Social Conflict and the Predatory State[*]

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Abstract

What kind of society is most profitable to govern—one that is internally fractionalized or one that is not? Conventional wisdom and the logic of collective action suggest that a predatory state may benefit from fractionalization, as internal conflict distracts the population from collective resistance against expropriation. Using a formal model, I show that the profits of social division depend critically on the state's revenue base. Internal conflict does not just reduce subjects' incentive to resist, but also to engage in economically productive activity. All else equal, a state that taxes the products of the society's labor benefits from a unified population. Conversely, a state whose objective is to control a fixed stock of wealth, such as natural resources, benefits from internal divisions. The incentive for a colonial empire, military occupier, or burgeoning state to extend its authority over a fractionalized population therefore depends critically on economic factors.

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The object of a predatory state, whether a colonial empire or a kleptocratic dictatorship, is to profit from power. Collective resistance by the subject population against a predatory regime or its extractive policies threatens the profitability of rule. Consequently, political practitioners—like the Roman empire and, later, Machiavelli—have sought to understand how governments can exploit mutual hostilities or fears between factions of their subjects to discourage collective action among them. More recently, scholars in political science, political economy, and history have studied the strategies by which rulers manipulate social divisions in order to maintain their positions and preserve their access to rents (e.g., Acemoglu, Verdier and Robinson 2004; Padró i Miquel 2007; Debs 2007; Burbank and Cooper 2010).

In this paper, rather than analyze specific strategies of divide-and-rule politics, I seek to establish a simple theoretical baseline about the relationship between social fractionalization and predatory governance. What kind of society is most profitable to govern—one that is internally fractionalized or one that is not? International relations scholars study interactions in which the boundaries of authority may shift and the shape of the society governed by the state is at least in part a matter of choice. To answer questions like the following, it is important to have a baseline understanding of how social fractionalization affects the profitability of rule. How did ethnic distinctions among colonized populations affect imperial competition and the timing of decolonization (Michalopoulos and Papaioannou 2013, 2016)? How do internal schisms within separatist movements affect central states' willingness to fight to preserve control (Cunningham 2011; Lacina 2015)? How does the success and profitability of a military occupation depend on the social relations of the occupied population (Liberman 1993; Edelstein 2004)? More broadly, which boundaries between peoples are politically sustainable (Carter and Goemans 2011)? What kinds of populations are attractive targets for conquest?

I model the political economy of predatory governance in a fractionalized society, identifying an economic variable—namely, the source from which government revenues derive—as the key determinant of whether the profitability of rule increases or decreases with the ex-

tent of social fractionalization. I distinguish between labor-financed governments, which extract revenues from the economic output of the subject population (e.g., staple crops), and capital-financed governments, whose revenues derive from some fixed source of wealth (e.g., offshore oil). Internal fractionalization is profitable for capital-financed governments, but not for labor-financed ones. Fractionalization reduces the incentives for collective action and thus for resistance against the predatory state, as studied in existing accounts of divide-and-rule politics. But by the same token, internal division also reduces the incentive to engage in economically productive activity. This countervailing effect is irrelevant for a capital-financed government, but for a labor-financed government it—surprisingly—always more than offsets the benefits of deterring collective action. Using a formal model, I explain why this is the case, and why the downsides of fractionalization might be difficult to detect through causal observation.

The model consists of a simple interaction between a predatory ruler, whose sole aim is rent-seeking, and the various factions of the society being ruled. First, the government chooses a tax rate. Then, each separate political faction divides its labor between economically productive activity, collective resistance against government expropriation, and internal conflict that alters the distribution of output among themselves. The government is purely predatory, with no direct stake in the outcome of the struggle over post-tax output among subject factions. However, internal conflict may indirectly benefit the government, insofar as it reduces resistance; it may also harm the government, insofar as it reduces production. The most important exogenous parameters in the analysis are the government's revenue base (labor or capital) and the extent of fractionalization (modeled as the number of distinct political factions). I analyze how these affect the model's key endogenous variables: the amount of internal conflict among factions and the predatory state's overall revenue.

The analysis reaches four major conclusions. The first—a critical building block for the next three—is that the equilibrium level of internal conflict increases with the extent of social

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fractionalization.[1] Participation in internal conflict increases a faction's own share of post-tax output at the expense of the others, making it a private good from the faction's perspective. By contrast, resistance against government predation is a public good, as it reduces the effective tax rate for all factions. In addition, because all output is subject to appropriation by other factions through internal conflict, economic production also plays the role of a public good here—it increases the size of the pie that the factions compete over. Therefore, by the usual logic of collective action (Olson 1965), both resistance and production decrease with the number of factions, with countervailing effects on the predatory state's revenues, while internal conflict increases.

The second major finding is that a labor-financed government's revenues always decrease with the extent of social fractionalization. For a predatory state that extracts the fruits of its subjects' labor, it is more profitable to rule a unified society than one with many schisms. As noted above, fractionalization has countervailing effects on the components of government revenue. Greater fractionalization produces more internal conflict, at the expense of both production (to the government's detriment) and resistance (to its benefit). Because the benefit of internal conflict relative to collective resistance increases with the extent of fractionalization, so too does the tax rate the government can impose without counterproductively provoking resistance. But while a labor-financed government receives a larger proportion of the pie in more fractionalized societies, this is always more than offset by how much the pie shrinks due to lower production. With greater fractionalization, the higher tax rate and the greater relative benefit of internal conflict combine to reduce the subjects' production incentives enough that the government ends up worse off overall.

The third key result is that the relationship between fractionalization and revenue is reversed for capital-financed governments. A predatory state whose revenue comes from a fixed stock of wealth is better off the more internally divided its subject population is.

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[1]This finding might appear to contradict empirical studies showing that civil war propensity does not increase monotonically with indices of ethnic fractionalization (Montalvo and Reynal-Querol 2005). However, the model implicitly assumes each faction is politically organized, whereas standard measures of fractionalization increase with the number of small but politically irrelevant identities.

This too follows from the fact that internal conflict increases with fractionalization, thereby decreasing collective resistance and economic production among subjects. For a capital-financed government, the decrease in production is irrelevant. Therefore, it is straightforward that fractionalization, by depressing the incentive for collective resistance against extraction, benefits a capital-financed government. The societal conditions that favor social conflict are profitable for a capital-financed government, but unprofitable for a labor-financed one.

In studies of colonialism, empire, and territorial conflict, what matters is not only the profitability of ruling a particular society, but also the ability of an outsider to gain control in the first place. To this end, the final major conclusion of the analysis is that an outsider's chance of gaining control increases with the extent of fractionalization, regardless of whether the outsider seeks to extract the society's labor or capital. To obtain this result, I extend the baseline model to have a prior stage in which an outsider contends for power against existing political factions, which must divide their effort between repelling the outsider and competing amongst themselves. The logic of this result mirrors that of the benefits of fractionalization for a capital-financed government: during the stage of contention for power, internal conflict only comes at the expense of collective resistance against the outsider. Even when a more divided society is not more profitable to rule, it is easier to conquer.

If internal fractionalization makes predatory states worse off in at least some circumstances, as this paper claims, why would practitioners and scholars not have thought so earlier? The formal analysis indicates how casual observation might be misleading. First, as noted above, the optimal tax rate increases with fractionalization even for labor-financed governments. What decreases is total production and therefore overall revenue, which is even more difficult to measure.[2] Second, at the optimal tax rate for a labor-financed government, the level of internal conflict is relatively high. It just happens that the policy which minimizes resistance is the one that maximizes internal conflict. The government would benefit from a change in structural conditions that reduced the overall incentives for social

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[2]For example, see Frankema's (2010) analysis of revenue extraction in the British empire.

conflict, but such changes would naturally be more difficult to observe. The benefit of the formal model is that it allows us to reason through these kinds of counterfactuals, while illustrating pitfalls to avoid (e.g., focusing on extractive demands rather than final revenues) for successive empirical work.

Although my main focus is on the effects of fractionalization, I also consider a societal feature that is more plausibly manipulable by the government—the effectiveness of labor spent on internal conflict, or how little manpower it takes to achieve a particular level of strength in the contest among factions. This conflict effectiveness is an exogenous parameter in the formal model, but it is easy to imagine a state increasing it by arming the population, or decreasing it by protecting factions' property claims against one another. I find that the effect of conflict effectiveness on the equilibrium level of social conflict is ambiguous, as it increases the marginal incentive to participate in conflict but decreases the labor burden of doing so. Whenever an increase in conflict effectiveness leads to more social conflict on the whole, such an increase works to the benefit of capital-financed governments and the detriment of labor-financed governments. Consequently, under reasonable circumstances, we should expect predatory states whose revenue base comes from subjects' economic output to be more willing to provide for secure property rights—or other foundations for social order—than those whose revenue derives from a fixed resource stock.

1   Related Literature

This paper contributes to a longstanding body of theory on the state as an extractive or predatory institution (North 1981; Tilly 1985; Levi 1989; Olson 1993). In particular, partially mirroring the analyses by Bates, Greif and Singh (2002) and North, Wallis and Weingast (2009), the theory presented in this paper concerns the relationship between predatory government, social violence, and social order. In this section, I highlight the most closely related strands of existing work and discuss how my analysis innovates on prior research.

Existing political economy models of divide-and-rule politics show how a policy of division might benefit an autocratic ruler. Acemoglu, Verdier and Robinson (2004) identify divide-and-rule politics as an explanation for the persistence of kleptocracies. In their model, a kleptocrat facing a divided opposition can credibly threaten to divert resources to a favored group in case of a challenge, thereby deterring a challenge in the first place. Padró i Miquel (2007) presents a similar model, except in which rulers' leverage comes from fear of political succession turning over power to a different group. Debs (2007) introduces an informational mechanism, finding that it is in a government's interest to manipulate the media in a way that polarizes social preferences over policy. These theories usefully characterize various strategies of rule in divided societies. They present richer models of governance than the one in this paper, at the expense of a much more restrictive analysis of what the population can do and how subjects interact with one another. My analysis differs by allowing for inefficient internal conflict among the social factions, and by focusing on how the structural incentives for such conflict affect the profitability of rule.

The model of conflict within the society draws from a wide literature on the political economy of appropriation and group conflict (Hirshleifer 1991; Skaperdas 1992; Grossman and Kim 1995; Azam 2002; Dal Bó and Dal Bó 2011; Silve 2017). Existing work studies the tradeoff between production and appropriation in societies with imperfect protection of property rights. Following this line of research, particularly Hirshleifer (1991) and Skaperdas (1992), I model internal conflict as a contest in which the size of the prize shrinks with the amount of effort devoted to the contest, reflecting how appropriative competition draws labor away from socially productive activities. In order to study how this internal conflict affects the policies of an predatory ruler (and vice versa), I extend these models of horizontal competition to have a vertical aspect. I introduce a government that seeks to expropriate from the society as a whole, and I allow the political factions within the society to devote labor to resisting this expropriation. All else equal, predatory tax policies decrease the level of internal conflict compared to the baseline environment considered in previous models, as

higher taxes increase groups' incentive to partake in collective resistance instead of internal conflict. A government that is financed by labor and therefore benefits from internal harmony may also prefer to enact policies, such as legal protection of claims to property, that further reduce the amount of internal conflict. The effect of predation by a capital-financed government is more ambiguous, however, as such states may seek to arm competing factions or otherwise increase social conflict, offsetting the direct effects of predatory taxation on internal conflict.

The relationship between resource dependence and internal conflict is a longstanding concern in the empirical civil war literature (for a review, see Ross 2015). In a close analogue of the distinction I draw between labor-financed and capital-financed governments, Dube and Vargas (2013) find that participation in civil war in Colombia falls as labor-intensive commodities like coffee become more economically important, but rises with the price of capital-intensive commodities like oil. My theory suggests that there may also be an resource curse for communal violence—as opposed to rebellion against the state—stemming from resource-dependent predatory governments' incentives to foster the structural conditions that favor internal conflict. Insofar as communal rivalries are a stronger cause of separatist conflicts than center-seeking wars (Lacina 2015), recent studies finding that the oil curse is concentrated among such conflicts (Paine 2016; Hunziker and Cederman 2017) provide suggestive evidence in favor of this claim, as does the finding that mineral wealth is associated with acts of violence at a local level (Berman et al. 2017).

Finally, as predation and exploitation of social divisions were both hallmarks of colonialism,[3] this paper contributes to the recent literature on the long-run consequences of colonial legacies and the mechanisms through which they operate. Engerman and Sokoloff (1997) argue that differences in precolonial factor endowments explain variation in subsequent growth patterns. Acemoglu, Johnson and Robinson (2001) argue, to the contrary, that this variation is best explained by whether colonial political institutions were primarily extractive and

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[3]For a wide-ranging historical analysis of how empires managed social difference, see Burbank and Cooper (2010).

therefore left a weak foundation for property rights protection. This paper suggests how the *interaction* of factor endowments and extractive politics may explain long-run variation in governance and growth. In particular, I find that predatory states that draw rents from the population's labor typically have an incentive to protect subjects' property rights against threats from each other, while those that profit from fixed natural resource stocks do not. My analysis of how predatory states' governance strategies vary with internal fractionalization is also related to recent empirical findings that precolonial political institutions and colonial boundaries splitting ethnic groups have long-run effects on conflict and economic performance (Michalopoulos and Papaioannou 2013, 2016).

2 The Model

I begin with a model of a predatory government that seeks to profit from the product of the population's labor. The players are the government, denoted G, and a set of N identical factions within society, denoted $\mathcal{N} = \{1, \ldots, N\}$. Let $i \in \mathcal{N}$ denote a generic faction.

The interaction proceeds in two stages. First, the government chooses a tax rate, $t \in [0, 1]$. The tax rate is the same for all factions; I relax this assumption in an extension later in the paper. Second, after observing the tax rate, each faction simultaneously allocates its labor among activities that affect the level and distribution of economic output. These are production, denoted p_i; resistance against the government, r_i; and internal conflict (or competition), c_i. Each faction has a finite amount of labor, meaning its allocation must satisfy the constraint

$$\frac{p_i}{\pi^p} + \frac{r_i}{\pi^r} + \frac{c_i}{\pi^c} = \frac{L}{N}, \tag{1}$$

where $L > 0$ denotes the total size of the population and each $\pi^p, \pi^r, \pi^c > 0$ denotes the society's productivity in the respective activity.[4] For example, the greater π^r is, the less

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[4]I write each faction's labor as a fraction of L so that I can take comparative statics on the number of factions while holding fixed the total size of the society. In the Appendix, I derive equilibrium existence and uniqueness results for the more general case in which factions may differ in their size and productivities.

labor is required to produce the same amount of resistance; total resistance cannot exceed $\pi^r L$. After these choices are made, the game ends and each player receives her payoffs.

Each player's goal, including the government's, is to capture as much economic output as possible. Production, the first potential outlet for labor, creates valuable output that the government and the factions contend over. I assume output simply equals the total amount of labor devoted to production,

$$f(p) = \sum_{i=1}^{N} p_i, \tag{2}$$

where $p = (p_1, \ldots, p_N)$ denotes the vector of each faction's production choice. The linear production technology rules out complementarity between different factions' economic production (see Silve 2017). This is a reasonable assumption for the most common forms of labor extraction by predatory states, such as staple crop production and mining of raw materials or precious metals, but would be less applicable to more developed economies. As the goal of each player is to consume as much as possible, each player's utility is ultimately a fraction of $f(p)$. The other choice variables—the tax rate, resistance, and internal conflict—determine what these fractions are.

