appendix, I consider the special symmetric case of the model discussed in the text, in which each $\pi_i^p = \pi^p$, $\pi_i^r = \pi^r$, $\pi_i^c = \pi^c$, and $L_i = L/N$. Let $\Gamma(t)$ denote the labor allocation subgame with tax rate t in the model with a labor-financed government, and let $\Gamma_X(t)$ denote the same with a capital-financed government.

An important initial result for the symmetric case is that in every equilibrium of every labor allocation subgame, every faction spends the same amount on internal conflict.

[18] The only complication in applying the usual argument is that the model is discontinuous at $c = 0$ in case $\phi(0) = 0$. However, by the same arguments as in the proof of Lemma 1, if $\phi(0) = 0$ there cannot be a sequence $(t^k, (p^k, r^k, c^k))$ in the graph of E such that $c^k \to 0$.

Lemma 7. *If the game is symmetric and (p, r, c) is an equilibrium of $\Gamma(t)$ or $\Gamma_X(t)$, then $c_i = c_j$ for all $i, j \in \mathcal{N}$.*

Proof. Because the game is symmetric, there exists an equilibrium (p', r', c') in which $(p'_i, r'_i, c'_i) = (p_j, r_j, c_j)$, $(p'_j, r'_j, c'_j) = (p_i, r_i, c_i)$, and $(p'_k, r'_k, c'_k) = (p_k, r_k, c_k)$ for all $k \in \mathcal{N} \setminus \{i, j\}$. Proposition 13 then implies $c_i = c_j$. $\square$

This means we can characterize an equilibrium of the labor allocation subgame in terms of just three variables: total production, total resistance, and the (common across factions) individual allocation to internal conflict. Using the characterization result of Lemma 4, we can identify an equilibrium as a solution to some subset of the following system of equations derived from the first-order conditions. Each Q^{xy} represents marginal equality of the returns to x and y, while Q^b is the budget constraint. I write these as functions of t as well as the exogenous parameters $\pi = (\pi^p, \pi^r, \pi^c, L, N)$ to allow for comparative statics via implicit differentiation:

$$Q^{pr}(P, R, C; t, \pi) = \pi^p(1 - tg(R)) + \pi^r tPg'(R) = 0, \tag{17}$$

$$Q^{pc}(P, R, C; t, \pi) = \frac{\pi^p}{P} - \frac{N-1}{N}\pi^c\hat{\phi}'(C) = 0, \tag{18}$$

$$Q^{rc}(P, R, C; t, \pi) = \frac{\pi^r tg'(R)}{1 - tg(R)} + \frac{N-1}{N}\pi^c\hat{\phi}'(C) = 0, \tag{19}$$

$$Q^b(P, R, C; t, \pi) = L - \frac{P}{\pi^p} - \frac{R}{\pi^r} - \frac{NC}{\pi^c} = 0. \tag{20}$$

The condition (19) is redundant when the government is labor-financed and (17) and (18) both hold, but it is useful for characterizing equilibrium with a capital-financed government.

A.4 Proof of Proposition 1

The quantities defined in Proposition 1 are as follows. $(\tilde{R}_X(t), \tilde{c}_X(t))$ is the solution to the system

$$Q^{rc}(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi) = \frac{\pi^r tg'(\tilde{R}_X(t))}{1 - tg(\tilde{R}_X(t))} + \frac{N-1}{N}\pi^c\hat{\phi}'(\tilde{c}_X(t)) = 0, \tag{21}$$

$$Q^b(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi) = L - \frac{\tilde{R}_X(t)}{\pi^r} - \frac{N\tilde{c}_X(t)}{\pi^c} = 0. \tag{22}$$

The cutpoint tax rates are

$$\hat{t}_0^X = \frac{\eta\pi^c\hat{\phi}'(\pi^c L/N)}{\eta\pi^c\hat{\phi}'(\pi^c L/N) - \pi^r g'(0)}, \tag{23}$$

$$\hat{t}_1^X = \frac{\eta\pi^c\hat{\phi}'(0)}{\eta\pi^c g(\pi^r L)\hat{\phi}'(0) - \pi^r g'(\pi^r L)}, \tag{24}$$

where $\eta = (N-1)/N$. Observe that

$$\begin{aligned}
\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} &= \eta\pi^c\hat{\phi}'(\pi^c L/N) = \pi^c \frac{\partial \log \omega_i((\pi^c L/N)\mathbf{1}_N)}{\partial c_i}, \\
\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1^X, (\pi^r L/N)\mathbf{1}_N)}{\partial r_i} &= \eta\pi^c\hat{\phi}'(0) \qquad\quad = \pi^c \frac{\partial \log \omega_i(0)}{\partial c_i}
\end{aligned}$$

for each $i \in \mathcal{N}$, where $\mathbf{1}_N$ is an N-vector of 1s.

Proposition 1. *If the government is capital-financed, every labor allocation subgame has a unique equilibrium. There exists a tax rate $\hat{t}_0^X \in (0,1)$ such that each $r_i = 0$ in equilibrium if and only if $t \leq \hat{t}_0^X$. There exists $\hat{t}_1^X > \hat{t}_0^X$ such that each $c_i = 0$ in equilibrium if and only if $t \geq \hat{t}_1^X$. For $t \in (\hat{t}_0^X, \hat{t}_1^X)$, in equilibrium each $r_i = \tilde{R}_X(t)/N > 0$ (strictly increasing in t) and each $c_i = \tilde{c}_X(t) > 0$ (strictly decreasing).*

Proof. $\Gamma_X(t)$ has an equilibrium (Proposition 12), and there exists $c_X^*(t)$ such that each $c_i = c_X^*(t)$ in all of its equilibria (Proposition 13 and Lemma 7). The budget constraint then implies each $r_i = \pi^r(L/N - c_X^*(t)/\pi^c)$ in every equilibrium of $\Gamma_X(t)$, so the equilibrium is unique.

Let (r, c) be the equilibrium of $\Gamma_X(t)$. To simplify expressions in what follows, let $C = c_i = c_X^*(t)$ and $R = \sum_i r_i = \pi^r(L - NC/\pi^c)$. If $t \leq \hat{t}_0^X$ and $R > 0$, then the first-order conditions give

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} = \pi^c \frac{\partial \log \omega_i((\pi^c L/N)\mathbf{1}_N)}{\partial c_i} \leq \pi^c \frac{\partial \log \omega_i(c)}{\partial c_i}$$

for each $i \in \mathcal{N}$. But this implies each $r_i = 0$, contradicting $R > 0$. Therefore, if $t \leq \hat{t}_0^X$, then $R = 0$. Similarly, if $t > \hat{t}_0^X$ and $R = 0$, then each $c_i = \pi^c L/N$ and thus

$$\pi^c \frac{\partial \log \omega_i(c)}{\partial c_i} = \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i}.$$

But this implies each $c_i = 0$, a contradiction. Therefore, if $t > \hat{t}_0^X$, then $R > 0$. The proof that $C > 0$ if and only if $t < \hat{t}_1^X$ is analogous.