Resistance, the second way the factions can expend their labor, determines how much the government can actually collect in taxes. Given the nominal tax rate t and the resistance allocations $r = (r_1, \ldots, r_N)$, let $\tau(t, r)$ denote the proportion of economic output that goes to the government, or the effective tax rate, and let $\bar{\tau} = 1 - \tau$ denote the share that remains to be divided among the factions. Resistance determines what proportion of the nominal tax rate the government can collect:

$$\tau(t, r) = t \times g\left(\sum_{i=1}^{N} r_i\right), \tag{3}$$

where $g : [0, \pi^r L] \to [0, 1]$ is a strictly decreasing function. Given total resistance $R = \sum_{i=1}^{N} r_i$, the function $g(R)$ may represent either the proportion of t that the government can collect or the probability that it collects t as opposed to nothing. As regularity conditions

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to ensure the existence of an equilibrium and ease its characterization, I assume g is twice continuously differentiable, convex, and log-concave.[5] Convexity implies that resistance has diminishing returns, so the factions face a classic collective action problem (Olson 1965): the more each faction expects the others to contribute to resistance, the less it prefers to contribute itself. I also assume the government fully collects the announced tax rate if there is no resistance, so $g(0) = 1$. The linear function $g(R) = 1 - R/\pi^r L$ is one example of the many functions that satisfy these conditions.

Resistance consists of any activity that directly reduces the government's ability to extract the population's economic output. The most obvious example is anti-government violence, such as in the 1791 revolt against French rule in Saint-Domingue or the 1857 Indian mutiny against the British East India Company. But resistance can also take more subtle forms. Scott (2008) describes means of "everyday resistance" employed by peasants against elites, including acts as simple as dragging one's feet. There are also overt but nonviolent forms of tax evasion, like the rampant smuggling of silver out of Spain's American colonies to circumvent the royal monopoly on bullion imports (Scammell 1989, 28).

Internal conflict, the third and final outlet for the factions' labor, determines the share each group receives of what is left over after the government takes its cut. I model the internal conflict over output as a contest, in which each faction expends costly effort to increase its share of the pie. Following Hirshleifer (1991) and Skaperdas (1992), I assume the cost of participation in the contest is an opportunity cost—the more labor a faction spends increasing its own share of the pie through conflict, the less it has to spend increasing the total size of the pie through production or reducing the effective tax rate through resistance. Given the conflict allocations $c = (c_1, \ldots, c_N)$, a faction's share of post-tax output is given by the contest success function

$$\omega_i(c) = \frac{\phi(c_i)}{\sum_{j=1}^{N} \phi(c_j)}, \tag{4}$$

where $\phi : [0, \pi^c L/N] \to \mathbb{R}_+$ is strictly increasing. Factions that devote more effort to internal

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[5]Given the first condition, the latter two are equivalent to $0 \leq g''(R) \leq g'(R)^2/g(R)$ for all $R \in [0, \pi^r L]$.

conflict end up with larger shares of the output. If every faction spends the same amount, or they all spend nothing, they all end up with equal shares, $\omega_i(c) = 1/N$.[6] Again, as regularity conditions to ensure equilibrium existence, I assume ϕ is twice continuously differentiable and log-concave. Both of the most popular contest success functions satisfy these criteria: the ratio form, with $\phi(c_i) = c_i$, and the difference form, with $\phi(c_i) = e^{c_i}$ (Hirshleifer 1989).

The contest determines the division of all of the post-tax output among the factions. In this sense the model features an environment with weak protection of property rights, in which possession is determined through appropriation (Skaperdas 1992).[7] By separating resistance and internal conflict into separate choices, the model assumes that effort spent resisting government predation does not help a faction in resource competition with other groups, and vice versa. In reality, some activities, such as building fortifications, may serve both purposes. However, it is analytically useful to focus on the case in which there is a stark separation between the two activities. Most importantly, the negative effect of fractionalization on incentives for collective action is strongest in an environment without spillovers between collective resistance and internal conflict. The less complementarity there is between these two activities, the more the government should be able to profit from fractionalization. Therefore, my assumption of no complementarity makes it all the more surprising that I identify conditions under which fractionalization decreases the revenue of the predatory state.

Each faction's utility is simply the amount of economic output it receives. This is a function of how much is produced, how much the government extracts through taxation, and the faction's standing in the internal contest. Together, these yield faction i's utility function,

$$u_i(t, p, r, c) = \omega_i(c) \times \bar{\tau}(t, r) \times f(p). \tag{5}$$

The multiplicative payoff structure is similar to that of Hirshleifer (1991) and Skaperdas

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[6]If $\phi(0) = 0$, in which case (4) is not well-defined at $c = 0$, let each $\omega_i(0) = 1/N$.

[7]For a model of conflict with (endogenously) partial property rights, see Grossman and Kim (1995).

(1992), which I extend to incorporate extraction by a predatory government and collective resistance against that extraction.

The government in the model is predatory insofar as its motivation is to increase revenues for its own consumption (Levi 1989). Its utility is how much of the economic output it receives, accounting for reductions in the effective tax rate due to resistance:

$$u_G(t, p, r, c) = \tau(t, r) \times f(p). \tag{6}$$

The government does not use tax revenues to provide public goods or redistribute wealth within society. In addition, the government does not have any preferences over the distribution of post-tax consumption among the factions—internal conflict does not directly enter its utility, though it matters indirectly insofar as it reduces production or resistance. Given the government's predatory incentives and lack of connection to the internal factions, the model is particularly well suited to study the extractive politics of empires, colonial powers, and military occupiers.

The model is a multistage game of complete information, so the appropriate solution concept is subgame perfect equilibrium. In the remainder of the analysis, I refer to the subgame in which the factions choose divisions of labor after learning the tax rate as the labor allocation subgame. I find the equilibrium by backward induction, first characterizing the equilibrium of the labor allocation subgame following each $t \in [0, 1]$, then solving for the government's optimal choice of tax rate given the factions' equilibrium responses.

This model sets up stark strategic tradeoffs for both the factions and the government. Each faction must choose between increasing the total size of the post-tax pie, namely through production or resistance, and securing its own share of that pie through internal conflict. They face a collective action problem, insofar as production and resistance are collectively beneficial while one's share of the internal contest only has private benefits. For the government, the key tradeoff comes in how it calibrates its extractive demand. Holding

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fixed the behavior of the factions, the government would always prefer a higher tax rate. But a high rate may in fact be counterproductive if it diverts social effort away from production and into resistance.

In analyzing the model, I focus on two parameters related to the idea of social order. The first is fractionalization, represented by the number of factions, N. Each faction is treated as a unitary actor in the model—i.e., a faction is a political unit that has solved its internal collective action problems well enough to coordinate on a division of labor that maximizes the group's total welfare. Therefore, a greater number of factions corresponds to a more politically fractionalized society. As I show in the analysis below, the equilibrium level of internal conflict increases with the level of fractionalization. A key question for the analysis is whether this internal conflict, and thus fractionalization itself, works to the benefit of the government. When is it more profitable to govern a fractionalized society rife with internal conflict, versus a more unified society that has better overcome its own collective action problems?

The second parameter I focus on is the factions' productivity in internal conflict, π^c, which gauges how easily the factions can appropriate from each other. I refer to π^c as conflict effectiveness, as it reflects how effectively a faction can translate its labor into strength in the internal conflict. The lower the value of π^c, the greater the opportunity cost of appropriating from other factions as opposed to engaging in production or resistance. Unlike the number of factions, which would be relatively difficult to manipulate directly, it is plausible that the government could shape the relative return to appropriation, at least at the margin. The introduction of new military technology, such as horses or firearms, might increase the ratio of effective strength to labor spent, thereby corresponding to an increase in π^c. On the other hand, stronger protection of the factions' property rights (against each other, not against state expropriation) would correspond to a decrease in π^c. In the analysis of the model, I look for conditions under which the government benefits from lower conflict effectiveness, examining how these relate to the promotion of social order.

3 Labor-Financed Governments

In the baseline model set up in the previous section, the sole source of economic value is what the population produces. If the population does not produce anything, the government comes away with nothing. In this baseline case, I say the government is *labor-financed*, as its expropriation targets the output of the population's labor. In the next section, I modify the model, replacing endogenous production with an exogenously fixed resource stock, in order to examine what changes when the government is *capital-financed* instead.

I focus on how taxation, fractionalization, and the relative labor cost of internal conflict (i.e., conflict effectiveness) affect social order in equilibrium, and how this in turn affects how much the government can extract. In order to focus on the most substantively relevant applications of the model, I impose the following assumption throughout this section:

Assumption 1 (State of Nature). $(N-1)/N > \phi(0)/\phi'(0)\pi^c L$.

This assumption holds if and only if the baseline level of internal conflict—what would occur in equilibrium if the government imposed no taxes—is nonzero.[8] If the State of Nature assumption does not hold, then for every tax rate $t \in [0,1]$, the equilibrium of the subsequent labor allocation subgame entails each faction spending nothing on internal conflict. These cases, in which internal conflict is effectively impossible regardless of the tax rate, are of relatively little substantive interest for the analysis of the interplay between government predation and social order.

3.1 Division of Labor

I begin by solving for each faction's equilibrium division of labor in the labor allocation subgame, after the government chooses a tax rate $t \in [0,1]$. Remember that each faction divides its labor between production, p_i; resistance, r_i; and internal conflict, c_i.

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[8]Assumption 1 holds only if $N > 1$, as the right-hand side of the condition is non-negative.

In an equilibrium of the labor allocation subgame, each faction's division of labor maximizes its own payoff, taking as fixed the other factions' actions. In an equilibrium labor allocation, each faction must devote its labor to the activity (or activities) with the greatest marginal benefit per unit of labor. If it expends labor on two or more activities, the marginal benefits from the two must be equal. For example, if there is an equilibrium (p, r, c) in which faction i expends labor on both production and internal conflict, the following condition must hold:

$$\pi^p \frac{\partial u_i(t, p, r, c)}{\partial p_i} = \pi^c \frac{\partial u_i(t, p, r, c)}{\partial c_i}.$$

If the marginal benefit of production were greater than that of internal conflict, then the faction could strictly increase its consumption by shifting a bit of labor out of internal conflict and into production, violating the condition of equilibrium.

The marginal benefit of one activity depends on the values of the others. For example, labor spent on internal conflict, c_i, increases a faction's share of the output that is left over after taxation. Production and resistance increase the amount of post-tax output and, consequently, raise the marginal benefit of internal conflict. Because of these interdependencies, the equilibrium labor allocation involves a mixture of activities. There cannot be an equilibrium with high production and no internal conflict, because the return to internal conflict would be too great for the factions to refrain. But there also cannot be an equilibrium with high internal conflict and no production, because then the conflict would have no benefit.

The benefits of resistance against government taxation depend not only on the level of production and a faction's share in the internal conflict, but also on the tax rate itself. Naturally, the more the government demands, the more there is to be gained from resistance. In the extreme case of no taxes, $t = 0$, resistance has no effect on the outcome. Each faction's labor would be better spent either on increasing total output through production or increasing its own share of output through internal conflict. Therefore, the equilibrium division of labor following $t = 0$ entails no resistance. More generally, when taxes are low enough, the marginal benefit of resistance remains too low to justify diverting effort from

production or internal conflict, and there is no resistance in equilibrium.

Without resistance, the equilibrium division of labor when taxes are low entails a mixture of production and internal conflict. Because the factions are identical in terms of size and productivity, in equilibrium each devotes the same amount of labor to internal conflict. As a result, each ends up with an equal share of the pie, $\omega_i(c) = 1/N$. The following proposition states the form of the equilibrium in this low-tax case.[9] I call this the baseline equilibrium, as it represents what would occur if the government imposed no taxes at all.

Proposition 1 (Baseline Equilibrium). *There is a tax rate $\hat{t}_0 \in (0, 1)$ such that $\sum_i r_i = 0$ in every equilibrium of the labor allocation subgame if and only if $t \leq \hat{t}_0$. Every subgame with $t \leq \hat{t}_0$ has the same unique equilibrium, in which $\sum_i p_i = \bar{P}_0 > 0$ and each $c_i = \bar{c}_0 > 0$.*

Even when the tax rate is zero, the equilibrium outcome is Pareto inefficient for the factions. Every faction would receive the same share, $1/N$, of a larger pie if they devoted all their labor to production. A kind of prisoner's dilemma logic explains why this Pareto efficient allocation of labor is not sustainable: if no faction planned to spend on the internal conflict, then any single faction could obtain a large share by spending relatively little. Under Assumption 1, the temptation is large enough that every faction has an incentive to deviate from a strategy profile with no conflict.

Holding the tax rate fixed below the baseline equilibrium cutoff, $t \leq \hat{t}_0$, the government benefits from greater social order—i.e., less internal conflict. To be clear, the government does not intrinsically care about internal conflict. However, when there is no resistance, any internal conflict must come at the expense of production, to the government's detriment. For $t \leq \hat{t}_0$, the government's payoff is $t\bar{P}_0$. Therefore, to analyze how fractionalization and conflict effectiveness affect the government's profits, I take comparative statics on baseline equilibrium production with respect to these parameters.

The comparative statics on the number of factions, N, which I interpret as fractionaliza-

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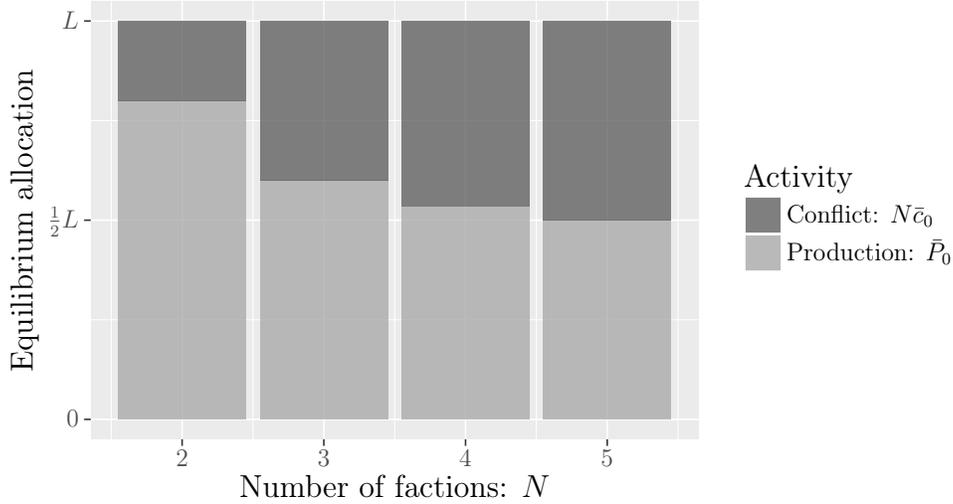
[9]All proofs are in the Appendix.

Figure 1. Division of labor in the baseline equilibrium (Proposition 1) as a function of the number of factions.

tion, are straightforward—more factions means less production. The following remark states this result, which Figure 1 illustrates.[10]

Remark 1. *Total production in the baseline equilibrium, $\bar{P}_0$, is strictly decreasing in the number of factions, N.*

This result follows from the same logic as the classic finding that public good provision decreases with group size (Olson 1965). In the baseline equilibrium, after the internal conflict among factions, each faction only recoups $1/N$ of what it produces. Because of appropriation by the other groups, a faction only partially internalizes the benefits of its own production, less so as the number of factions increases. But each faction always fully internalizes the benefits of the labor it devotes to internal conflict, regardless of the number of other factions. Therefore, as N increases, the relative return to internal conflict increases, so the equilibrium division of labor entails less production. For any fixed tax rate that results in the baseline equilibrium, the government is better off when the society is less fractionalized.

The effects of conflict effectiveness, π^c, on the baseline equilibrium are more complicated.

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[10]All figures use the parameters $\pi^p = \pi^r = \pi^c = 1$ and $L = 2.5$, and the functional forms $g(R) = 1 - R/\pi^r L$ and $\phi(c_i) = e^{c_i}$.

Specifically, a marginal increase in π^c, which represents a decrease in the opportunity cost of expending labor on internal conflict, has two cross-cutting effects. The first, which I call the *incentive effect*, is to raise the marginal benefit per unit of labor of internal conflict. Since the marginal benefits of conflict and production must be equal in the baseline equilibrium, on its own this leads to greater conflict and less production. However, the second effect of increasing π^c, which I call the *labor-saving effect*, works the other way. If a faction's conflict effectiveness increases, it can achieve the same amount of effective strength, $\phi(c_i)$, with a smaller labor force. It may use some of the freed-up labor to even further push its advantage in the internal conflict, but it may also devote some to production.

The overall effect of π^c on economic output, and thus the government's payoff in the baseline equilibrium, depend on whether the incentive effect or the labor-saving effect dominates. When the incentive effect is stronger, a marginal increase in π^c (e.g., arming the population or weakening protection of property rights) leads to more internal conflict and lower total production, reducing government revenues. The opposite is true when the labor-saving effect is stronger. The following condition characterizes the relative strength of the two effects. For any $C > 0$, I say the incentive effect outweighs the labor-saving effect at C if

$$\frac{d \log \phi(C)}{dC} + C \frac{d^2 \log \phi(C)}{dC^2} > 0,$$

and the labor-saving effect outweighs the incentive effect if the opposite inequality holds.[11]

Remark 2. *Total production in the baseline equilibrium, $\bar{P}_0$, strictly decreases with a marginal increase in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

The 17th-century experience of the Dutch East India Company in the Sulawesi region of present-day Indonesia, which I return to below, illustrates how a change in conflict effec-

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[11]In Lemma 12 in the Appendix, I verify that the incentive effect always outweighs the labor-saving effect for the "difference" contest success function ($\phi(C) = \theta e^{\lambda C}$), and the two effects are exactly offsetting in case of a "ratio" contest success function ($\phi(C) = \theta C^\lambda$).

tiveness alters division of labor choices (Henley 2004). In the absence of a "stranger king" to mediate disputes, effort devoted to appropriation was relatively effective and therefore common. However, once the Company began enforcing claims to property against raiding by rival neighbors, the return to labor spent raiding plunged, corresponding to a decrease in π^c. Consequently, Sulawesi groups spent relatively more effort on economic production, increasing Company profits.

These comparative statics results show that, for any fixed tax rate below the threshold, a labor-financed predatory state benefits from the structural conditions that favor social order. Revenues always decrease with fractionalization, and they decrease with conflict effectiveness unless the labor-saving effect is so strong that the amount of labor devoted to internal conflict decreases with π^c. However, these results are conditional on taxes being so low that the government does not face any resistance. When internal conflict can only take place at the expense of economic output, of course the government prefers less internal conflict. Before inferring from these results that the government benefits from social order, I must show that the equilibrium tax rate results in no resistance, and thus the baseline division of labor.

In fact, the location of the cutpoint tax rate, $\hat{t}_0$, itself depends on fractionalization and conflict effectiveness. This cutpoint—the most the government can demand in taxes without engendering resistance—is the point at which the factions are at the margin indifferent about diverting some labor away from their baseline equilibrium allocations and into resistance. Because there are diminishing marginal returns to production, this indifference occurs at a relatively low tax rate when $\bar{P}_0$ is relatively high. Therefore, factors that decrease baseline production (like fractionalization) increase the cutpoint tax rate.

Remark 3. *The cutpoint tax rate,*

$$\hat{t}_0 = \frac{\pi^p}{\pi^p - \pi^r \bar{P}_0 g'(0)},$$

is strictly increasing in fractionalization, N. It strictly increases with a marginal increase in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.

This finding complicates the picture of how social order affects the predatory state's revenues. On one hand, for any fixed tax rate below the threshold, a marginal increase in fractionalization or conflict effectiveness (if it increases internal conflict at the expense of production) reduces the amount the government extracts. However, these same effects push the threshold upward. When the incentive for social strife increases, the economic pie shrinks, but the government can extract a greater portion of it without engendering resistance. I return to the interplay of these cross-cutting effects below when I analyze the government's equilibrium choice of tax rate.

For higher tax rates, the baseline equilibrium is no longer sustainable. Once the tax rate crosses the threshold, $t > \hat{t}_0$, the marginal return to resistance is too high for the factions to prefer spending nothing on resistance. Resistance increases gradually with taxes and may reach a point where it crowds out internal conflict entirely. However, there is always positive production. Resistance pushes the effective tax rate down, so there remains some incentive to produce even at the maximal nominal tax rate, $t = 1$, unlike in the usual Laffer curve. The following proposition states the form of the equilibrium above the baseline; Figure 2 illustrates equilibrium labor allocations as a function of the tax rate and the number of factions.

Proposition 2 (Resistance Equilibrium). *There is a tax rate $\hat{t}_1 > \hat{t}_0$ such that in every equilibrium of the labor allocation subgame with tax rate t:*

- *If $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = \tilde{P}_1(t) > 0$ (weakly decreasing in t), $\sum_i r_i = \tilde{R}_1(t) > 0$ (strictly increasing), and each $c_i = \tilde{c}_1(t) > 0$ (strictly decreasing).*

- *If $t \geq \hat{t}_1$, then $\sum_i p_i = \tilde{P}_2(t) > 0$ (strictly decreasing in t), $\sum_i r_i = \tilde{R}_2(t) > 0$ (strictly increasing), and each $c_i = 0$.*

A high tax rate galvanizes the population, giving the factions an incentive to act in concert to reduce government expropriation. For example, we see excessive extraction by a colonial empire leading to unified resistance both among the American colonies during the Stamp Act crisis of 1765 and between creoles and Indians during a contemporaneous tax revolt in Quito (Elliott 2007, 310–314).

3.2 Optimal Tax Rate

I now solve for a labor-financed government's choice of tax rate. Given the equilibrium responses to each potential choice, as characterized above, the government faces a tradeoff. A higher tax rate has an obvious benefit—the government gets a greater share of the output, all else equal. But all else is not equal. Greater tax rates are met with greater resistance, reducing the government's effective share of output. Moreover, total output itself changes, shrinking as the factions devote more labor to resistance and thereby less to production when taxes are greater.

These tradeoffs create a Laffer curve, such that the highest tax rate does not maximize the government's revenue. At low tax rates, namely those that result in the baseline equilibrium characterized by Proposition 1, the tax rate has no marginal effect on the factions' behavior. Therefore, the government's payoff strictly increases with the tax rate in this low range, as it receives a larger share of the same pie. Beyond that, however, greater taxes are self-defeating. As taxes increase above $\hat{t}_0$, the increase in resistance and the concomitant decrease in production are large enough to offset the gain the government might get from demanding a greater proportional share. As the following proposition states, the equilibrium tax rate is $\hat{t}_0$, the highest at which there is no resistance.

Proposition 3 (Optimal Tax Rate). *If the government is labor-financed, there is an equilibrium in which the government chooses the greatest tax rate that engenders no resistance, $t = \hat{t}_0$. If g or ϕ is strictly log-concave, this is the unique equilibrium tax rate.*
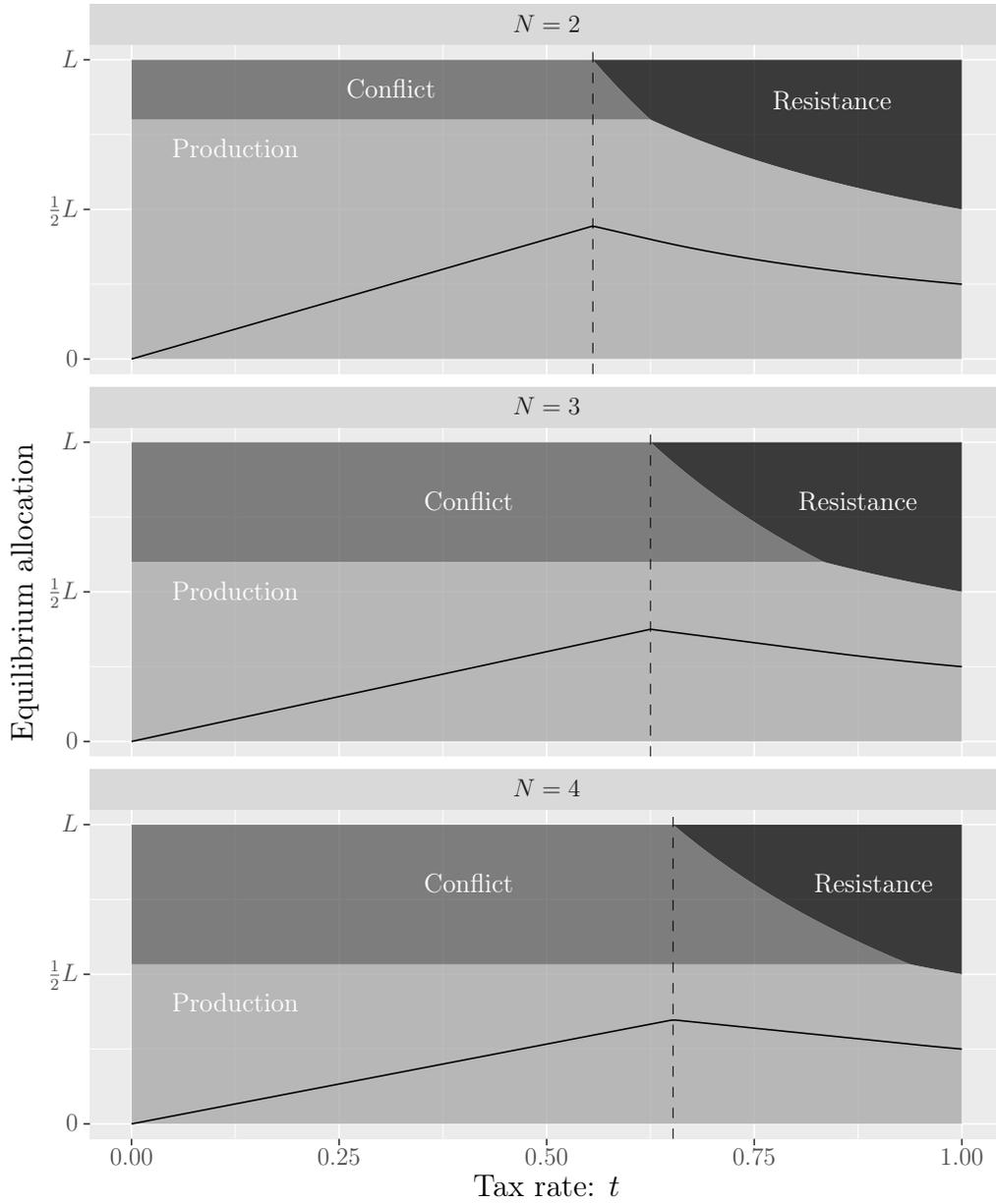
Figure 2. Equilibrium labor allocations, as described in Propositions 1 and 2, as a function of the tax rate and the number of factions. The solid curve is the government's payoff as a function of the tax rate, and the dashed line is the equilibrium tax rate.

To see why it is optimal for the government to avoid engendering resistance, consider a tax rate $t' > \hat{t}_0$ that does lead to resistance. This will result in an effective tax rate of $\tau(t', r) < t$, with some of the increase in resistance coming at the expense of production. It would be better for the government to make the announced tax rate equal the effective one, $t'' = \tau(t', r)$, and reap the gains of the additional production. Of course, if $t'' > \hat{t}_0$, there will still be positive resistance at t'' and the government will not recoup the full share. Nonetheless, as I show in the proof of this proposition in the Appendix, the increase in production by moving to a lower rate is great enough to be profitable for the government.

At a glance, this result might appear to imply that the government benefits from internal disorder. It is evident from Figure 2 that the equilibrium tax rate corresponds to a high point for internal conflict. But this correlation is not exactly causal. Where internal conflict is at its maximum, production is also at its maximum, and resistance is at its minimum. To analyze whether a labor-financed government benefits from social order or disorder, it would be more apt to imagine an exogenous shock to fractionalization or the opportunity cost of internal conflict.

To better understand how social order affects the government's ability to extract economic surplus, I return to the main structural determinants of internal conflict—fractionalization (N) and conflict effectiveness (π^c). I showed above that these have cross-cutting effects on the baseline equilibrium. Because the equilibrium tax rate is the cutpoint $\hat{t}_0$, per Proposition 3, the government's equilibrium payoff is this fraction of baseline production, $\hat{t}_0 \bar{P}_0$. Fractionalization and conflict effectiveness (when the incentive effect outweighs the labor-saving effect) increase the cutpoint tax rate, per Remark 3, but decrease baseline production, per Remarks 1 and 2. In other words, as the structural incentives for internal violence increase, the government ends up with a larger share of a smaller pie in equilibrium. The critical question is which of these effects dominates. On the whole, would a predatory government financed by labor output prefer to rule a society that is more or less prone to internal conflict? Surprisingly, I find that it is always the latter. As the structural determinants of

internal conflict increase, the government's equilibrium payoff decreases.

Proposition 4. *A labor-financed government's equilibrium payoff is strictly decreasing in the number of factions, N. It strictly decreases with a marginal increase in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

The most striking result here is that additional fractionalization always makes the government worse off. Why does the decrease in production always more than offset the increase in the equilibrium tax rate? Once again, the answer lies in the relative marginal benefits of production, resistance, and internal conflict for the factions. At the equilibrium tax rate $\hat{t}_0$, each faction's payoff is $(1 - \hat{t}_0)\bar{P}_0/N$ and the government's payoff is $\hat{t}_0\bar{P}_0$. The marginal benefit of production at the equilibrium point is therefore

$$\pi^p \frac{\partial u_i}{\partial p_i} = \frac{\pi^p}{N}(1 - \hat{t}_0),$$

which decreases with the tax rate, $\hat{t}_0$. The marginal benefit of resistance in equilibrium is

$$\pi^r \frac{\partial u_i}{\partial r_i} = \frac{-\pi^r g'(0)}{N}\hat{t}_0\bar{P}_0,$$

where $g'(0) < 0$ represents how quickly the effective tax rate shrinks with a marginal increase in resistance. Notice that the marginal benefit to resistance increases with the government's payoff, $\hat{t}_0\bar{P}_0$. Because the equilibrium tax rate pushes the factions just to the point where resistance would become profitable, in equilibrium the marginal benefits of production and resistance must be equal even though no resistance takes place. Equality of the above expressions is equivalent to

$$\pi^p(1 - \hat{t}_0) = -\pi^r g'(0)\hat{t}_0\bar{P}_0.$$

Remark 3 shows that fractionalization increases $\hat{t}_0$ and thereby decreases the marginal return to production (the left-hand side of the above expression). Therefore, in order to maintain

the equality of marginal benefits, production $\bar{P}_0$ must decrease enough with fractionalization that the government's payoff, $\hat{t}_0 \bar{P}_0$, also decreases. Fractionalization directly reduces the benefits of resistance, but it also reduces the benefits of production. Therefore, even as fractionalization increases, the threat that the factions will divert labor from production into resistance remains strong enough to prevent the government from profiting.