For $t \in (\hat{t}_0^X, \hat{t}_1^X)$, the first-order conditions imply that R and C solve $Q^{rc}(0, R, C; t, \pi) = Q^b(0, R, C; t, \pi) = 0$; therefore, $R = \tilde{R}_X(t)$ and $C = \tilde{c}_X(t)$. To reduce clutter in what follows, I omit the evaluation point $(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi)$ from all partial derivative expressions. The Jacobian of the system defining $(\tilde{R}_X(t), \tilde{c}_X(t))$ is

$$\begin{aligned}
\mathbf{J}_X &= \begin{bmatrix} \partial Q^{rc}/\partial R & \partial Q^{rc}/\partial C \\ \partial Q^b/\partial R & \partial Q^b/\partial C \end{bmatrix} \\
&= \begin{bmatrix} \pi^r t \dfrac{g''(\tilde{R}_X(t)) - t g(\tilde{R}_X(t))^2 \hat{g}''(\tilde{R}_X(t))}{(1 - t g(\tilde{R}_X(t)))^2} & \eta\pi^c\hat{\phi}''(\tilde{c}_X(t)) \\ -1/\pi^r & -N/\pi^c \end{bmatrix}
\end{aligned}$$

where $\eta = (N-1)/N$ and $\hat{g} = \log g$. Its determinant is

$$|\mathbf{J}_X| = \frac{\pi^c}{\pi^r}\left(\eta\hat{\phi}''(\tilde{c}_X(t)) - N\pi^r t\frac{g''(\tilde{R}_X(t)) - tg(\tilde{R}_X(t))^2\hat{g}''(\tilde{R}_X(t))}{(1-tg(\tilde{R}_X(t)))^2}\right) < 0.$$

By the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{d\tilde{R}_X(t)}{dt} &= \frac{\begin{vmatrix} -\partial Q^{rc}/\partial t & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial t & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X|} \\
&= \frac{\begin{vmatrix} -\pi^r g'(\tilde{R}_X(t))/(1-tg(\tilde{R}_X(t)))^2 & \eta\pi^c\hat{\phi}''(\tilde{c}_X(t)) \\ 0 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X|} \\
&= \frac{N\pi^r g'(\tilde{R}_X(t))}{\pi^c(1-tg(\tilde{R}_X(t)))^2|\mathbf{J}_X|} \\
&> 0,
\end{aligned}$$

as claimed. The budget constraint then implies $d\tilde{c}_X(t)/dt < 0$, as claimed. □

A.5 Proof of Propositions 2 and 3

Before proving the results, I separately derive the comparative statics of $\hat{t}_0^X$ and $\tilde{R}_X(t)$ in N and π^c.

Lemma 8. *The lower cutpoint $\hat{t}_0^X$ is strictly increasing in the number of factions, N. It is locally decreasing in the effectiveness of conflict, π^c, if and only if*

$$\hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N}\hat{\phi}''\left(\frac{\pi^c L}{N}\right) \geq 0.$$

Proof. Recall that

$$\hat{t}_0^X = \frac{((N-1)/N)\pi^c\hat{\phi}'(\pi^c L/N)}{((N-1)/N)\pi^c\hat{\phi}'(\pi^c L/N) - \pi^r g'(0)}.$$

Observe that $g'(0) < 0$, $(N-1)/N$ is strictly increasing in N, and $\hat{\phi}'(\pi^c L/N)$ is weakly increasing in N. Therefore, $\hat{t}_0^X$ is strictly increasing in N. Notice that

$$\frac{\partial}{\partial\pi^c}\left[\pi^c\hat{\phi}'\left(\frac{\pi^c L}{N}\right)\right] = \hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N}\hat{\phi}''\left(\frac{\pi^c L}{N}\right),$$

so $\hat{t}_0^X$ is locally increasing in π^c if and only if the above expression is positive. □

Lemma 9. *For fixed $t \in (\hat{t}_0^X, \hat{t}_1^X)$, total resistance, $\tilde{R}_X(t)$, is strictly decreasing in the number* *of factions, N. It is locally decreasing in the effectiveness of conflict, π^c, if and only if*

$$\hat{\phi}'(\tilde{c}_X(t)) + \tilde{c}_X(t)\hat{\phi}''(\tilde{c}_X(t)) \geq 0.$$

Proof. I will treat N as if it were continuous in order to obtain comparative statics by implicit differentiation. Throughout the proof I write $\tilde{R}_X(t)$ and $\tilde{c}_X(t)$ as functions of (N, π^c).

I first consider comparative statics in N. To reduce clutter in what follows, I omit the evaluation point $(0, \tilde{R}_X(t; N, \pi^c), \tilde{c}_X(t; N, \pi^c); t, \pi)$ from all partial derivative expressions. By the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial N} &= \frac{\begin{vmatrix} -\partial Q^{rc}/\partial N & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial N & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|} \\
&= \frac{\begin{vmatrix} -\pi^c \hat{\phi}'(\tilde{c}_X(t; N, \pi^c))/N^2 & ((N-1)/N)\pi^c \hat{\phi}''(\tilde{c}_X(t; N, \pi^c)) \\ \tilde{c}_X(t; N, \pi^c)/\pi^c & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|} \\
&= \frac{\hat{\phi}'(\tilde{c}_X(t; N, \pi^c)) - (N-1)\tilde{c}_X(t; N, \pi^c)\hat{\phi}''(\tilde{c}_X(t; N, \pi^c))}{N|\mathbf{J}_X(t; N, \pi^c)|} \\
&< 0,
\end{aligned}$$

as claimed, where $|\mathbf{J}_X(t; N, \pi^c)| < 0$ is defined as in the proof of Proposition 1.

I now consider comparative statics in π^c. Again by the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial \pi^c} &= \frac{\begin{vmatrix} -\partial Q^{rc}/\partial \pi^c & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial \pi^c & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|} \\
&= \frac{\begin{vmatrix} -((N-1)/N)\hat{\phi}'(\tilde{c}_X(t; N, \pi^c)) & ((N-1)/N)\pi^c \hat{\phi}''(\tilde{c}_X(t; N, \pi^c)) \\ -N\tilde{c}_X(t; N, \pi^c)/(\pi^c)^2 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|} \\
&= \frac{(N-1)\left(\hat{\phi}'(\tilde{c}_X(t; N, \pi^c)) + \tilde{c}_X(t; N, \pi^c)\hat{\phi}''(\tilde{c}_X(t; N, \pi^c))\right)}{\pi^c|\mathbf{J}_X(t; N, \pi^c)|}.
\end{aligned}$$

Therefore, $\partial \tilde{R}_X(t; N, \pi^c)/\partial \pi^c \leq 0$ if and only if

$$\hat{\phi}'(\tilde{c}_X(t; N, \pi^c)) + \tilde{c}_X(t; N, \pi^c)\hat{\phi}''(\tilde{c}_X(t; N, \pi^c)) \geq 0,$$

as claimed. □

The propositions, which I prove together, follow mainly from these lemmas.

Proposition 2. *A capital-financed government's equilibrium payoff is increasing in the number of factions, N.*

Proposition 3. *If there is a unique equilibrium tax rate t^*, the government's equilibrium payoff is locally increasing in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect (i.e., Equation 7 holds) at the corresponding equilibrium level of internal conflict.*

Proof. Throughout the proof I write various equilibrium quantities, including the cutpoints $\hat{t}_0^X$ and $\hat{t}_1^X$, as functions of (N, π^c). Let the government's equilibrium payoff as a function of these parameters be

$$u_G^*(N, \pi^c) = \max_{t \in [0,1]} t \times g(R^*(t; N, \pi^c)) \times X.$$

I begin with the comparative statics on N. First, suppose $t = \hat{t}_0^X(N', \pi^c)$ is an equilibrium for all N' in a neighborhood of N.[19] Then $u_G^*(N', \pi^c) = \hat{t}_0^X(N', \pi^c) \times X$ in a neighborhood of N, which by Lemma 8 is strictly increasing in N'. Next, suppose there is an equilibrium with $t \in (\hat{t}_0^X(N', \pi^c), \hat{t}_1^X(N', \pi^c))$ for all N' in a neighborhood of N. Then, by the envelope theorem,

$$\frac{\partial u_G^*(N, \pi^c)}{\partial N} = g'(\tilde{R}_X(t; N, \pi^c)) \frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial N} \times X > 0,$$

where the inequality follows from Lemma 9. Finally, suppose $\hat{t}_1^X(N', \pi^c) < 1$ and $t = 1$ is an equilibrium for all N' in a neighborhood of N. Then $u_G^*(N, \pi^c) = g(\pi^r L) \times X$ is locally constant in N, and thus weakly increasing.