The upshot of Proposition 4 is that social order increases the profitability of extraction from the population's labor. For a labor-financed government, it is better off governing a society where structural conditions are favorable to social order. If we think of these conditions as at least partially endogenous, this implies that it is in the government's interest to promote social order if the costs of doing so are low enough. For example, the government might seek to divert labor away from looting and into productive economic activity by enforcing subjects' claims to property against appropriation by other factions.

The aforementioned example of the Dutch East India Company in Sulawesi illustrates the logic of Proposition 4. When the Company arrived in Sulawesi in the 17th century, the region was beset with raiding and other violence, largely between neighboring rival villages (Schouten 1998). Instead of encouraging these conflicts, the Dutch sought to reduce looting and protect property rights. Warring parties regularly called on the Dutch to arbitrate their disputes, making the Company a kind of "stranger king" in Sulawesi society. And the Dutch found it in their interest to do so, as "any conflict quickly tended to interfere with the production and supply of the Minahasan rice which ... formed the Company's main economic interest in the area" (Henley 2004, 105). In other words, with its revenues in Sulawesi funded by agricultural output, the ruling class did not profit from social strife, and in fact sought to reduce it.

Other examples abound. On the frontiers of Latin Christendom prior to Frankish conquest in the late Middle Ages, "direct predation ... was not the occasional excess of the lawless but the prime activity of the free adult male population" (Bartlett 1993, 303). By establishing free villages and improving the protection of property rights, the immigrant

Frankish nobility was able to profit by directing labor into productive activity rather than banditry. Later in Europe, the Ottoman empire maintained an advantage in trade and revenue in part by maintaining peace among the diverse religious and cultural groups that constituted its subjects (Burbank and Cooper 2010, 132–133). In yet another example, the Mughal empire that preceded British rule on the Indian subcontinent "defined their task as to keep an ordered balance between the different forces which constituted Indian society" (Wilson 2016, 17).

For a labor-financed government, the costs of social conflict outweigh the benefits. Internal conflict may distract the population from resistance against taxation, but it also reduces the incentives for economic production. On the whole, it is more profitable to control a society that is more unified, and therefore more productive. The benefits of social order might be difficult to detect from casual observation, however. The optimal policy choice for the government involves a relatively high level of internal conflict among the factions—not because this conflict is beneficial in itself, but because that policy happens also to be the one that maximizes production and minimizes resistance.

4   Capital-Financed Governments

The analysis so far has considered the case in which the government's objective is to expropriate the product of the population's labor; if the population does not work, then there is no profit to be made. I have shown that the government has an incentive to promote social order under these conditions. In this section, I consider an alternative political economy, in which the main source of value—what the government and the factions want to appropriate—is a fixed stock whose value does not depend on labor inputs from the subject population. For short, I call this a capital-financed government. I find that whereas a labor-financed government benefits from social order, a capital-financed government benefits instead from chaos.

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The clearest example of this type of revenue source is natural resources, particularly oil. Oil exploitation is capital-intensive, often takes place offshore, and can be conducted by workers imported from abroad (Le Billon 2013, 28–30). In the absence of mass resistance, a government can extract value from oil deposits regardless of local labor contributions. Land itself may also play the role of a fixed resource that is valued in its own right. Whereas Spanish settlers in South America acquired property to be worked by native labor, English settlers in North America sought sparsely populated territories and fought to expel Indians where they settled (Elliott 2007, 36–38). In the terms I use here, the Spanish colonizers were labor-financed, and the English colonizers were capital-financed.

The strategic tradeoffs for a capital-financed government are significantly different than in the case considered above. For a labor-financed government, internal conflict has cross-cutting effects: it reduces resistance, allowing the government to impose a higher effective tax rate, but it also reduces production. The situation is simpler for a capital-financed government. Because the value of the resources available to expropriate does not depend on the society's labor inputs, there is no longer a tradeoff between internal conflict and the size of the pie. Internal conflict merely helps prevent the factions from cooperating to resist government expropriation. Consequently, a capital-financed government benefits from fractionalization and will profit from structural conditions that heighten social conflict.

To model the political economy of capital extraction, I make a simple change to the baseline model. In the baseline model, the total size of the pie is $f(p)$, the endogenous result of production by the factions. If the government is capital-financed, however, the value of the economic product available for expropriation is fixed at the exogenous value $X > 0$.[12] The government's choice is still the tax rate $t \in [0, 1]$. With production out of the picture,[13] the factions now only choose to allocate their labor between resistance, r_i, and internal conflict,

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[12]Different actors might value the same resources differently. For example, control of oil fields might be more valuable in an absolute sense to the government than to rebel groups (Le Billon 2013, 29–30). The results of the analysis would not change if each actor valued the pie at a potentially different level $X_i > 0$, since this would simply entail the multiplication of each actor's utility function by a positive constant.

[13]The results would be similar substantively but more cumbersome to derive if economic output were the combination of exogenous resources and endogenous production, such as $f(p) = X + \sum_i p_i$.

c_i. The budget constraint for each faction is still given by (1), with p_i fixed to 0. Utility functions in the model with a capital-financed government are

$$u_i(t, r, c) = \omega_i(c) \times \bar{\tau}(t, r) \times X,$$

$$u_G(t, r, c) = \tau(t, r) \times X,$$

so all players' payoffs sum to the total value X. This model of internal appropriation with a fixed resource stock is similar to that of Hodler (2006), extended to include an predatory government and potential resistance to it.

The relationship between the tax rate and the equilibrium division of labor under a capital-financed government mirrors that of the political economy studied in the previous section, except with production taken out. As the tax rate increases, so too does resistance, at the expense of internal conflict. The following proposition summarizes the equilibrium, and Figure 3 illustrates.

Proposition 5. *If the government is capital-financed, every labor allocation subgame has a unique equilibrium. There exists a tax rate $\hat{t}_0^X \in (0, 1)$ such that each $r_i = 0$ in equilibrium if and only if $t \leq \hat{t}_0^X$. There exists $\hat{t}_1^X > \hat{t}_0^X$ such that each $c_i = 0$ in equilibrium if and only if $t \geq \hat{t}_1^X$. For $t \in (\hat{t}_0^X, \hat{t}_1^X)$, in equilibrium each $r_i = \tilde{R}_X(t)/N > 0$ (strictly increasing in t) and each $c_i = \tilde{c}_X(t) > 0$ (strictly decreasing).*

Unlike in the baseline model, it is not necessarily true that there will be no resistance on the equilibrium path.[14] Whereas a labor-financed government always chooses the tax rate that results in maximal internal conflict (given equilibrium responses by the factions), a capital-financed government may not do so. But this is not because the incentives for social order are stronger for a capital-financed government. Instead, it simply reflects how the government's strategic tradeoffs change when it no longer values production by the

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[14]In Figure 3 the equilibrium tax rate is $\hat{t}_0^X$, mirroring the baseline model, but this is an artifact of the particular functional forms used to make the figures.
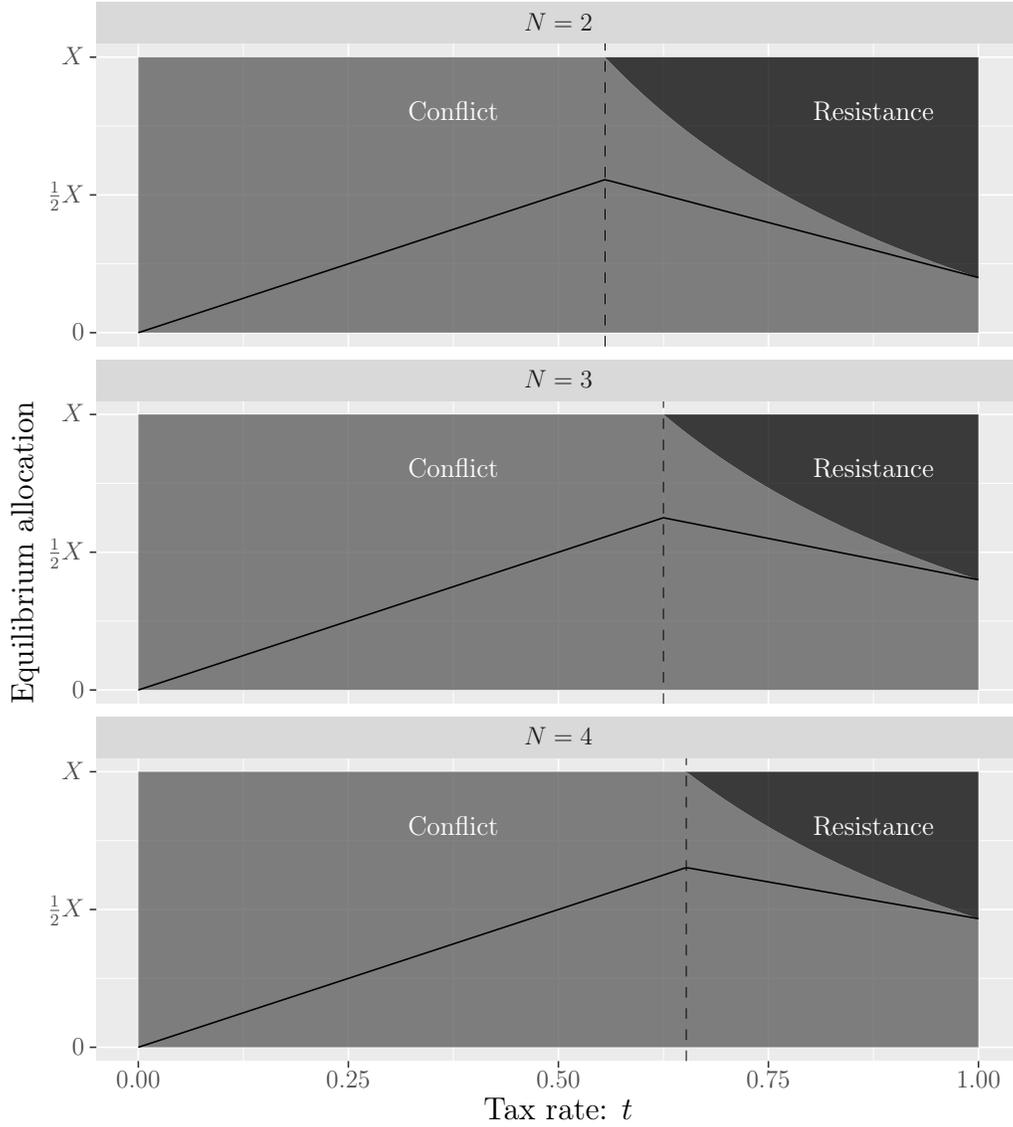
Figure 3. Equilibrium labor allocations with a capital-financed government, as described in Proposition 5, as a function of the tax rate and the number of factions. The solid curve is the government's payoff as a function of the tax rate, and the dashed line is the equilibrium tax rate. Parameters and functional forms are the same as in the previous figures, with $X = L = 2.5$.

population. Higher tax rates now have only one drawback (greater resistance), compared to two (that and less production) in the earlier case.

In fact, looking at the structural parameters that drive internal conflict, it is clear that capital-financed governments prefer social conflict over social order. Their incentives are therefore opposite those of labor-financed governments. For example, consider fractionalization, as represented by the number of factions, N. In an economy based on the population's labor, fractionalization leads to lower production (Remark 1) and thereby reduces how much the government can extract (Proposition 4), even though it also reduces the incentives for collective resistance. But when the source of value for the government is exogenously fixed, the negative effect of fractionalization on the population's economic activity is irrelevant, while its negative effect on resistance remains. Therefore, for a capital-financed government, additional fractionalization is a net benefit. The story is similar for conflict effectiveness, π^c: as long as the incentive effect dominates the labor-saving effect, meaning a marginal increase in conflict effectiveness results in greater social conflict in equilibrium, the capital-financed predatory state benefits from such an increase. The following proposition summarizes how a capital-financed government benefits from structural conditions that favor internal conflict.

Proposition 6. *A capital-financed government's equilibrium payoff is increasing in the number of factions, N. If there is a unique equilibrium tax rate t^*, the government's equilibrium payoff is locally increasing in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at the corresponding equilibrium level of internal conflict.*

Proposition 6, in combination with the results of the previous section, shows that the profitability of social conflict depends critically on the nature of the predatory state's revenue base. If the main source of value is the product of the population's labor, then fractionalization and internal disorder decrease the incentive to produce, ultimately reducing the profits of expropriation. But if the source of contention between the government and the population is some fixed pool of goods or resources, the opposite logic prevails. In this case, internal

conflict does not reduce the value of the pie, but it does keep the population distracted from resistance against the government.

The major empirical implication of these results is that the relationship between social order and the policies and profitability of extractive governance is conditional on the nature of what is being extracted. All else equal, when a predatory state is financed by the population's labor, we should expect it to impose policies that reduce conflict and promote internal order, at least at the margins. By reducing appropriation among various ethnic groups or political factions, these policies raise the overall productivity of the governed population, which is profitable for the government. Moreover, we should expect extractive governance to be more profitable and stable in polities where structural conditions favor internal order. For example, labor-financed imperial regimes should be less willing to expend resources to gain or maintain control over internally divided societies.

When the object of government extraction is a fixed source, such as a natural resource, we should expect these relationships to go the other direction. A capital-financed government will, at the margin, prefer policies that increase internal conflict and thereby reduce anti-government resistance, such as lax enforcement of competing groups' property rights. Colonies or occupations whose main objective is to secure control of some existing resource will be more successful when the population is more divided, or local conditions otherwise favor internal conflict.

5 Endogenous Inequality

The preceding analyses of government extraction have assumed that taxation affects all factions identically. In addition, in the model, all factions are *ex ante* identical in terms of their productivity and incentives. In this section, I briefly consider whether a labor-financed government might profit by creating inequality between groups, namely by taxing them at different rates. This extension is similar to the model of divide-and-rule politics in Acemoglu,

Verdier and Robinson (2004); the most important difference is that the factions can engage in costly conflict to appropriate from each other. The main result is that a labor-financed government cannot benefit by creating inequality—the logic of the baseline case carries over despite the government's ability to induce asymmetries in the factions' incentives.