I now consider the comparative statics on π^c. First, suppose $t = \hat{t}_0^X(N, \pi^{c\prime})$ is an equilibrium for all $\pi^{c\prime}$ in a neighborhood of π^c. Then $u_G^*(N, \pi^{c\prime}) = \hat{t}_0^X(N, \pi^{c\prime}) \times X$ in a neighborhood of $\pi^{c\prime}$, which by Lemma 8 is locally increasing at π^c if and only if

$$\hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N} \hat{\phi}''\left(\frac{\pi^c L}{N}\right) \geq 0.$$

Next, suppose there is a unique equilibrium with $t \in (\hat{t}_0^X(N, \pi^{c\prime}), \hat{t}_1^X(N, \pi^{c\prime}))$ for all $\pi^{c\prime}$ in a neighborhood of π^c. Then, by the envelope theorem,

$$\frac{\partial u_G^*(N, \pi^c)}{\partial \pi^c} = g'(\tilde{R}_X(t; N, \pi^c)) \frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial \pi^c} \times X.$$

This is positive if and only if $\hat{\phi}'(\tilde{c}_X(t)) + \tilde{c}_X(t)\hat{\phi}''(\tilde{c}_X(t)) \geq 0$, per Lemma 9. Finally, suppose $\hat{t}_1^X(N, \pi^{c\prime}) < 1$ and $t = 1$ is an equilibrium for all $\pi^{c\prime}$ in a neighborhood of π^c. Then $u_G^*(N, \pi^c) = g(\pi^r L) \times X$ is locally constant in π^c, and thus weakly increasing. □

I also prove the claim in footnote 14.

Lemma 10. *Let $\theta, \lambda > 0$. If $\phi(C) = \theta \exp(\lambda C)$, then the incentive effect outweighs the labor-saving effect for all $C \geq 0$. If $\phi(C) = \theta C^\lambda$, then the incentive and labor-saving effects are exactly offsetting for all $C > 0$.*

[19] There cannot be an equilibrium tax rate $t < \hat{t}_0^X$ or (if $\hat{t}_1^X < 1$) $t \in [\hat{t}_1^X, 1)$, as R^* is constant on $[0, \hat{t}_0^X]$ and on $[\hat{t}_1^X, 1]$.

Proof. First consider the difference contest success function, $\phi(C) = \theta \exp(\lambda C)$. Then $\hat{\phi}(C) = \log\theta + \lambda C$, $\hat{\phi}'(C) = \lambda$, and $\hat{\phi}''(C) = 0$ for all $C \geq 0$. Therefore,

$$\hat{\phi}'(C) + C\hat{\phi}''(C) = \lambda > 0.$$

Now consider the ratio contest success function, $\phi(C) = \theta C^{\lambda}$. Then $\hat{\phi}(C) = \log\theta + \lambda\log C$, $\hat{\phi}'(C) = \lambda/C$, and $\hat{\phi}''(C) = -\lambda/C^2$. Therefore,

$$\hat{\phi}'(C) + C\hat{\phi}''(C) = \frac{\lambda}{C} + C\left(\frac{-\lambda}{C^2}\right) = 0. \qquad \square$$

A.6 Proof of Proposition 4

The quantities defined in Proposition 4 are as follows. $(\bar{P}_0, \bar{c}_0)$ is the solution to the system

$$Q^{pc}(\bar{P}_0, 0, \bar{c}_0; t, \pi) = \frac{\pi^p}{\bar{P}_0} - \frac{N-1}{N}\pi^c\hat{\phi}'(\bar{c}_0) = 0, \tag{25}$$

$$Q^{b}(\bar{P}_0, 0, \bar{c}_0; t, \pi) = L - \frac{\bar{P}_0}{\pi^p} - \frac{N\bar{c}_0}{\pi^c} = 0. \tag{26}$$

$(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t))$ is the solution to the system

$$Q^{pr}(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = \pi^p(1 - tg(\tilde{R}_1(t)) + \pi^r t\tilde{P}_1(t)g'(\tilde{R}_1(t)) = 0, \tag{27}$$

$$Q^{pc}(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = \frac{\pi^p}{\tilde{P}_1(t)} - \frac{N-1}{N}\pi^c\hat{\phi}'(\tilde{c}_1(t)) = 0, \tag{28}$$

$$Q^{b}(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = L - \frac{\tilde{P}_1(t)}{\pi^p} - \frac{\tilde{R}_1(t)}{\pi^r} - \frac{N\tilde{c}_1(t)}{\pi^c} = 0. \tag{29}$$

$(\tilde{P}_2(t), \tilde{R}_2(t))$ is the solution to the system

$$Q^{pr}(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi) = \pi^p(1 - tg(\tilde{R}_2(t)) + \pi^r t\tilde{P}_2(t)g'(\tilde{R}_2(t)) = 0, \tag{30}$$

$$Q^{b}(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi) = L - \frac{\tilde{P}_2(t)}{\pi^p} - \frac{\tilde{R}_2(t)}{\pi^r} = 0. \tag{31}$$

The first cutpoint tax rate is

$$\hat{t}_0 = \frac{\pi^p}{\pi^p - \pi^r\bar{P}_0 g'(0)}. \tag{32}$$

Lemma 11 below shows that $\bar{P}_0 > 0$ and therefore, since $g'(0) < 0$, that $\hat{t}_0 < 1$. The second cutpoint tax rate is

$$\hat{t}_1 = \frac{\pi^p}{\pi^p g(\bar{R}_1) - \pi^r\bar{P}_1 g'(\bar{R}_1)}, \tag{33}$$

where

$$\bar{P}_1 = \frac{N}{N-1}\frac{\pi^p}{\pi^c}\frac{1}{\hat{\phi}'(0)}, \tag{34}$$

$$\bar{R}_1 = \pi^r \left(L - \frac{\bar{P}_1}{\pi^p} \right). \tag{35}$$

The next three lemmas give conditions on the tax rate under which there is positive production, resistance, and internal conflict in the equilibrium of the labor allocation subgame. Jointly, these lemmas constitute the bulk of the proof of Proposition 4. The proofs rely on the following equalities:

$$\begin{aligned} \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, 0)}{\partial r_i} &= -\pi^r \frac{\hat{t}_0 g'(0)}{1 - \hat{t}_0} &&= \frac{\pi^p}{\bar{P}_0}, \\ \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, (\bar{R}_1/N)\mathbf{1}_N)}{\partial r_i} &= -\pi^r \frac{\hat{t}_1 g'(\bar{R}_1)}{1 - \hat{t}_1 g(\bar{R}_1)} &&= \frac{\pi^p}{\bar{P}_1} \end{aligned}$$

for all $i \in \mathcal{N}$.

Lemma 11. *If the game is symmetric, Assumption 1 holds, and (p, r, c) is an equilibrium of $\Gamma(t)$, then $0 < \sum_i p_i \leq \bar{P}_0 < \pi^p L$.*

Proof. Assumption 1 implies

$$Q^{pc}(\pi^p L, 0, 0; 0, \pi) = \frac{1}{L} - \frac{N-1}{N} \pi^c \hat{\phi}'(0) < 0.$$

Since Q^{pc} is decreasing in P and weakly increasing in C, this gives $\bar{P}_0 < \pi^p L$.