In the model with *asymmetric taxation*, the government is labor-financed and taxes each faction's production separately. To keep the analysis simple, I assume throughout the extension that $N = 2$. The government chooses a pair of tax rates, t_1 and t_2, where each $t_i \in [0,1]$ as before. The factions then respond as before, by allocating their labor among production, resistance, and internal conflict, (p_i, r_i, c_i), subject to the budget constraint (1). I consider tax schemes such that $t_1 \geq t_2$; as the factions remain identical *ex ante*, this restriction is without loss of generality. The utility functions for the government and the factions are now

$$u_G(t, p, r, c) = \tau(t_1, r)p_1 + \tau(t_2, r)p_2,$$

$$u_i(t, p, r, c) = \omega_i(c) \left[\bar{\tau}(t_1, r)p_1 + \bar{\tau}(t_2, r)p_2 \right].$$

If the government chooses the same tax rate for both groups, $t_1 = t_2$, then each player's utility is the same as in the original model with that rate. Throughout the analysis of this extension, I impose an additional technical condition on the function that translates c_i into effective strength in the internal conflict: I assume ϕ'/ϕ is convex.[15]

To analyze the extension, I first consider how the factions would respond to the choice of unequal tax rates. Naturally, as taxation reduces the marginal benefit of production, the faction that is taxed more produces less in equilibrium. The more highly taxed faction then shifts some of the labor it would have spent on economic production into resistance and internal conflict. This has the counterintuitive implication that the equilibrium payoff for the more-taxed faction is no less than that of the less-taxed faction. By reducing a group's incentive to produce, the government increases its incentive to appropriate from the other

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[15]The baseline assumptions imply that ϕ'/ϕ is positive and decreasing, so convexity is natural. The difference and ratio functional forms described above in footnote 11 both satisfy this condition.

group, resulting in it taking home a disproportionate share of the total post-tax output. This result is reminiscent of the "paradox of power" characterized by Hirshleifer (1991), wherein seemingly weaker groups expend disproportionate effort on appropriation. The following proposition summarizes the equilibrium responses to unequal taxation.

Proposition 7. *In the game with asymmetric taxation, if the government chooses $t_1 > t_2$, then $p_1 \leq p_2$, $r_1 \geq r_2$, and $c_1 \geq c_2$ in any equilibrium of the subsequent labor allocation subgame.*

The factions' responses show why asymmetric taxation is ultimately unprofitable for a labor-financed government. There is obviously no profit to be made from the more highly taxed faction, as it reduces its production in response to the greater taxation. But as the more-taxed faction increases its appropriative efforts, the less-taxed faction also loses some of its incentive to engage in productive activity. The decrease in the less-taxed faction's incentive to resist does not make up the difference, as the marginal benefit of resistance remains a function of the government's overall payoff. Ultimately, then, the government is no better off having the ability to set unequal tax rates across groups.

Proposition 8. *Asymmetric taxation does not raise the equilibrium payoff of a labor-financed government.*

This brief extension demonstrates that the earlier results for labor-financed extraction do not depend on the assumption of equal tax rates across factions. All else equal, a labor-financed government benefits from social order and has no incentive to create inequality where none exists before. What this extension does not answer is how asymmetric tax rates would interact with *ex ante* asymmetries in productivity or size among factions, a topic that is beyond the scope of the present analysis.

6 Conquest

The preceding analysis takes the identity of the ruler as fixed. I have shown that the relationship between social conflict and the profitability of rule depends on the type of economic product from which the ruler's rents derive. I now briefly consider the process of taking control, prior to the selection of the tax rate and subsequent division of society's labor. When an outside force seeks to usurp authority, is it more likely to succeed when the population is more divided?

Whereas internal fractionalization is only conditionally beneficial for predatory governance, namely when the government is capital-financed, it unconditionally increases the prospects of an outsider seeking to gain control in the first place. The intuition behind this result mirrors the logic of the finding above that fractionalization increases the tax rate the government can impose without engendering resistance (Remark 3). Resistance against an outsider's attempt to take control is effectively a public good. As the number of factions increases, the incentive to provide this public good rather than to fend for oneself decreases (Olson 1965). Therefore, an outsider can more easily take control of a divided society than a unified one.

In the *conquest model*, a set of N factions compete with each other and with an outsider, denoted O, for the chance to be the government in the future. The incremental value of being the government is $v(N) > 0$, which may increase with N (when capital is the main source of revenue) or decrease (when labor is the main source of revenue). Each faction has L/N units of labor, which it may divide between two activities: $s_i \geq 0$, to prevent the outsider from taking over; and $d_i \geq 0$, to influence its own chance of becoming the government if the outsider fails. Each faction's budget constraint is[16]

$$s_i + d_i = \frac{L}{N}. \tag{7}$$

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[16]The assumption of unit productivity for each activity is without loss of generality. The model here with functional forms $\chi(S) = \tilde{\chi}(\pi^s S)$ and $\psi(D) = \tilde{\psi}(\pi^d D)$ is isomorphic to a model with the common budget constraint $s_i/\pi^s + d_i/\pi^d = L/N$ and functional forms $\tilde{\chi}$ and $\tilde{\psi}$.

The success of the attempted takeover depends on how much the factions spend to combat the outsider. I assume the outsider's military strength is a fixed value, $\bar{s}_O > 0$, so the outsider is not a strategic player here. The assumption that the outsider's strength is exogenous is of course a simplification, but it is plausible in situations where the outsider marshals its forces before fully understanding the internal political situation—such as in Cortés's incursion into the Mexican mainland, and other early maritime colonial ventures.[17] The probability that the outsider becomes the government is

$$\frac{\bar{s}_O}{\bar{s}_O + \chi(\sum_{i=1}^{N} s_i)},\tag{8}$$

where $\chi : [0, L] \to \mathbb{R}_+$ represents the translation of society's labor into its strength against the outsider. In case the outsider fails, the probability that faction i becomes the government is

$$\frac{\psi(d_i)}{\sum_{j=1}^{N} \psi(d_j)},\tag{9}$$

where $\psi : [0, L/N] \to \mathbb{R}_+$ represents the translation of an individual faction's labor into its proportional chance of success against other factions. As with the function ϕ in the original model, I assume χ and ψ are strictly increasing and log-concave.

The factions simultaneously choose how to allocate their labor, subject to the budget constraint (7). A faction's utility function is

$$u_i(s, d) = \frac{\psi(d_i)}{\sum_{j=1}^{N} \psi(d_j)} \times \frac{\chi(\sum_{j=1}^{N} s_j)}{\bar{s}_O + \chi(\sum_{j=1}^{N} s_j)} \times v(N),\tag{10}$$

where $s = (s_1, \ldots, s_N)$ and $d = (d_1, \ldots, d_N)$.

The strategic tradeoff for the factions here is analogous to the tradeoff between resistance and internal conflict in the above models of post-conquest political economy. Critically, the relative marginal benefit of fighting the outsider declines as the number of factions

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[17]With some additional restrictions on the parameters of the model, the results of the conquest game would be essentially the same if the outsider's strength were chosen endogenously.

increases. When the number of factions is large, any individual faction's chance of becoming the government if the outsider loses is small, which in turn reduces its incentive to contribute to the collective effort against the government. Consequently, as the following result states, the outsider is more likely to win the more divided the society is.

Proposition 9. *In the conquest model, the probability that the outsider wins is increasing in the number of factions, N.*

To be clear, unlike some of the earlier results, Proposition 9 does not address how fractionalization affects the outsider's overall welfare in equilibrium. In particular, if the outsider's ultimate objective is to extract the population's labor, there is no guarantee that the increase in the chance of winning due to greater fractionalization would offset the decrease in $v(N)$. Proposition 9, in combination with the earlier results, implies that a society that is easy to conquer may nevertheless be difficult to govern. Specifically, fractionalization benefits a labor-financed government in the conquest stage, but not in the governance stage. Only for a capital-financed government, which benefits from internal chaos even while governing, does fractionalization have the same effect on ease of conquest and the profitability of rule.

The conquest model captures how Cortés benefited from internal divisions in Aztec society. He was able to conquer with significantly less military support than would have been necessary otherwise, because the incumbent regime also had to contend with its internal enemies (Elliott 2007; Burkholder and Johnson 2015). The Dutch East India Company similarly exploited internal divisions when initially establishing its foothold in present-day Indonesia (Scammell 1989, 20). In pure military competition, the political economy issues that arise in extraction of labor output—namely, the tradeoff between internal conflict and economic productivity—are sidelined. Only after establishing control does internal fractionalization become a potential problem for the predatory state.

7 Conclusion

I have characterized the political economy of predatory rule in a divided society. The main result is that the profitability of fractionalization and social division depends on the nature of the economic product the ruler wishes to extract. When it is the output of the population's labor, the ruler is better off when society is less deeply divided. The opposite is true for a capital-financed government that seeks to expropriate from an exogenously fixed source of value. Additionally, regardless of the ultimate extractive aims, internal fractionalization increases the likelihood of gaining control in the first place.

Besides its contributions to the literatures on social conflict and the political economy of colonialism, the model here also has applications in the study of state formation. Early states are typically thought of as predatory institutions (Tilly 1985; Olson 1993; Sánchez de la Sierra 2018). Studies of the relationship between political development and economic development have focused on the state's ability to commit to limited extraction (North 1981, e.g.,), but have focused less on the state's incentives to protect appropriation *among* its subjects. My results suggest that it sometimes may be profitable for such a state to refrain from establishing total sovereignty—to allow some banditry to take place within its sphere of ostensible authority. If the proto-state's main revenue source is the output of its subjects, then eliminating internal appropriation and establishing full sovereignty would indeed be optimal if possible. But if it benefits mainly from control of some fixed resource, such as control of an economically or strategically important waterway, the early state may be better off tolerating some infighting so as to minimize the chance of a serious competitor emerging from within.

My results also have implications for the study of international conflict and its causes. The most influential theory of conflict posits that war is the result of bargaining failure (Fearon 1995). Although international relations theorists have made tremendous progress identifying how and why bargaining might break down, the theoretical literature has relatively little to say about the issues that states bargain over. The model here provides a novel explanation

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of why some territory is more valuable than others—an important question, in light of the centrality of territorial disputes as a cause of war (Goertz and Diehl 1992). In particular, there is an important interaction between natural resource wealth and social fractionalization. Internal divisions should increase the value of resource-rich territory, but decrease the value of territory with relatively few natural resources. Consequently, in the empirical record, we should expect fractionalization to have differential effects on the probability of a crisis breaking out. Similarly, the theory provides a political economy foundation for the regularity that new interstate borders tend to follow previous administrative frontiers (Carter and Goemans 2011). Boundary changes that split ethnic groups may threaten a predatory state's revenues even if its territorial holdings increase overall, namely by increasing communal violence (see also Michalopoulos and Papaioannou 2016).

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A   Appendix to "Social Conflict and the Predatory State"

Contents

A.1   Additional Notation

Throughout the appendix, let $\Phi(c) = \sum_i \phi(c_i)$. We have $\log \omega_i(c) = \log \phi(c_i) - \log \Phi(c)$ and thus

$$
\begin{aligned}
\frac{\partial \log \omega_i(c)}{\partial c_i} &= \frac{\phi'(c_i)}{\phi(c_i)} - \frac{\phi'(c_i)}{\Phi(c)} \\
&= \frac{\phi'(c_i)}{\phi(c_i)} \left(1 - \frac{\phi(c_i)}{\Phi(c)} \right) \\
&= \hat{\phi}'(c_i)(1 - \omega_i(c)),
\end{aligned}
$$

where $\hat{\phi} = \log \phi$. Because ϕ is strictly increasing and log-concave, $\hat{\phi}' > 0$ and $\hat{\phi}'' \leq 0$.

A.2   Equilibrium Existence and Uniqueness

For the existence and uniqueness results, I consider a more general version of the model presented in the text. I allow groups to be asymmetric in their size and productivities, which entails generalizing each faction i's budget constraint (1) to

$$
\frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} = L_i, \tag{11}
$$

where $\pi_i^p, \pi_i^r, \pi_i^c, L_i > 0$. In addition, the results here do not depend on Assumption 1.

Let $\Gamma(t)$ denote the subgame that follows the government's selection of t, in which the factions simultaneously decide how to allocate their labor. Let $\sigma_i = (p_i, r_i, c_i)$ be a strategy for faction i in the subgame, and let

$$
\Sigma_i = \left\{ (p_i, r_i, c_i) \,\left|\, \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} = L_i \right. \right\}
$$

1

denote the strategy space. Let $\sigma = (\sigma_1, \dots, \sigma_N)$ and $\Sigma = \bigtimes_{i=1}^{N} \Sigma_i$.

I begin by proving that a Nash equilibrium exists in each subgame. The task is complicated by the potential discontinuity of the factions' payoffs, namely at $c = 0$ when $\phi(0) = 0$. I rely on Reny's (1999) conditions for the existence of pure strategy equilibria in a discontinuous game. The key condition is *better-reply security*—informally, that at least one player can assure a strict benefit by deviating from any non-equilibrium strategy profile, even if the other players make slight deviations.

Lemma 1. $\Gamma(t)$ *is better-reply secure.*

Proof. Let $U^t : \Sigma \to \mathbb{R}_+^N$ be the vector payoff function for the factions in $\Gamma(t)$, so that $U^t(\sigma) = (u_1(t, \sigma), \dots, u_N(t, \sigma))$. Take any convergent sequence in the graph of U^t, call it $(\sigma^k, U^t(\sigma^k)) \to (\sigma^*, U^*)$, such that σ^* is not an equilibrium of $\Gamma(t)$. Because production and the effective tax rate are continuous in (p, r), we have

$$U_i^* = w_i^* \times \bar{\tau}(t, r^*) \times f(p^*)$$

for each i, where $w_i^* \geq 0$ and $\sum_{i=1}^{N} w_i^* = 1$. I must show there is a player i who can secure a payoff $\bar{U}_i > U_i^*$ at σ^*; i.e., there exists $\bar{\sigma}_i \in \Sigma_i$ such that $u_i(t, \bar{\sigma}_i, \sigma_{-i}') \geq \bar{U}_i$ for all σ_{-i}' in a neighborhood of σ_{-i}^* (Reny 1999, 1032).

If $N = 1$ or $\Phi(c^*) > 0$, then U^t is continuous in a neighborhood of σ^*, so the conclusion is immediate. If $\bar{\tau}(t, r^*) \times f(p^*) = 0$, then each $U_i^* = 0$ and each faction can assure a strictly greater payoff by deviating to a strategy with positive production, resistance, and conflict. For the remaining cases, suppose $N > 1$, $\bar{\tau}(t, r^*) \times f(p^*) > 0$, and $\Phi(c^*) = 0$, the latter of which implies $c^* = 0$ and $\phi(0) = 0$. Since $N > 1$, there is a faction i such that $w_i^* < 1$. Take any $\epsilon \in (0, (1 - w_i^*)/2)$ and any $\delta_1 > 0$ such that

$$\bar{\tau}(t, r') \times f(p') \geq (w_i^* + 2\epsilon) \times \bar{\tau}(t, r^*) \times f(p^*)$$

for all σ' in a δ_1-neighborhood of σ^*. Since $w_i^* + 2\epsilon < 1$ and $\bar{\tau}(t, r) \times f(p)$ is continuous in (p, r), such a δ_1 exists. Then let $\bar{\sigma}_i = (\bar{p}_i, \bar{r}_i, \bar{c}_i)$ be any strategy in a δ_1-neighborhood of σ_i^* such that $\bar{c}_i > 0$. Because $c_{-i}^* = 0$ and ϕ is continuous, there exists $\delta_2 > 0$ such that

$$\omega_i(\bar{c}_i, c_{-i}') = \frac{\phi(\bar{c}_i)}{\phi(\bar{c}_i) + \sum_{j \in \mathcal{N} \setminus \{i\}} \phi(c_j')} \geq \frac{w_i^* + \epsilon}{w_i^* + 2\epsilon}$$

for all σ_{-i}' in a δ_2-neighborhood of σ_{-i}^*. Therefore, for all σ_{-i}' in a $\min\{\delta_1, \delta_2\}$-neighborhood of σ_{-i}^*, we have

$$u_i(t, \bar{\sigma}_i, \sigma_{-i}') \geq (w_i^* + \epsilon) \times \bar{\tau}(t, r^*) \times f(p^*) > U_i^*,$$

establishing the claim. $\square$

The other main condition for equilibrium existence is that each faction's utility function be quasiconcave in its own actions. I prove this by showing that the logarithm of a faction's utility function is concave in its actions.

Lemma 2. $\Gamma(t)$ *is log-concave.*

Proof. Take any (p, r, c) such that $u_i(t, p, r, c) > 0$, and let $P = \sum_j p_j$ and $R = \sum_j r_j$. First, assume $\sum_{j \neq i} \phi(c_j) > 0$, so that u_i is continuously differentiable in (p_i, r_i, c_i). We have

$$\frac{\partial \log u_i(t, p, r, c)}{\partial p_i} = \frac{1}{P},$$

$$\frac{\partial \log u_i(t, p, r, c)}{\partial r_i} = \frac{-tg'(R)}{1 - tg(R)},$$

$$\frac{\partial \log u_i(t, p, r, c)}{\partial c_i} = \hat{\phi}'(c_i)(1 - \omega_i(c)),$$

and therefore

$$\frac{\partial^2 \log u_i(t, p, r, c)}{\partial p_i^2} = \frac{-1}{P^2} < 0,$$

$$\frac{\partial^2 \log u_i(t, p, r, c)}{\partial r_i^2} = \frac{-tg''(R)(1 - tg(R)) - (tg'(R))^2}{(1 - tg(R))^2} \leq 0,$$

$$\frac{\partial^2 \log u_i(t, p, r, c)}{\partial c_i^2} = \hat{\phi}''(c_i)(1 - \omega_i(c)) - \hat{\phi}'(c_i)\frac{\partial \omega_i(c)}{\partial c_i} \leq 0,$$

$$\frac{\partial^2 \log u_i(t, p, r, c)}{\partial p_i \partial r_i} = \frac{\partial^2 \log u_i(t, p, r, c)}{\partial p_i \partial c_i} = \frac{\partial^2 \log u_i(t, p, r, c)}{\partial r_i \partial c_i} = 0,$$

so $\log u_i$ is concave in (p_i, r_i, c_i). By the same token, $\bar{\tau}(t, r) \times f(p)$ is log-concave in (p, r) regardless of whether $\sum_{j \neq i} \phi(c_j) > 0$.

Now assume $\sum_{j \neq i} \phi(c_j) = 0$. Take any (p'_i, r'_i, c'_i) such that $u_i(t, p', r', c') > 0$, where $(p', r', c') = ((p'_i, p_{-i}), (r'_i, r_{-i}), (c'_i, c_{-i}))$. Take any $\alpha \in [0, 1]$, and let $(p^\alpha, r^\alpha, c^\alpha) = \alpha(p, r, c) + (1 - \alpha)(p', r', c')$. If $c_i = c'_i = 0$, then $\omega_i(c^\alpha) = \omega_i(c) = \omega_i(c') = 1/N$ and thus

$$\log u_i(t, p^\alpha, r^\alpha, c^\alpha) = \log \frac{1}{N} + \log \bar{\tau}(t, r^\alpha) + \log f(p^\alpha)$$

$$\geq \log \frac{1}{N} + \alpha \left(\log \bar{\tau}(t, r) + \log f(p)\right)$$

$$+ (1 - \alpha)\left(\log \bar{\tau}(t, r') + \log f(p')\right)$$

$$= \alpha \log u_i(t, p, r, c) + (1 - \alpha) \log u_i(t, p', r', c'),$$

where the inequality follows from the log-concavity of $\bar{\tau}(t, r) \times f(p)$ in (p, r). If $c_i > 0$ and $c'_i = 0$, then $\omega_i(c^\alpha) = \omega_i(c) = 1$, $\omega_i(c') = 1/N$, and thus

$$\log u_i(t, p^\alpha, r^\alpha, c^\alpha) = \log \bar{\tau}(t, r^\alpha) + \log f(p^\alpha)$$

$$\geq \alpha \left(\log \bar{\tau}(t, r) + \log f(p)\right)$$

$$+ (1 - \alpha)\left(\log \frac{1}{N} + \log \bar{\tau}(t, r') + \log f(p')\right)$$

$$= \alpha \log u_i(t, p, r, c) + (1 - \alpha) \log u_i(t, p', r', c').$$

The same argument holds in case $c_i = 0$ and $c'_i > 0$. It is easy to see that the same conclusion

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holds if $c_i > 0$ and $c'_i > 0$, in which case $\omega_i(c^\alpha) = \omega_i(c) = \omega_i(c') = 1$. Therefore, $\log u_i$ is concave in (p_i, r_i, c_i). $\qquad\square$

Equilibrium existence follows immediately from the two preceding lemmas.

Proposition 10. $\Gamma(t)$ *has a pure strategy equilibrium.*

Proof. The strategy space Σ is compact, each payoff function u_i is bounded on Σ, and $\Gamma(t)$ is better-reply secure (Lemma 1) and quasiconcave (Lemma 2). Therefore, a pure strategy equilibrium exists (Reny 1999, Theorem 3.1). $\qquad\square$

I now turn to the question of uniqueness. I show that although $\Gamma(t)$ may have multiple equilibria, these equilibria are identical in terms of three essential characteristics: total production, $\sum_i p_i$; total resistance, $\sum_i r_i$; and the vector of individual expenditures on internal conflict, c.

To prove essential uniqueness, I must characterize the equilibrium more fully than I have up to this point. The following result rules out equilibria in which (1) a faction's share in the internal competition is zero or (2) a faction could raise its share to one by an infinitesimal change in strategy.

Lemma 3. *If $N > 1$, then each $\phi(c_i) > 0$ in any equilibrium of $\Gamma(t)$.*

Proof. Assume $N > 1$, and let (p, r, c) be a strategy profile of $\Gamma(t)$ in which $c_i = 0$ for some $i \in \mathcal{N}$. The claim holds trivially if $\phi(0) > 0$, so assume $\phi(0) = 0$. If $\Phi(c) > 0$ or $\bar{\tau}(t, r) \times f(p) = 0$, then $u_i(t, p, r, c) = 0$. But i could ensure a strictly positive payoff with any strategy that allocated nonzero labor to production, resistance, and conflict, so (p, r, c) is not an equilibrium. Conversely, suppose $\Phi(c) = 0$, which implies $c_j = 0$ for all $j \in \mathcal{N}$, and $\bar{\tau}(t, r) \times f(p) > 0$. Then $u_i(t, p, r, c) = (\bar{\tau}(t, r) \times f(p))/N$. But i could obtain a payoff arbitrarily close to $\bar{\tau}(t, r) \times f(p)$ by diverting an infinitesimal amount of labor away from production or resistance and into internal conflict, so (p, r, c) is not an equilibrium. $\qquad\square$

This result is important because it implies the game is continuously differentiable in the neighborhood of any equilibrium. Equilibria can therefore be characterized in terms of first-order conditions.

Lemma 4. (p', r', c') *is an equilibrium of $\Gamma(t)$ if and only if, for each $i \in \mathcal{N}$,*

$$p'_i \left(\pi_i^p \frac{\partial \log f(p')}{\partial p_i} - \mu_i \right) = 0, \tag{12}$$

$$r'_i \left(\pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} - \mu_i \right) = 0, \tag{13}$$

$$c'_i \left(\pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} - \mu_i \right) = 0, \tag{14}$$

$$\frac{p'_i}{\pi_i^p} + \frac{r'_i}{\pi_i^r} + \frac{c'_i}{\pi_i^c} - L_i = 0, \tag{15}$$

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where

$$\mu_i = \max\left\{\pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i}, \pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i}\right\}.$$

Proof. In equilibrium, each faction's strategy must solve the constrained maximization problem

$$\max_{p_i, r_i, c_i} \quad \log u_i(t, p, r, c)$$

$$\text{s.t.} \quad \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} - L_i = 0,$$

$$p_i \geq 0, r_i \geq 0, c_i \geq 0.$$

It follows from Lemma 3 that each u_i is C^1 in (p_i, r_i, c_i) in a neighborhood of any equilibrium. This allows use of the Karush–Kuhn–Tucker conditions to characterize solutions of the above problem. The "only if" direction holds because (12)–(15) are the first-order conditions for the problem and the linearity constraint qualification holds. The "if" direction holds because $\log u_i$ is concave in (p_i, r_i, c_i), per Lemma 2. □

A weak welfare optimality result follows almost immediately from this equilibrium characterization. If (p', r', c') is an equilibrium of $\Gamma(t)$, then there is no other equilibrium (p'', r'', c'') such that $c'' = c'$ and $\bar{\tau}(t, r'') \times f(p'') > \bar{\tau}(t, r') \times f(p')$. In other words, taking as fixed the factions' allocations toward internal conflict, there is no inefficient misallocation of labor between production and resistance.

Corollary 1. *If (p', r', c') is an equilibrium of $\Gamma(t)$, then (p', r') solves*

$$\max_{p, r} \quad \log \bar{\tau}(t, r) + \log f(p)$$

$$\text{s.t.} \quad \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} = L_i - \frac{c_i'}{\pi_i^c}, \qquad i = 1, \ldots, N,$$

$$p_i \geq 0, r_i \geq 0, \qquad\qquad i = 1, \ldots, N.$$

Proof. This is a C^1 concave maximization problem with linear constraints, so the Karush–Kuhn–Tucker first-order conditions are necessary and sufficient for a solution. The result then follows from Lemma 4. □

I next prove that if post-tax output is weakly greater in one equilibrium of $\Gamma(t)$ than another, then each of the two individual components (production and the factions' total share) is weakly greater. The proof relies on the fact that if $c_i' \leq c_i''$ and $\omega_i(c') \leq \omega_i(c'')$, then

$$\frac{\partial \log \omega_i(c')}{\partial c_i} = \hat{\phi}'(c_i')(1 - \omega_i(c')) \geq \hat{\phi}'(c_i'')(1 - \omega_i(c'')) = \frac{\partial \log \omega_i(c'')}{\partial c_i}.$$

If in addition $\omega_i(c') < \omega_i(c'')$, the inequality is strict.

Lemma 5. *If (p', r', c') and (p'', r'', c'') are equilibria of $\Gamma(t)$ such that $\bar{\tau}(t, r') \times f(p') \geq \bar{\tau}(t, r'') \times f(p'')$, then $\bar{\tau}(t, r') \geq \bar{\tau}(t, r'')$ and $f(p') \geq f(p'')$.*

Proof. Suppose the claim of the lemma does not hold, so there exist equilibria such that $\bar{\tau}(t, r') \times f(p') \geq \bar{\tau}(t, r'') \times f(p'')$ but $\bar{\tau}(t, r') < \bar{\tau}(t, r'')$. Together, these inequalities imply $f(p') > f(p'')$. (The proof in case $\bar{\tau}(t, r') > \bar{\tau}(t, r'')$ and $f(p') < f(p'')$ is analogous.)

I will first establish that $p_i' > 0$ implies $r_i'' = 0$. Per Lemma 4 and the log-concavity of f and $\bar{\tau}$, $p_i' > 0$ implies

$$\pi_i^p \frac{\partial \log f(p'')}{\partial p_i} > \pi_i^p \frac{\partial \log f(p')}{\partial p_i} \geq \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} > \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i}.$$

Therefore, again by Lemma 4, $r_i'' = 0$.

Next, I establish that $\Phi(c'') > \Phi(c')$. Since $f(p') > f(p'')$, there is a faction $i \in \mathcal{N}$ such that $p_i' > p_i''$. As this implies $r_i'' = 0$, the budget constraint gives $c_i'' > c_i'$. If $\Phi(c'') \leq \Phi(c')$, then $\omega_i(c'') > \omega_i(c')$ and thus by Lemma 4

$$\pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} > \pi_i^c \frac{\partial \log \omega_i(c'')}{\partial c_i} \geq \pi_i^p \frac{\partial \log f(p'')}{\partial p_i} > \pi_i^p \frac{\partial \log f(p')}{\partial p_i}.$$

But this implies $p_i' = 0$, a contradiction. Therefore, $\Phi(c'') > \Phi(c')$.

Using these intermediate results, I can now establish the main claim by contradiction. Since $\bar{\tau}(t, r'') > \bar{\tau}(t, r')$, there is a faction $j \in \mathcal{N}$ such that $r_j'' > r_j'$. This implies $p_j' = 0$, so the budget constraint gives $c_j'' < c_j'$. Since $\Phi(c'') > \Phi(c')$, this in turn gives $\omega_j(c'') < \omega_j(c')$ and thus

$$\pi_j^c \frac{\partial \log \omega_j(c'')}{\partial c_j} > \pi_j^c \frac{\partial \log \omega_j(c')}{\partial c_j} \geq \pi_j^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j} > \pi_j^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j}.$$

But this implies $r_j'' = 0$, a contradiction. $\square$

I can now state and prove the essential uniqueness of the equilibrium of each labor allocation subgame.

Proposition 11. *If (p', r', c') and (p'', r'', c'') are equilibria of $\Gamma(t)$, then $f(p') = f(p'')$, $\bar{\tau}(t, r') = \bar{\tau}(t, r'')$, and $c' = c''$.*

Proof. First I prove that $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$. Suppose not, so that, without loss of generality, $\bar{\tau}(t, r') \times f(p') > \bar{\tau}(t, r'') \times f(p'')$. Then Lemma 5 implies $\bar{\tau}(t, r') \geq \bar{\tau}(t, r'')$ and $f(p') \geq f(p'')$, at least one strictly so, and thus

$$\max \left\{ \pi_i^p \frac{\partial \log f(p'')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i} \right\} \geq \max \left\{ \pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} \right\}$$

for all $i \in \mathcal{N}$, strictly so for some $j \in \mathcal{N}$. Since $\bar{\tau}(t, r'') \times f(p'') < \bar{\tau}(t, r') \times f(p')$, it follows from Corollary 1 that the set $\mathcal{N}^+ = \{ i \in \mathcal{N} \mid c_i'' > c_i' \}$ is nonempty. For any $i \in \mathcal{N}^+$ such that $\omega_i(c'') > \omega_i(c')$,

$$\pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} > \pi_i^c \frac{\partial \log \omega_i(c'')}{\partial c_i}$$

$$\geq \max\left\{\pi_i^p \frac{\partial \log f(p'')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i}\right\}$$

$$\geq \max\left\{\pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i}\right\}.$$

But this implies $p_i' = r_i' = 0$, contradicting $c_i'' > c_i'$. So $\omega_i(c'') \leq \omega_i(c')$ for all $i \in \mathcal{N}^+$. Since $\mathcal{N}^+$ is nonempty and the conflict shares are increasing in c_i and sum to one, this can hold only if $\mathcal{N}^+ = \mathcal{N}$ and $\omega_i(c'') = \omega_i(c')$ for all $i \in \mathcal{N}$. This implies

$$\pi_j^c \frac{\partial \log \omega_j(c')}{\partial c_j} \geq \pi_j^c \frac{\partial \log \omega_j(c'')}{\partial c_j}$$

$$\geq \max\left\{\pi_j^p \frac{\partial \log f(p'')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j}\right\}$$

$$> \max\left\{\pi_j^p \frac{\partial \log f(p')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j}\right\},$$

which in turn implies $p_j' = r_j' = 0$, contradicting $c_j'' > c_j'$. I conclude that $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$ and thus, by Lemma 5, $\bar{\tau}(t, r') = \bar{\tau}(t, r'')$ and $f(p') = f(p'')$.