Let $P = \sum_i p_i$, and suppose $P > \bar{P}_0$. The budget constraint and Lemma 7 then give $c_i = C < \bar{c}_0$ for each $i \in \mathcal{N}$. But then we have

$$\pi^c \frac{\partial \log \omega_i(c)}{\partial c_i} \geq \frac{N-1}{N} \pi^c \hat{\phi}'(\bar{c}_0) = \frac{\pi^p}{\bar{P}_0} > \pi^p \frac{\partial \log f(p)}{\partial p_i}$$

for each $i \in \mathcal{N}$. By Lemma 4, this implies each $p_i = 0$, a contradiction. Therefore, $P \leq \bar{P}_0$.

Finally, since $P = 0$ implies each $u_i(t, p, r, c) = 0$, but any faction can assure itself a positive payoff with any $(p_i, r_i, c_i) \gg 0$, in equilibrium $P > 0$. □

Lemma 12. *If the game is symmetric, Assumption 1 holds, and (p, r, c) is an equilibrium of $\Gamma(t)$, then $\sum_i r_i > 0$ if and only if $t > \hat{t}_0$.*

Proof. Let $P = \sum_i p_i$ and $R = \sum_i r_i$. To prove the "if" direction, suppose $t > \hat{t}_0$ and $R = 0$. Since $P \leq \bar{P}_0$, this implies each $c_i = C > 0$; the first-order conditions of Lemma 4 then give $P = \bar{P}_0$ and $C = \bar{c}_0$. It follows that

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, r)}{\partial r_i} = \frac{\pi^p}{\bar{P}_0} = \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

This implies each $p_i = 0$, a contradiction.

To prove the "only if" direction, suppose $t \leq \hat{t}_0$ and $R > 0$. For each $i \in \mathcal{N}$,

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, 0)}{\partial r_i} = \frac{\pi^p}{\bar{P}_0} \leq \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

This implies each $r_i = 0$, a contradiction. □

Lemma 13. *If the game is symmetric and Assumption 1 holds, then $\hat{t}_1 > \hat{t}_0$. If, in addition, (p, r, c) is an equilibrium of $\Gamma(t)$, then each $c_i > 0$ if and only if $t < \hat{t}_1$.*

Proof. To prove that $\hat{t}_1 > \hat{t}_0$, note that

$$\bar{P}_1 = \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(0)} \leq \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(\bar{c}_0)} = \bar{P}_0$$

by log-concavity of ϕ. This implies $\bar{R}_1 > 0$, so $g(\bar{R}_1) < g(0) = 1$ and $g'(0) \leq g'(\bar{R}_1) < 0$. Therefore,

$$\pi^p - \pi^r \bar{P}_0 g'(0) > \pi^p g(\bar{R}_1) - \pi^r \bar{P}_1 g'(\bar{R}_1) > 0,$$

which implies $\hat{t}_1 > \hat{t}_0$.

Let $P = \sum_i p_i$ and $R = \sum_i r_i$. To prove the "if" direction of the second statement, suppose $t \geq \hat{t}_1$ and some $c_i > 0$. By Lemma 7, $c_j = c_i = C > 0$ for each $j \in \mathcal{N}$. Since $P > 0$ by Lemma 11, the first-order conditions give

$$P = \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(C)} \geq \bar{P}_1.$$

The budget constraint then gives $R < \bar{R}_1$ and thus

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, (\bar{R}_1/N)\mathbf{1}_N)}{\partial r_i} = \frac{\pi^p}{\bar{P}_1} \geq \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

But this implies each $p_i = 0$, a contradiction.

To prove the "only if" direction, suppose $t < \hat{t}_1$ and each $c_i = 0$. The first-order conditions then give $P \leq \bar{P}_1$, so $R \geq \bar{R}_1 > 0$ by the budget constraint. This in turn gives

$$\pi^p \frac{\partial \log f(p)}{\partial p_i} = \frac{\pi^p}{P} \geq \frac{\pi^p}{\bar{P}_1} \geq \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i}$$

for each $i \in \mathcal{N}$. But this implies each $r_i = 0$, a contradiction. □

The last thing we need to prove the propositions is how the labor allocations change with the tax rate when $t > \hat{t}_0$.

Lemma 14. *Let the game be symmetric and Assumption 1 hold. For all $t \in (\hat{t}_0, \hat{t}_1)$,*

$$\frac{d\tilde{P}_1(t)}{dt} = \frac{-(N-1)\pi^p \pi^c \hat{\phi}''(\tilde{c}_1(t))}{N \pi^r t \Delta_1(t)} \leq 0, \tag{36}$$

$$\frac{d\tilde{R}_1(t)}{dt} = \frac{-\pi^p\left(N\pi^p/\pi^c\tilde{P}_1(t)^2 - (N-1)\pi^c\hat{\phi}''(\tilde{c}_1(t))/N\pi^p\right)}{t\Delta_1(t)} > 0, \tag{37}$$

$$\frac{d\tilde{c}_1(t)}{dt} = \frac{(\pi^p)^2}{\pi^r t\tilde{P}_1(t)^2\Delta_1(t)} < 0, \tag{38}$$

where

$$\begin{aligned}\Delta_1(t) &= \left(\pi^p tg'(\tilde{R}_1(t)) - \pi^r t\tilde{P}_1(t)g''(\tilde{R}_1(t))\right)\left(\frac{N\pi^p}{\pi^c\tilde{P}_1(t)^2} - \frac{N-1}{N}\frac{\pi^c}{\pi^p}\hat{\phi}''(\tilde{c}_1(t))\right)\\ &\quad - \frac{N-1}{N}\pi^c tg'(\tilde{R}_1(t))\hat{\phi}''(\tilde{c}_1(t))\\ &< 0.\end{aligned} \tag{39}$$

For all $t > \hat{t}_1$,

$$\frac{d\tilde{P}_2(t)}{dt} = \frac{-\pi^p}{\pi^r t\Delta_2(t)} < 0, \tag{40}$$

$$\frac{d\tilde{R}_2(t)}{dt} = \frac{1}{t\Delta_2(t)} > 0, \tag{41}$$

where

$$\Delta_2(t) = \frac{\pi^r}{\pi^p}t\tilde{P}_2(t)g''(\tilde{R}_2(t)) - 2tg'(\tilde{R}_2(t)) > 0. \tag{42}$$

Proof. Throughout the proof, let $\eta = (N-1)/N$.

First consider $t \in (\hat{t}_0, \hat{t}_1)$. To reduce clutter in what follows, I omit the evaluation point $(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi)$ from all partial derivative expressions. The Jacobian of the system of equations that defines $(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t))$ is

$$\begin{aligned}\mathbf{J}_1(t) &= \begin{bmatrix}\partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R & \partial Q^{pr}/\partial C\\ \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial R & \partial Q^{pc}/\partial C\\ \partial Q^{b}/\partial P & \partial Q^{b}/\partial R & \partial Q^{b}/\partial C\end{bmatrix}\\ &= \begin{bmatrix}\pi^r tg'(\tilde{R}_1(t)) & \pi^r t\tilde{P}_1(t)g''(\tilde{R}_1(t)) - \pi^p tg'(\tilde{R}_1(t)) & 0\\ -\pi^p/\tilde{P}_1(t)^2 & 0 & -\eta\pi^c\hat{\phi}''(\tilde{c}_1(t))\\ -1/\pi^p & -1/\pi^r & -N/\pi^c\end{bmatrix}.\end{aligned}$$