Next, I prove that $c' = c''$. Suppose not, so $c' \neq c''$. Without loss of generality, suppose $\Phi(c') \geq \Phi(c'')$. Since $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$ yet $c' \neq c''$, by Corollary 1 there exists $i \in \mathcal{N}$ such that $c_i' > c_i''$ and $j \in \mathcal{N}$ such that $c_j' < c_j''$. It follows from $\Phi(c') \geq \Phi(c'')$ that $\omega_j(c') < \omega_j(c'')$ and therefore

$$\pi_j^c \frac{\partial \log \omega_j(c')}{\partial c_j} > \pi_j^c \frac{\partial \log \omega_j(c'')}{\partial c_j}$$

$$\geq \max\left\{\pi_j^p \frac{\partial \log f(p'')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j}\right\}$$

$$= \max\left\{\pi_j^p \frac{\partial \log f(p')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j}\right\}.$$

But this implies $p_j' = r_j' = 0$, contradicting $c_j'' > c_j'$. $\qquad \square$

Proposition 11 allows me to write the equilibrium values of total production, total resistance, and individual conflict allocations as functions of the tax rate. For each $t \in [0, 1]$, let $P^*(t) = P$ if and only if there is an equilibrium (p, r, c) of $\Gamma(t)$ such that $\sum_i p_i = P$. Let the functions $R^*(t)$ and $c^*(t)$, the latter of which is vector-valued, be defined analogously.

The only remaining step to prove the existence of an equilibrium in the full game is to show that an optimal tax rate exists. An important consequence of Proposition 11 is that the optimal tax rate (if one exists) does not depend on the equilibrium that is selected in each labor allocation subgame, since the government's payoff depends only on total production and resistance. The main step toward proving the existence of an optimal tax rate is to show that total production and resistance are continuous in t.

Lemma 6. P^*, R^*, and c^* are continuous.

Proof. Define the equilibrium correspondence $E : [0,1] \rightrightarrows \Sigma$ by

$$E(t) = \{(p,r,c) \,|\, (p,r,c) \text{ is an equilibrium of } \Gamma(t)\}.$$

Standard arguments (e.g., Fudenberg and Tirole 1991, 30–32) imply that E has a closed graph.[18] This in turn implies E is upper hemicontinuous, as its codomain, Σ, is compact. Let $F : \Sigma \to \mathbb{R}_+^{N+2}$ be the function defined by $F(p,r,c) = (\sum_i p_i, \sum_i r_i, c)$. Since F is continuous as a function, it is upper hemicontinuous as a correspondence. Then we can write the functions in the lemma as the composition of F and E:

$$(P^*(t), R^*(t), c^*(t)) = \{F(p,r,c) \,|\, (p,r,c) \in E(t)\} = (F \circ E)(t).$$

As the composition of upper hemicontinuous correspondences, (P^*, R^*, c^*) is upper hemicontinuous (Aliprantis and Border 2006, Theorem 17.23). Then, as an upper hemicontinuous correspondence that is single-valued (per Proposition 11), (P^*, R^*, c^*) is continuous as a function. $\qquad\square$

Continuity of total production and resistance in the tax rate imply that the government's payoff is continuous in the tax rate, so an equilibrium exists.

Proposition 12. *There is a pure strategy equilibrium.*

Proof. For each labor allocation subgame $\Gamma(t)$, let $\sigma^*(t)$ be a pure strategy equilibrium of $\Gamma(t)$. Proposition 10 guarantees the existence of these equilibria. By Proposition 11, the government's payoff from any $t \in [0,1]$ is

$$u_G(t, \sigma^*(t)) = t \times g(R^*(t)) \times P^*(t).$$

This expression is continuous in t, per Lemma 6, and therefore attains its maximum on the compact interval $[0,1]$. A maximizer t^* exists, and the pure strategy profile $(t^*, (\sigma^*(t))_{t \in [0,1]})$ is an equilibrium. $\qquad\square$

A.3 Proof of Propositions 1 and 2

In these and all remaining proofs, I consider the special symmetric case of the model discussed in the text, in which each $\pi_i^p = \pi^p$, $\pi_i^r = \pi^r$, $\pi_i^c = \pi^c$, and $L_i = L/N$. An important initial result for the symmetric case is that in every equilibrium of every labor allocation subgame, every faction spends the same amount on internal conflict.

Lemma 7. *If the game is symmetric and (p,r,c) is an equilibrium of $\Gamma(t)$, then $c_i = c_j$ for all $i, j \in \mathcal{N}$.*

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[18]The only complication in applying the usual argument is that the model is discontinuous at $c = 0$ in case $\phi(0) = 0$. However, by the same arguments as in the proof of Lemma 1, if $\phi(0) = 0$ there cannot be a sequence $(t^k, (p^k, r^k, c^k))$ in the graph of E such that $c^k \to 0$.

Proof. Consider an equilibrium in which $c_i > c_j$. This implies $\omega_i(c) > \omega_j(c)$ and therefore

$$\pi^c \frac{\partial \log \omega_j(c)}{\partial c_j} > \pi^c \frac{\partial \log \omega_i(c)}{\partial c_i}$$

$$\geq \max \left\{ \pi^p \frac{\partial \log f(p)}{\partial p_i}, \pi^r \frac{\partial \log \bar{\tau}(t,r)}{\partial r_i} \right\}$$

$$= \max \left\{ \pi^p \frac{\partial \log f(p)}{\partial p_j}, \pi^r \frac{\partial \log \bar{\tau}(t,r)}{\partial r_j} \right\}.$$

Lemma 4 then gives $p_j = r_j = 0$. But since i and j have the same budget constraint, this contradicts $c_i > c_j$. $\qquad\square$

In the equilibrium of each labor allocation subgame, total production, total resistance, and the amount each faction spends on conflict (the same for all factions by Lemma 7) solve some subset of the following system of equations. These equations give the conditions for equal marginal benefits per unit of labor across activities, as well as the budget constraint (1). I write them as functions of t as well as the exogenous parameters $\pi = (\pi^p, \pi^r, \pi^c, L, N)$ to allow for comparative statics via implicit differentiation:

$$Q^{pr}(P, R, C; t, \pi) = \pi^p(1 - tg(R)) + \pi^r tP g'(R) \;\; = 0, \tag{16}$$

$$Q^{pc}(P, R, C; t, \pi) = \frac{\pi^p}{P} - \frac{N-1}{N} \pi^c \hat{\phi}'(C) \qquad\quad = 0, \tag{17}$$

$$Q^{rc}(P, R, C; t, \pi) = \frac{\pi^r tg'(R)}{1 - tg(R)} + \frac{N-1}{N} \pi^c \hat{\phi}'(C) = 0, \tag{18}$$

$$Q^b(P, R, C; t, \pi) = L - \frac{P}{\pi^p} - \frac{R}{\pi^r} - \frac{NC}{\pi^c} \qquad = 0. \tag{19}$$

The condition (18) is redundant when (16) and (17) both hold, but I use it later when modeling a capital-financed government.

The quantities defined in Propositions 1 and 2 are as follows. $(\bar{P}_0, \bar{c}_0)$ is the solution to the system

$$Q^{pc}(\bar{P}_0, 0, \bar{c}_0; t, \pi) = \frac{\pi^p}{\bar{P}_0} - \frac{N-1}{N} \pi^c \hat{\phi}'(\bar{c}_0) = 0, \tag{20}$$

$$Q^b(\bar{P}_0, 0, \bar{c}_0; t, \pi) = L - \frac{\bar{P}_0}{\pi^p} - \frac{N\bar{c}_0}{\pi^c} \qquad = 0. \tag{21}$$

$(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t))$ is the solution to the system

$$Q^{pr}(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = \pi^p(1 - tg(\tilde{R}_1(t)) + \pi^r t\tilde{P}_1(t)g'(\tilde{R}_1(t)) = 0, \tag{22}$$

$$Q^{pc}(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = \frac{\pi^p}{\tilde{P}_1(t)} - \frac{N-1}{N} \pi^c \hat{\phi}'(\tilde{c}_1(t)) \qquad = 0, \tag{23}$$

$$Q^b(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = L - \frac{\tilde{P}_1(t)}{\pi^p} - \frac{\tilde{R}_1(t)}{\pi^r} - \frac{N\tilde{c}_1(t)}{\pi^c} \qquad = 0. \tag{24}$$

$(\tilde{P}_2(t), \tilde{R}_2(t))$ is the solution to the system

$$Q^{pr}(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi) = \pi^p(1 - tg(\tilde{R}_2(t)) + \pi^r t\tilde{P}_2(t)g'(\tilde{R}_2(t)) = 0, \qquad (25)$$

$$Q^b(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi) = L - \frac{\tilde{P}_2(t)}{\pi^p} - \frac{\tilde{R}_2(t)}{\pi^r} = 0. \qquad (26)$$

The first cutpoint tax rate is

$$\hat{t}_0 = \frac{\pi^p}{\pi^p - \pi^r \bar{P}_0 g'(0)}. \qquad (27)$$

Lemma 8 below shows that $\bar{P}_0 > 0$ and therefore, since $g'(0) < 0$, that $\hat{t}_0 < 1$. The second cutpoint tax rate is

$$\hat{t}_1 = \frac{\pi^p}{\pi^p g(\bar{R}_1) - \pi^r \bar{P}_1 g'(\bar{R}_1)}, \qquad (28)$$

where

$$\bar{P}_1 = \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(0)}, \qquad (29)$$

$$\bar{R}_1 = \pi^r \left(L - \frac{\bar{P}_1}{\pi^p} \right). \qquad (30)$$

The next three lemmas give conditions on the tax rate under which there is positive production, resistance, and internal conflict in the equilibrium of the labor allocation subgame. Jointly, these lemmas constitute the bulk of the proof of Propositions 1 and 2. The proofs rely on the following equalities:

$$\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, 0)}{\partial r_i} = -\pi^r \frac{\hat{t}_0 g'(0)}{1 - \hat{t}_0} = \frac{\pi^p}{\bar{P}_0},$$

$$\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, (\bar{R}_1/N)\mathbf{1}_N)}{\partial r_i} = -\pi^r \frac{\hat{t}_1 g'(\bar{R}_1)}{1 - \hat{t}_1 g(\bar{R}_1)} = \frac{\pi^p}{\bar{P}_1}$$

for all $i \in \mathcal{N}$, where $\mathbf{1}_N$ is the N-vector each of whose elements equals one.

Lemma 8. *If the game is symmetric, Assumption 1 holds, and (p, r, c) is an equilibrium of $\Gamma(t)$, then $0 < \sum_i p_i \le \bar{P}_0 < \pi^p L$.*

Proof. Assumption 1 implies

$$Q^{pc}(\pi^p L, 0, 0; 0, \pi) = \frac{1}{L} - \frac{N-1}{N} \pi^c \hat{\phi}'(0) < 0.$$

Since Q^{pc} is decreasing in P and weakly increasing in C, this gives $\bar{P}_0 < \pi^p L$.

Let $P = \sum_i p_i$, and suppose $P > \bar{P}_0$. The budget constraint and Lemma 7 then give $c_i = C < \bar{c}_0$ for each $i \in \mathcal{N}$. But then we have

$$\pi^c \frac{\partial \log \omega_i(c)}{\partial c_i} \ge \frac{N-1}{N} \pi^c \hat{\phi}'(\bar{c}_0) = \frac{\pi^p}{\bar{P}_0} > \pi^p \frac{\partial \log f(p)}{\partial p_i}$$

for each $i \in \mathcal{N}$. By Lemma 4, this implies each $p_i = 0$, a contradiction. Therefore, $P \leq \bar{P}_0$.

Finally, since $P = 0$ implies each $u_i(t, p, r, c) = 0$, but any faction can assure itself a positive payoff with any $(p_i, r_i, c_i) \gg 0$, in equilibrium $P > 0$. $\qquad\square$

Lemma 9. *If the game is symmetric, Assumption 1 holds, and (p, r, c) is an equilibrium of $\Gamma(t)$, then $\sum_i r_i > 0$ if and only if $t > \hat{t}_0$.*

Proof. Let $P = \sum_i p_i$ and $R = \sum_i r_i$. To prove the "if" direction, suppose $t > \hat{t}_0$ and $R = 0$. Since $P \leq \bar{P}_0$, this implies each $c_i = C > 0$; the first-order conditions of Lemma 4 then give $P = \bar{P}_0$ and $C = \bar{c}_0$. It follows that

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, r)}{\partial r_i} = \frac{\pi^p}{\bar{P}_0} = \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

This implies each $p_i = 0$, a contradiction.

To prove the "only if" direction, suppose $t \leq \hat{t}_0$ and $R > 0$. For each $i \in \mathcal{N}$,

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, 0)}{\partial r_i} = \frac{\pi^p}{\bar{P}_0} \leq \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

This implies each $r_i = 0$, a contradiction. $\qquad\square$

Lemma 10. *If the game is symmetric and Assumption 1 holds, then $\hat{t}_1 > \hat{t}_0$. If, in addition, (p, r, c) is an equilibrium of $\Gamma(t)$, then each $c_i > 0$ if and only if $t < \hat{t}_1$.*

Proof. To prove that $\hat{t}_1 > \hat{t}_0$, note that

$$\bar{P}_1 = \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(0)} \leq \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(\bar{c}_0)} = \bar{P}_0$$

by log-concavity of ϕ. This implies $\bar{R}_1 > 0$, so $g(\bar{R}_1) < g(0) = 1$ and $g'(0) \leq g'(\bar{R}_1) < 0$. Therefore,

$$\pi^p - \pi^r \bar{P}_0 g'(0) > \pi^p g(\bar{R}_1) - \pi^r \bar{P}_1 g'(\bar{R}_1) > 0,$$

which implies $\hat{t}_1 > \hat{t}_0$.

Let $P = \sum_i p_i$ and $R = \sum_i r_i$. To prove the "if" direction of the second statement, suppose $t \geq \hat{t}_1$ and some $c_i > 0$. By Lemma 7, $c_j = c_i = C > 0$ for each $j \in \mathcal{N}$. Since $P > 0$ by Lemma 8, the first-order conditions give

$$P = \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(C)} \geq \bar{P}_1.$$

The budget constraint then gives $R < \bar{R}_1$ and thus

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, (\bar{R}_1/N)\mathbf{1}_N)}{\partial r_i} = \frac{\pi^p}{\bar{P}_1} \geq \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

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But this implies each $p_i = 0$, a contradiction.

To prove the "only if" direction, suppose $t < \hat{t}_1$ and each $c_i = 0$. The first-order conditions then give $P \leq \bar{P}_1$, so $R \geq \bar{R}_1 > 0$ by the budget constraint. This in turn gives

$$\pi^p \frac{\partial \log f(p)}{\partial p_i} = \frac{\pi^p}{P} \geq \frac{\pi^p}{\bar{P}_1} \geq \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i}$$

for each $i \in \mathcal{N}$. But this implies each $r_i = 0$, a contradiction. $\qquad\square$

The last thing we need to prove the propositions is how the labor allocations change with the tax rate when $t > \hat{t}_0$.