It is easy to verify that $|\mathbf{J}_1(t)| = \Delta_1(t) < 0$. Notice that

$$\begin{aligned}\frac{\partial Q^{pr}}{\partial t} &= \pi^r\tilde{P}_1(t)g'(\tilde{R}_1(t)) - \pi^p g(\tilde{R}_1(t))\\ &= \pi^r\left(-\frac{\pi^p(1 - tg(\tilde{R}_1(t)))}{\pi^r tg'(\tilde{R}_1(t))}\right)g'(\tilde{R}_1(t)) - \pi^p g(\tilde{R}_1(t))\\ &= -\frac{\pi^p}{t}.\end{aligned}$$

Then, by the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{d\tilde{P}_1(t)}{dt} &= \frac{\begin{vmatrix} -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial R & \partial Q^{pr}/\partial C \\ -\partial Q^{pc}/\partial t & \partial Q^{pc}/\partial R & \partial Q^{pc}/\partial C \\ -\partial Q^{b}/\partial t & \partial Q^{b}/\partial R & \partial Q^{b}/\partial C \end{vmatrix}}{|\mathbf{J}_1(t)|} \\
&= \frac{-\eta\pi^p\pi^c\hat{\phi}''(\tilde{c}_1(t))}{\pi^r t\Delta_1(t)} \\
&\leq 0, \\
\frac{d\tilde{R}_1(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial C \\ \partial Q^{pc}/\partial P & -\partial Q^{pc}/\partial t & \partial Q^{pc}/\partial C \\ \partial Q^{b}/\partial P & -\partial Q^{b}/\partial t & \partial Q^{b}/\partial C \end{vmatrix}}{|\mathbf{J}_1(t)|} \\
&= \frac{-\pi^p\left(N\pi^p/\pi^c\tilde{P}_1(t)^2 - \eta\pi^c\hat{\phi}''(\tilde{c}_1(t))/\pi^p\right)}{t\Delta_1(t)} \\
&> 0, \\
\frac{d\tilde{c}_1(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R & -\partial Q^{pr}/\partial t \\ \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial R & -\partial Q^{pc}/\partial t \\ \partial Q^{b}/\partial P & \partial Q^{b}/\partial R & -\partial Q^{b}/\partial t \end{vmatrix}}{|\mathbf{J}_1(t)|} \\
&= \frac{(\pi^p)^2}{\pi^r t\tilde{P}_1(t)^2\Delta_1(t)} \\
&< 0,
\end{aligned}$$

as claimed.

Now consider $t > \hat{t}_1$. Again to reduce clutter in what follows, I omit the evaluation point $(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi)$ from all partial derivative expressions. The Jacobian of the system of equations that defines $(\tilde{P}_2(t), \tilde{R}_2(t))$ is

$$\begin{aligned}
\mathbf{J}_2(t) &= \begin{bmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R \\ \partial Q^{b}/\partial P & \partial Q^{b}/\partial R \end{bmatrix} \\
&= \begin{bmatrix} \pi^r t g'(\tilde{R}_2(t)) & \pi^r t\tilde{P}_2(t)g''(\tilde{R}_2(t)) - \pi^p t g'(\tilde{R}_2(t)) \\ -1/\pi^p & -1/\pi^r \end{bmatrix}.
\end{aligned}$$

It is easy to verify that $|\mathbf{J}_2(t)| = \Delta_2(t) > 0$. As before, $\partial Q^{pr}/\partial t = -\pi^p/t$. So by the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{d\tilde{P}_2(t)}{dt} &= \frac{\begin{vmatrix} -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial R \\ -\partial Q^{b}/\partial t & \partial Q^{b}/\partial R \end{vmatrix}}{|\mathbf{J}_2(t)|} \\
&= \frac{-\pi^p}{\pi^r t\Delta_2(t)}
\end{aligned}$$

$$
\begin{aligned}
&< 0, \\
\frac{d\tilde{R}_2(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & -\partial Q^{pr}/\partial t \\ \partial Q^{b}/\partial P & -\partial Q^{b}/\partial t \end{vmatrix}}{|\mathbf{J}_2(t)|} \\
&= \frac{1}{t\Delta_2(t)} \\
&> 0,
\end{aligned}
$$

as claimed. □

The proof of Proposition 4 follows almost immediately from these lemmas.

Proposition 4. *Assume the government is labor-financed. There exist tax rates $\hat{t}_0 \in (0,1)$ and $\hat{t}_1 > \hat{t}_0$ such that in every equilibrium of the labor allocation with tax rate t:*

- *If $t \leq \hat{t}_0$, then each $p_i = \bar{P}_0/N > 0$, each $r_i = 0$, and each $c_i = \bar{c}_0 > 0$.*
- *If $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = \tilde{P}_1(t) > 0$ (weakly decreasing in t), $\sum_i r_i = \tilde{R}_1(t) > 0$ (strictly increasing), and each $c_i = \tilde{c}_1(t) > 0$ (strictly decreasing).*
- *If $t \geq \hat{t}_1$, then $\sum_i p_i = \tilde{P}_2(t) > 0$ (strictly decreasing in t), $\sum_i r_i = \tilde{R}_2(t) > 0$ (strictly increasing), and each $c_i = 0$.*

Proof. For fixed t, every equilibrium of $\Gamma(t)$ has the same total production, total resistance, and individual conflict allocations, per Proposition 13. Consider any $t \in [0,1]$ and let (p,r,c) be an equilibrium of $\Gamma(t)$.

If $t \leq \hat{t}_0$, then $\sum_i p_i = P > 0$, $\sum_i r_i = 0$, and each $c_i = C > 0$ by Lemmas 11–13. The first-order conditions (Lemma 4) imply that P and C solve $Q^{pc}(P,0,C;t,\pi) = Q^b(P,0,C;t,\pi) = 0$; therefore, $P = \bar{P}_0$ and $C = \bar{c}_0$. Since each $r_i = 0$, each $p_i = \pi^p(L/N - \bar{c}_0/\pi^c) = \bar{P}_0/N$, so the equilibrium is unique.

Similarly, if $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = P > 0$, $\sum_i r_i = R > 0$, and each $c_i = C > 0$ by Lemmas 11–13. The first-order conditions then imply that these solve the system (17)–(20); therefore, $P = \tilde{P}_1(t)$, $R = \tilde{R}_1(t)$, and $C = \tilde{c}_1(t)$. The comparative statics on $\tilde{P}_1$, $\tilde{R}_1$, and $\tilde{c}_1$ follow from Lemma 14.

Finally, if $t \geq \hat{t}_1$, then $\sum_i p_i = P > 0$, $\sum_i r_i = R > 0$, and each $c_i = 0$ by Lemmas 11–13. The first-order conditions then imply that P and R solve $Q^{pr}(P,R,0;t,\pi) = Q^b(P,R,0;t,\pi) = 0$; therefore, $P = \tilde{P}_2(t)$ and $R = \tilde{R}_2(t)$. The comparative statics on $\tilde{P}_2$ and $\tilde{R}_2$ follow from Lemma 14. □

A.7 Proof of Proposition 5

Proposition 5. *If the government is labor-financed, there is an equilibrium in which the government chooses the greatest tax rate that engenders no resistance, $t = \hat{t}_0$. If g or ϕ is strictly log-concave, this is the unique equilibrium tax rate.*

Proof. As in the proof of Lemma 14, let $\eta = (N-1)/N$.

For each $t \in [0,1]$, fix an equilibrium $(p(t), r(t), c(t))$ of $\Gamma(t)$. By Propositions 4 and 13, the government's induced utility function is

$$u_G^*(t) = u_G(t, p(t), r(t), c(t)) = \begin{cases} t\bar{P}_0 & t \le \hat{t}_0, \\ tg(\tilde{R}_1(t))\tilde{P}_1(t) & \hat{t}_0 < t < \hat{t}_1, \\ tg(\tilde{R}_2(t))\tilde{P}_2(t) & t \ge \hat{t}_1. \end{cases}$$

It is immediate from the above expression that $u_G^*(t) < u_G^*(\hat{t}_0)$ for all $t < \hat{t}_0$.

Now consider $t \in (\hat{t}_0, \hat{t}_1)$. By Lemma 14,

$$\begin{aligned} \frac{du_G^*(t)}{dt} &= g(\tilde{R}_1(t))\tilde{P}_1(t) + tg'(\tilde{R}_1(t))\frac{d\tilde{R}_1(t)}{dt}\tilde{P}_1(t) + tg(\tilde{R}_1(t))\frac{d\tilde{P}_1(t)}{dt} \\ &= g(\tilde{R}_1(t))\tilde{P}_1(t) - \frac{\pi^p g'(\tilde{R}_1(t))\tilde{P}_1(t)\left(N\pi^p/\pi^c\tilde{P}_1(t)^2 - \eta\pi^c\hat{\phi}''(\tilde{c}_1(t))/\pi^p\right)}{\Delta_1(t)} \\ &\quad - \frac{\eta\pi^p\pi^c g(\tilde{R}_1(t))\hat{\phi}''(\tilde{c}_1(t))}{\pi^r\Delta_1(t)}, \end{aligned}$$

where $\Delta_1(t)$ is defined by (39). To reduce clutter in what follows, let $\tilde{P} = \tilde{P}_1(t)$, $\tilde{R} = \tilde{R}_1(t)$, and $\tilde{c} = \tilde{c}_1(t)$. Since $\Delta_1(t) < 0$, the sign of the above expression is the same as that of

$$\begin{aligned} &g'(\tilde{R})\tilde{P}\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) + \frac{\eta\pi^p\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})}{\pi^r} - g(\tilde{R})\tilde{P}\Delta_1(t) \\ &\quad = \tilde{P}g'(\tilde{R})\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) + \frac{\eta\pi^p\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})}{\pi^r} \\ &\qquad - \tilde{P}g(\tilde{R})\left(\pi^p tg'(\tilde{R}) - \pi^r t\tilde{P}g''(\tilde{R})\right)\left(\frac{N\pi^p}{\pi^c\tilde{P}^2} - \eta\frac{\pi^c}{\pi^p}\hat{\phi}''(\tilde{c})\right) \\ &\qquad + \eta\pi^c t\tilde{P}g(\tilde{R})g'(\tilde{R})\hat{\phi}''(\tilde{c}) \\ &\quad = \tilde{P}\left(g'(\tilde{R}) - \frac{g(\tilde{R})\left(\pi^p tg'(\tilde{R}) - \pi^r t\tilde{P}g''(\tilde{R})\right)}{\pi^p}\right)\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) \\ &\qquad + \eta\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})\left(\frac{\pi^p}{\pi^r} + t\tilde{P}g'(\tilde{R})\right) \\ &\quad = \frac{\tilde{P}}{g'(\tilde{R})}(1 - tg(\tilde{R}))\left(g'(\tilde{R})^2 - g(\tilde{R})g''(\tilde{R})\right)\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) \\ &\qquad + \eta\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})\left(\frac{\pi^p}{\pi^r}tg(\tilde{R})\right). \end{aligned}$$

The first term is weakly negative, strictly so if g is strictly log-concave. The second term is weakly negative, strictly so if ϕ is strictly log-concave. Therefore, $du_G^*(t)/dt \le 0$ for all $t \in (\hat{t}_0, \hat{t}_1)$, strictly so if g or ϕ is strictly log-concave. This implies $u_G^*(\hat{t}_0) \ge u_G^*(t)$ for all $t \in (\hat{t}_0, \hat{t}_1]$, strictly so if g or ϕ is strictly log-concave.

Finally, consider $t > \hat{t}_1$. Again by Lemma 14,

$$\begin{aligned}\frac{du_G^*(t)}{dt} &= g(\tilde{R}_2(t))\tilde{P}_2(t) + tg'(\tilde{R}_2(t))\frac{d\tilde{R}_2(t)}{dt}\tilde{P}_2(t) + tg(\tilde{R}_2(t))\frac{d\tilde{P}_2(t)}{dt} \\ &= g(\tilde{R}_2(t))\tilde{P}_2(t) + \frac{g'(\tilde{R}_2(t))\tilde{P}_2(t)}{\Delta_2(t)} - \frac{\pi^p g(\tilde{R}_2(t))}{\pi^r \Delta_2(t)},\end{aligned}$$

where $\Delta_2(t)$ is defined by (42). To reduce clutter in what follows, let $\tilde{P} = \tilde{P}_2(t)$ and $\tilde{R} = \tilde{R}_2(t)$. Since $\Delta_2(t) > 0$, the sign of the above expression is the same as that of

$$\begin{aligned}&\tilde{P}g(\tilde{R})\Delta_2(t) + \tilde{P}g'(\tilde{R}) - \frac{\pi^p g(\tilde{R})}{\pi^r} \\ &= \tilde{P}g(\tilde{R})\left(\frac{\pi^r t\tilde{P}g''(\tilde{R})}{\pi^p} - 2tg'(\tilde{R})\right) + \tilde{P}g'(\tilde{R}) - \frac{\pi^p g(\tilde{R})}{\pi^r} \\ &= \frac{\pi^p}{\pi^r tg'(\tilde{R})^2}\left((1-tg(\tilde{R}))^2\left(g(\tilde{R})g''(\tilde{R}) - g'(\tilde{R})^2\right) - \left(tg(\tilde{R}g'(\tilde{R}))\right)^2\right) \\ &< \frac{\pi^p(1-tg(\tilde{R}))^2\left(g(\tilde{R})g''(\tilde{R}) - g'(\tilde{R})^2\right)}{\pi^r tg'(\tilde{R})^2} \\ &\leq 0.\end{aligned}$$

Therefore, $u_G^*(\hat{t}_0) \geq u_G^*(\hat{t}_1) > u_G^*(t)$ for all $t > \hat{t}_1$.

Combining these findings, $u_G^*(\hat{t}_0) \geq u_G^*(t)$ for all $t \in [0,1] \setminus \{\hat{t}_0\}$, strictly so if g or ϕ is strictly log-concave. Therefore, there is an equilibrium in which $t = \hat{t}_0$, and every equilibrium has this tax rate if g or ϕ is strictly log-concave. □

A.8 Proof of Proposition 6

Proposition 6. *Equilibrium production with a labor-financed government, $\bar{P}_0$, is strictly decreasing in the number of factions, N. It is locally decreasing in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

Proof. I will treat N as if it were continuous in order to obtain comparative statics by implicit differentiation. Throughout the proof I write $\bar{P}_0$ and $\bar{c}_0$ as functions of (N, π^c).

Recall that $(\bar{P}_0(N,\pi^c), \bar{c}_0(N,\pi^c))$ is defined as the solution to (25) and (26). To reduce clutter in what follows, I omit the evaluation point $(\bar{P}_0(N,\pi^c), 0, \bar{c}_0(N,\pi^c); t, \pi)$ from all partial derivative expressions. The Jacobian of the system is

$$\mathbf{J}_0 = \begin{bmatrix}\partial Q^{pc}/\partial P & \partial Q^{pc}/\partial C \\ \partial Q^b/\partial P & \partial Q^b/\partial C\end{bmatrix} = \begin{bmatrix}-\pi^p/\bar{P}_0(N,\pi^c)^2 & -(N-1)\pi^c\hat{\phi}''(\bar{c}_0(N,\pi^c))/N \\ -1/\pi^p & -N/\pi^c\end{bmatrix},$$

with determinant

$$|\mathbf{J}_0| = \frac{N\pi^p}{\pi^c\bar{P}_0(N,\pi^c)^2} - \frac{N-1}{N}\frac{\pi^c}{\pi^p}\hat{\phi}''(\bar{c}_0(N,\pi^c)) > 0.$$

By the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{\partial \bar{P}_0(N,\pi^c)}{\partial N} &= \frac{\begin{vmatrix} -\partial Q^{pc}/\partial N & \partial Q^{pc}/\partial C \\ -\partial Q^b/\partial N & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_0|} \\
&= \frac{\begin{vmatrix} \pi^c \hat{\phi}'(\bar{c}_0(N,\pi^c))/N^2 & -(N-1)\pi^c \hat{\phi}''(\bar{c}_0(N,\pi^c))/N \\ \bar{c}_0(N,\pi^c)/\pi^c & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_0|} \\
&= \frac{1}{|\mathbf{J}_0|}\left(\frac{N-1}{N}\bar{c}_0(N,\pi^c)\hat{\phi}''(\bar{c}_0(N,\pi^c)) - \frac{\hat{\phi}'(\bar{c}_0(N,\pi^c))}{N}\right) \\
&< 0,
\end{aligned}$$

as claimed. Similarly,

$$\begin{aligned}
\frac{\partial \bar{P}_0(N,\pi^c)}{\partial \pi^c} &= \frac{\begin{vmatrix} -\partial Q^{pc}/\partial \pi^c & \partial Q^{pc}/\partial C \\ -\partial Q^b/\partial \pi^c & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_0|} \\
&= \frac{\begin{vmatrix} (N-1)\hat{\phi}'(\bar{c}_0(N,\pi^c))/N & -(N-1)\pi^c \hat{\phi}''(\bar{c}_0(N,\pi^c))/N \\ -N\bar{c}_0(N,\pi^c)/(\pi^c)^2 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_0|} \\
&= -\frac{N-1}{\pi^c|\mathbf{J}_0|}\left(\hat{\phi}'(\bar{c}_0(N,\pi^c)) + \bar{c}_0(N,\pi^c)\hat{\phi}''(\bar{c}_0(N,\pi^c))\right),
\end{aligned}$$

which is negative if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0(N,\pi^c)$. □

A.9 Proof of Proposition 7

Proposition 7. *The equilibrium tax rate of a labor-financed government,*

$$\hat{t}_0 = \frac{\pi^p}{\pi^p - \pi^r \bar{P}_0 g'(0)},$$

is strictly increasing in fractionalization, N. It strictly increases with a marginal increase in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.

Proof. Immediate from Proposition 6, as $\hat{t}_0$ is strictly decreasing in $\bar{P}_0$, and N and π^c enter the expression for $\hat{t}_0$ only via $\bar{P}_0$. □

A.10 Proof of Proposition 8

Proposition 8. *A labor-financed government's equilibrium payoff is strictly decreasing in the number of factions, N. It strictly decreases with a marginal increase in conflict effectiveness,*

π^c, *if and only if the incentive effect outweighs the labor-saving effect at* $\bar{c}_0$.

Proof. By Proposition 5, the government's equilibrium payoff is

$$\hat{t}_0\bar{P}_0 = \frac{\pi^p\bar{P}_0}{\pi^p - \pi^r\bar{P}_0 g'(0)} = \frac{\pi^p}{(\pi^p/\bar{P}_0) - \pi^r g'(0)}.$$

This expression is strictly increasing in $\bar{P}_0$. Since N and π^c only enter through the equilibrium value of $\bar{P}_0$, the claim follows from Proposition 6. $\square$

A.11 Proof of Proposition 9

Similar to above, let $\Gamma(t_1, t_2)$ denote the labor allocation subgame following the government's choice of the given tax rates. Notice that in the model with asymmetric taxation we have

$$\begin{aligned}
\frac{\partial u_i(t,p,r,c)}{\partial p_i} &= \frac{\phi(c_i)}{\phi(c_i)+\phi(c_j)}(1 - t_i g(r_i + r_j)),\\
\frac{\partial u_i(t,p,r,c)}{\partial r_i} &= \frac{\phi(c_i)}{\phi(c_i)+\phi(c_j)}(-g'(r_i + r_j))(t_i p_i + t_j p_j),\\
\frac{\partial u_i(t,p,r,c)}{\partial c_i} &= \frac{\phi'(c_i)\phi(c_j)}{(\phi(c_i)+\phi(c_j))^2}\left[(1 - t_i g(r_i + r_j))p_i + (1 - t_j g(r_i + r_j))p_j\right].
\end{aligned}$$

Proposition 9. *In the game with asymmetric taxation, if the government chooses* $t_1 > t_2$*, then* $p_1 \leq p_2$*,* $r_1 \geq r_2$*, and* $c_1 \geq c_2$ *in any equilibrium of the subsequent labor allocation subgame.*

Proof. First I will prove $c_1 \geq c_2$. To this end, suppose $c_i > c_j$; I will show this implies $t_i > t_j$ and thus $i = 1$. Log-concavity of ϕ implies $\phi'(c_i)\phi(c_j) \leq \phi'(c_j)\phi(c_i)$, so we have

$$\pi^c \frac{\partial u_i(t,p,r,c)}{\partial c_i} \leq \pi^c \frac{\partial u_j(t,p,r,c)}{\partial c_j}.$$

The first-order conditions of equilibrium then imply

$$\max\left\{\pi^p \frac{\partial u_i(t,p,r,c)}{\partial p_i}, \pi^r \frac{\partial u_i(t,p,r,c)}{\partial r_i}\right\} \leq \max\left\{\pi^p \frac{\partial u_j(t,p,r,c)}{\partial p_j}, \pi^r \frac{\partial u_j(t,p,r,c)}{\partial r_j}\right\}.$$

As $\phi(c_i) > \phi(c_j)$, this can hold only if $1 - t_i g(r_i + r_j) < 1 - t_j g(r_i + r_j)$; i.e., $t_i > t_j$.

I now prove $r_1 \geq r_2$. First suppose $c_1 > 0$. The first-order conditions of equilibrium, combined with the fact that $c_1 \geq c_2$, imply

$$\pi^c \frac{\partial u_2(t,p,r,c)}{\partial c_2} \geq \pi^c \frac{\partial u_1(t,p,r,c)}{\partial c_1} \geq \pi^r \frac{\partial u_1(t,p,r,c)}{\partial r_1} > \pi^r \frac{\partial u_2(t,p,r,c)}{\partial r_2}.$$

It then follows from the first-order conditions that $r_2 = 0$, which implies $r_1 \geq r_2$. On the other hand, suppose $c_1 = 0$, in which case $c_2 = 0$ per above. It is then immediate from the budget constraint that $r_1 \geq r_2$ if $p_1 = 0$, so suppose $p_1 > 0$. The first-order conditions and

$t_1 > t_2$ imply

$$\pi^p \frac{\partial u_2(t,p,r,c)}{\partial p_2} > \pi^p \frac{\partial u_1(t,p,r,c)}{\partial p_1} \geq \pi^r \frac{\partial u_1(t,p,r,c)}{\partial r_1} = \pi^r \frac{\partial u_2(t,p,r,c)}{\partial r_2}.$$

It then follows from the first-order conditions that $r_2 = 0$, which implies $r_1 \geq r_2$.

Finally, under the budget constraint, $c_1 \geq c_2$ and $r_1 \geq r_2$ imply $p_1 \leq p_2$. □

A.12 Proof of Proposition 10

Proposition 10. *Asymmetric taxation does not raise the equilibrium payoff of a labor-financed government.*

Proof. Assume $t_1 > t_2$, and let (p, r, c) be an equilibrium of $\Gamma(t_1, t_2)$. Let $\hat{t}_0$, $\bar{P}_0$, and $\bar{c}_0$ be defined as in Proposition 4. In addition, let $P = p_1 + p_2$, $R = r_1 + r_2$, and $C = c_1 + c_2$.

My first task is to prove $P \leq \bar{P}_0$. As any equilibrium entails $P > 0$, it follows from Proposition 9 that $p_2 > 0$. If $p_1 = 0$, in which case $P = p_2$, the first-order conditions for equilibrium imply

$$\pi^p \phi(c_2) \geq \frac{\pi^c \phi'(c_2)\phi(c_1)}{\phi(c_1) + \phi(c_2)} P.$$

Rearranging terms and applying the fact that $\phi(c_1) \geq \phi(c_2)$ (per Proposition 9) gives

$$P \leq \frac{\pi^p(\phi(c_1) + \phi(c_2))\phi(c_2)}{\pi^c \phi(c_1)\phi'(c_2)} \leq \frac{2\pi^p \phi(c_2)}{\pi^c \phi'(c_2)}.$$

Under this inequality, $P > \bar{P}_0$ would imply $c_2 > \bar{c}_0$, violating the budget constraint. Therefore, $P \leq \bar{P}_0$. Next, suppose $p_1 > 0$, so the first-order conditions for equilibrium imply

$$\begin{aligned} \pi^p(1 - t_1 g(R)) &\geq \frac{\pi^c \phi'(c_1)\phi(c_2)}{\phi(c_1)(\phi(c_1) + \pi(c_2))} \left[P - (t \cdot p)g(R)\right], \\ \pi^p(1 - t_2 g(R)) &\geq \frac{\pi^c \phi'(c_2)\phi(c_1)}{\phi(c_2)(\phi(c_1) + \pi(c_2))} \left[P - (t \cdot p)g(R)\right]. \end{aligned} \tag{43}$$

As $\log \phi$ is concave and its derivative is convex, $c_1 \geq c_2$ (per Proposition 9) implies $\phi'(c_1)/\phi(c_1) \leq \phi'(c_2)/\phi(c_2)$ and

$$\frac{1}{2}\left(\frac{\phi'(c_1)}{\phi(c_1)} + \frac{\phi'(c_2)}{\phi(c_2)}\right) \geq \frac{\phi'(C/2)}{\phi(C/2)}.$$

In addition, $p_1 \leq p_2$ and $t_1 > t_2$ imply

$$\left(\frac{t_1 + t_2}{2}\right) P \geq t_1 p_1 + t_2 p_2.$$

Summing the conditions in (43) and applying these inequalities gives

$$2\pi^p \left(1 - \frac{t_1 + t_2}{2} g(R)\right)$$

$$
\begin{aligned}
&\geq \pi^c \left[P - (t \cdot p) g(R)\right] \left[\left(\frac{\phi(c_2)}{\phi(c_1) + \phi(c_2)}\right) \frac{\phi'(c_1)}{\phi(c_1)} + \left(\frac{\phi(c_1)}{\phi(c_1) + \phi(c_2)}\right) \frac{\phi'(c_2)}{\phi(c_2)}\right] \\
&\geq \pi^c \left[P - (t \cdot p) g(R)\right] \left[\frac{1}{2}\left(\frac{\phi'(c_1)}{\phi(c_1)} + \frac{\phi'(c_2)}{\phi(c_2)}\right)\right] \\
&\geq \pi^c \left[P - (t \cdot p) g(R)\right] \left(\frac{\phi'(C/2)}{\phi(C/2)}\right) \\
&\geq \pi^c P \left(1 - \frac{t_1 + t_2}{2} g(R)\right) \left(\frac{\phi'(C/2)}{\phi(C/2)}\right)
\end{aligned}
$$

Simplifying and rearranging terms gives

$$
P \leq \frac{2\pi^p \phi'(C/2)}{\pi^c \phi(C/2)}.
$$

As in the previous case, the budget constraint then implies $P \leq \bar{P}_0$.

If $t_i g(R) \leq \hat{t}_0$ for each $i = 1, 2$ such that $p_i > 0$, then $P \leq \bar{P}_0$ implies $u_G(t, c, p, r) \leq \hat{t}_0 \bar{P}_0$, as claimed. So suppose there is a group i such that $p_i > 0$ and $t_i g(R) > \hat{t}_0$. The first-order conditions for equilibrium imply

$$
\pi^p (1 - t_i g(R)) \geq \pi^r (-g'(R))(t_i p_i + t_j p_j).
$$

This inequality, combined with log-concavity of g and the assumption that $t_i g(R) > \hat{t}_0$, gives

$$
\begin{aligned}
u_G(t, p, c, r) &= (t_i p_i + t_j p_j) g(R) \\
&\leq -\left(\frac{g(R)}{g'(R)}\right) \frac{\pi^p (1 - t_i g(R))}{\pi^r} \\
&< -\left(\frac{1}{g'(0)}\right) \frac{\pi^p (1 - \hat{t}_0)}{\pi^r} \\
&= \left(\frac{\pi^p}{\pi^p - \pi^r \bar{P}_0 g'(0)}\right) \bar{P}_0 \\
&= \hat{t}_0 \bar{P}_0,
\end{aligned}
$$

as claimed. □

A.13 Proof of Proposition 11

I begin by characterizing the equilibrium of the conquest game. Throughout the proofs, let $\hat{\chi} = \log \chi$ and $\hat{\psi} = \log \psi$. I will characterize equilibria in terms of the criterion function

$$
Q^{ds}(S; N) = \frac{N - 1}{N} (\psi(S) + \bar{s}_O) \hat{\chi}' \left(\frac{L - S}{N}\right) - \hat{\psi}'(S) \bar{s}_O, \tag{44}
$$

which is strictly increasing in both S and N.

Lemma 15. *The conquest game has a unique equilibrium in which each*

$$s_i = \begin{cases} 0 & Q^{ds}(0; N) \geq 0, \\ \tilde{S}(N)/N & Q^{ds}(0; N) < 0, Q^{ds}(L; N) > 0, \\ L/N & Q^{ds}(L; N) \leq 0, \end{cases}$$

and each $d_i = L/N - s_i$*, where* $\tilde{S}(N)$ *is the unique solution to* $Q^{ds}(\tilde{S}(N); N) = 0$*.*

Proof. Like the original game, the conquest game is log-concave, so a pure-strategy equilibrium exists can be characterized by first-order conditions. In addition, the proof of Lemma 7 carries over to the conquest game, so in equilibrium each $d_i = d_j$ for $i, j \in \mathcal{N}$. The claim then follows from the first-order conditions for maximization of each faction's utility. □

The proof of Proposition 11 follows from this equilibrium characterization.

Proposition 11. *In the conquest model, the probability that the outsider wins is increasing in the number of factions, N.*

Proof. I will prove that the equilibrium value of $\sum_i s_i$ decreases with the number of factions. Let (d, s) and (d', s') be the equilibria at N and N' respectively, where $N' > N$, and let $S = \sum_i s_i$ and $S' = \sum_i s'_i$. If $S = 0$, then $Q^{ds}(0; N) \geq 0$ and thus $Q^{ds}(0; N') \geq 0$, so $S' = 0$ as well. If $S \in (0, L)$, then $S = \tilde{S}(N)$, which implies $Q^{ds}(L; N) > 0$ and thus $Q^{ds}(L; N') > 0$. This in turn implies either $S' = \tilde{S}(N') < \tilde{S}(N)$ or $S' = 0 < \tilde{S}(N)$. Finally, if $S = L$, then it is trivial that $S' \leq S$. □