Lemma 11. *Let the game be symmetric and Assumption 1 hold. For all $t \in (\hat{t}_0, \hat{t}_1)$,*

$$\frac{d\tilde{P}_1(t)}{dt} = \frac{-(N-1)\pi^p \pi^c \hat{\phi}''(\tilde{c}_1(t))}{N\pi^r t \Delta_1(t)} \leq 0, \tag{31}$$

$$\frac{d\tilde{R}_1(t)}{dt} = \frac{-\pi^p \left(N\pi^p/\pi^c \tilde{P}_1(t)^2 - (N-1)\pi^c \hat{\phi}''(\tilde{c}_1(t))/N\pi^p\right)}{t\Delta_1(t)} > 0, \tag{32}$$

$$\frac{d\tilde{c}_1(t)}{dt} = \frac{(\pi^p)^2}{\pi^r t \tilde{P}_1(t)^2 \Delta_1(t)} < 0, \tag{33}$$

where

$$\Delta_1(t) = \left(\pi^p t g'(\tilde{R}_1(t)) - \pi^r t \tilde{P}_1(t) g''(\tilde{R}_1(t))\right) \left(\frac{N\pi^p}{\pi^c \tilde{P}_1(t)^2} - \frac{N-1}{N} \frac{\pi^c}{\pi^p} \hat{\phi}''(\tilde{c}_1(t))\right)$$
$$- \frac{N-1}{N} \pi^c t g'(\tilde{R}_1(t)) \hat{\phi}''(\tilde{c}_1(t)) \tag{34}$$
$$< 0.$$

For all $t > \hat{t}_1$,

$$\frac{d\tilde{P}_2(t)}{dt} = \frac{-\pi^p}{\pi^r t \Delta_2(t)} < 0, \tag{35}$$

$$\frac{d\tilde{R}_2(t)}{dt} = \frac{1}{t\Delta_2(t)} > 0, \tag{36}$$

where

$$\Delta_2(t) = \frac{\pi^r}{\pi^p} t \tilde{P}_2(t) g''(\tilde{R}_2(t)) - 2t g'(\tilde{R}_2(t)) > 0. \tag{37}$$

Proof. Throughout the proof, let $\eta = (N-1)/N$.

First consider $t \in (\hat{t}_0, \hat{t}_1)$. To reduce clutter in what follows, I omit the evaluation point $(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi)$ from all partial derivative expressions. The Jacobian of the system

of equations that defines $(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t))$ is

$$
\begin{aligned}
\mathbf{J}_1(t) &= \begin{bmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R & \partial Q^{pr}/\partial C \\ \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial R & \partial Q^{pc}/\partial C \\ \partial Q^{b}/\partial P & \partial Q^{b}/\partial R & \partial Q^{b}/\partial C \end{bmatrix} \\
&= \begin{bmatrix} \pi^r t g'(\tilde{R}_1(t)) & \pi^r t \tilde{P}_1(t) g''(\tilde{R}_1(t)) - \pi^p t g'(\tilde{R}_1(t)) & 0 \\ -\pi^p/\tilde{P}_1(t)^2 & 0 & -\eta\pi^c\hat{\phi}''(\tilde{c}_1(t)) \\ -1/\pi^p & -1/\pi^r & -N/\pi^c \end{bmatrix}.
\end{aligned}
$$

It is easy to verify that $|\mathbf{J}_1(t)| = \Delta_1(t) < 0$. Notice that

$$
\begin{aligned}
\frac{\partial Q^{pr}}{\partial t} &= \pi^r \tilde{P}_1(t) g'(\tilde{R}_1(t)) - \pi^p g(\tilde{R}_1(t)) \\
&= \pi^r \left(-\frac{\pi^p(1 - t g(\tilde{R}_1(t)))}{\pi^r t g'(\tilde{R}_1(t))} \right) g'(\tilde{R}_1(t)) - \pi^p g(\tilde{R}_1(t)) \\
&= -\frac{\pi^p}{t}.
\end{aligned}
$$

Then, by the implicit function theorem and Cramer's rule,

$$
\begin{aligned}
\frac{d\tilde{P}_1(t)}{dt} &= \frac{\begin{vmatrix} -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial R & \partial Q^{pr}/\partial C \\ -\partial Q^{pc}/\partial t & \partial Q^{pc}/\partial R & \partial Q^{pc}/\partial C \\ -\partial Q^{b}/\partial t & \partial Q^{b}/\partial R & \partial Q^{b}/\partial C \end{vmatrix}}{|\mathbf{J}_1(t)|} \\
&= \frac{-\eta\pi^p\pi^c\hat{\phi}''(\tilde{c}_1(t))}{\pi^r t \Delta_1(t)} \\
&\leq 0, \\
\frac{d\tilde{R}_1(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial C \\ \partial Q^{pc}/\partial P & -\partial Q^{pc}/\partial t & \partial Q^{pc}/\partial C \\ \partial Q^{b}/\partial P & -\partial Q^{b}/\partial t & \partial Q^{b}/\partial C \end{vmatrix}}{|\mathbf{J}_1(t)|} \\
&= \frac{-\pi^p \left(N\pi^p/\pi^c \tilde{P}_1(t)^2 - \eta\pi^c\hat{\phi}''(\tilde{c}_1(t))/\pi^p \right)}{t\Delta_1(t)} \\
&> 0, \\
\frac{d\tilde{c}_1(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R & -\partial Q^{pr}/\partial t \\ \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial R & -\partial Q^{pc}/\partial t \\ \partial Q^{b}/\partial P & \partial Q^{b}/\partial R & -\partial Q^{b}/\partial t \end{vmatrix}}{|\mathbf{J}_1(t)|} \\
&= \frac{(\pi^p)^2}{\pi^r t \tilde{P}_1(t)^2 \Delta_1(t)} \\
&< 0,
\end{aligned}
$$

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as claimed.

Now consider $t > \hat{t}_1$. Again to reduce clutter in what follows, I omit the evaluation point $(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi)$ from all partial derivative expressions. The Jacobian of the system of equations that defines $(\tilde{P}_2(t), \tilde{R}_2(t))$ is

$$
\mathbf{J}_2(t) = \begin{bmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R \\ \partial Q^b/\partial P & \partial Q^b/\partial R \end{bmatrix}
$$
$$
= \begin{bmatrix} \pi^r t g'(\tilde{R}_2(t)) & \pi^r t \tilde{P}_2(t) g''(\tilde{R}_2(t)) - \pi^p t g'(\tilde{R}_2(t)) \\ -1/\pi^p & -1/\pi^r \end{bmatrix}.
$$

It is easy to verify that $|\mathbf{J}_2(t)| = \Delta_2(t) > 0$. As before, $\partial Q^{pr}/\partial t = -\pi^p/t$. So by the implicit function theorem and Cramer's rule,

$$
\frac{d\tilde{P}_2(t)}{dt} = \frac{\begin{vmatrix} -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial R \\ -\partial Q^b/\partial t & \partial Q^b/\partial R \end{vmatrix}}{|\mathbf{J}_2(t)|}
$$
$$
= \frac{-\pi^p}{\pi^r t \Delta_2(t)}
$$
$$
< 0,
$$
$$
\frac{d\tilde{R}_2(t)}{dt} = \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & -\partial Q^{pr}/\partial t \\ \partial Q^b/\partial P & -\partial Q^b/\partial t \end{vmatrix}}{|\mathbf{J}_2(t)|}
$$
$$
= \frac{1}{t \Delta_2(t)}
$$
$$
> 0,
$$

as claimed. □

The proofs of Propositions 1 and 2 follow almost immediately from these lemmas. I prove them jointly.

Proposition 1 (Baseline Equilibrium). *There is a tax rate $\hat{t}_0 \in (0, 1)$ such that $\sum_i r_i = 0$ in every equilibrium of the labor allocation subgame if and only if $t \leq \hat{t}_0$. Every subgame with $t \leq \hat{t}_0$ has the same unique equilibrium, in which $\sum_i p_i = \bar{P}_0 > 0$ and each $c_i = \bar{c}_0 > 0$.*

Proposition 2 (Resistance Equilibrium). *There is a tax rate $\hat{t}_1 > \hat{t}_0$ such that in every equilibrium of the labor allocation subgame with tax rate t:*

- *If $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = \tilde{P}_1(t) > 0$ (weakly decreasing in t), $\sum_i r_i = \tilde{R}_1(t) > 0$ (strictly increasing), and each $c_i = \tilde{c}_1(t) > 0$ (strictly decreasing).*

- *If $t \geq \hat{t}_1$, then $\sum_i p_i = \tilde{P}_2(t) > 0$ (strictly decreasing in t), $\sum_i r_i = \tilde{R}_2(t) > 0$ (strictly increasing), and each $c_i = 0$.*

Proof. For fixed t, every equilibrium of $\Gamma(t)$ has the same total production, total resistance,

and individual conflict allocations, per Proposition 11. Consider any $t \in [0, 1]$ and let (p, r, c) be an equilibrium of $\Gamma(t)$.

If $t \leq \hat{t}_0$, then $\sum_i p_i = P > 0$, $\sum_i r_i = 0$, and each $c_i = C > 0$ by Lemmas 8–10. The first-order conditions (Lemma 4) imply that P and C solve $Q^{pc}(P, 0, C; t, \pi) = Q^b(P, 0, C; t, \pi) = 0$; therefore, $P = \bar{P}_0$ and $C = \bar{c}_0$. Since each $r_i = 0$, each $p_i = \pi^p(L/N - \bar{c}_0/\pi^c) = \bar{P}_0/N$, so the equilibrium is unique.

Similarly, if $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = P > 0$, $\sum_i r_i = R > 0$, and each $c_i = C > 0$ by Lemmas 8–10. The first-order conditions then imply that these solve the system (16)–(19); therefore, $P = \tilde{P}_1(t)$, $R = \tilde{R}_1(t)$, and $C = \tilde{c}_1(t)$. The comparative statics on $\tilde{P}_1$, $\tilde{R}_1$, and $\tilde{c}_1$ follow from Lemma 11.

Finally, if $t \geq \hat{t}_1$, then $\sum_i p_i = P > 0$, $\sum_i r_i = R > 0$, and each $c_i = 0$ by Lemmas 8–10. The first-order conditions then imply that P and R solve $Q^{pr}(P, R, 0; t, \pi) = Q^b(P, R, 0; t, \pi) = 0$; therefore, $P = \tilde{P}_2(t)$ and $R = \tilde{R}_2(t)$. The comparative statics on $\tilde{P}_2$ and $\tilde{R}_2$ follow from Lemma 11. □

A.4 Proof of Proposition 3

Proposition 3 (Optimal Tax Rate). *If the government is labor-financed, there is an equilibrium in which the government chooses the greatest tax rate that engenders no resistance, $t = \hat{t}_0$. If g or ϕ is strictly log-concave, this is the unique equilibrium tax rate.*

Proof. As in the proof of Lemma 11, let $\eta = (N-1)/N$.

For each $t \in [0, 1]$, fix an equilibrium $(p(t), r(t), c(t))$ of $\Gamma(t)$. By Propositions 1, 2, and 11, the government's induced utility function is

$$u_G^*(t) = u_G(t, p(t), r(t), c(t)) = \begin{cases} t\bar{P}_0 & t \leq \hat{t}_0, \\ tg(\tilde{R}_1(t))\tilde{P}_1(t) & \hat{t}_0 < t < \hat{t}_1, \\ tg(\tilde{R}_2(t))\tilde{P}_2(t) & t \geq \hat{t}_1. \end{cases}$$

It is immediate from the above expression that $u_G^*(t) < u_G^*(\hat{t}_0)$ for all $t < \hat{t}_0$.

Now consider $t \in (\hat{t}_0, \hat{t}_1)$. By Lemma 11,

$$\frac{du_G^*(t)}{dt} = g(\tilde{R}_1(t))\tilde{P}_1(t) + tg'(\tilde{R}_1(t))\frac{d\tilde{R}_1(t)}{dt}\tilde{P}_1(t) + tg(\tilde{R}_1(t))\frac{d\tilde{P}_1(t)}{dt}$$

$$= g(\tilde{R}_1(t))\tilde{P}_1(t) - \frac{\pi^p g'(\tilde{R}_1(t))\tilde{P}_1(t)\left(N\pi^p/\pi^c\tilde{P}_1(t)^2 - \eta\pi^c\hat{\phi}''(\tilde{c}_1(t))/\pi^p\right)}{\Delta_1(t)}$$

$$- \frac{\eta\pi^p\pi^c g(\tilde{R}_1(t))\hat{\phi}''(\tilde{c}_1(t))}{\pi^r\Delta_1(t)},$$

where $\Delta_1(t)$ is defined by (34). To reduce clutter in what follows, let $\tilde{P} = \tilde{P}_1(t)$, $\tilde{R} = \tilde{R}_1(t)$, and $\tilde{c} = \tilde{c}_1(t)$. Since $\Delta_1(t) < 0$, the sign of the above expression is the same as that of

$$g'(\tilde{R})\tilde{P}\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) + \frac{\eta\pi^p\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})}{\pi^r} - g(\tilde{R})\tilde{P}\Delta_1(t)$$

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$$= \tilde{P}g'(\tilde{R})\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) + \frac{\eta\pi^p\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})}{\pi^r}$$

$$- \tilde{P}g(\tilde{R})\left(\pi^p tg'(\tilde{R}) - \pi^r t\tilde{P}g''(\tilde{R})\right)\left(\frac{N\pi^p}{\pi^c\tilde{P}^2} - \eta\frac{\pi^c}{\pi^p}\hat{\phi}''(\tilde{c})\right)$$

$$+ \eta\pi^c t\tilde{P}g(\tilde{R})g'(\tilde{R})\hat{\phi}''(\tilde{c})$$

$$= \tilde{P}\left(g'(\tilde{R}) - \frac{g(\tilde{R})\left(\pi^p tg'(\tilde{R}) - \pi^r t\tilde{P}g''(\tilde{R})\right)}{\pi^p}\right)\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right)$$

$$+ \eta\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})\left(\frac{\pi^p}{\pi^r} + t\tilde{P}g'(\tilde{R})\right)$$

$$= \frac{\tilde{P}}{g'(\tilde{R})}(1 - tg(\tilde{R}))\left(g'(\tilde{R})^2 - g(\tilde{R})g''(\tilde{R})\right)\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right)$$

$$+ \eta\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})\left(\frac{\pi^p}{\pi^r}tg(\tilde{R})\right).$$

The first term is weakly negative, strictly so if g is strictly log-concave. The second term is weakly negative, strictly so if ϕ is strictly log-concave. Therefore, $du_G^*(t)/dt \leq 0$ for all $t \in (\hat{t}_0, \hat{t}_1)$, strictly so if g or ϕ is strictly log-concave. This implies $u_G^*(\hat{t}_0) \geq u_G^*(t)$ for all $t \in (\hat{t}_0, \hat{t}_1]$, strictly so if g or ϕ is strictly log-concave.

Finally, consider $t > \hat{t}_1$. Again by Lemma 11,

$$\frac{du_G^*(t)}{dt} = g(\tilde{R}_2(t))\tilde{P}_2(t) + tg'(\tilde{R}_2(t))\frac{d\tilde{R}_2(t)}{dt}\tilde{P}_2(t) + tg(\tilde{R}_2(t))\frac{d\tilde{P}_2(t)}{dt}$$

$$= g(\tilde{R}_2(t))\tilde{P}_2(t) + \frac{g'(\tilde{R}_2(t))\tilde{P}_2(t)}{\Delta_2(t)} - \frac{\pi^p g(\tilde{R}_2(t))}{\pi^r\Delta_2(t)},$$

where $\Delta_2(t)$ is defined by (37). To reduce clutter in what follows, let $\tilde{P} = \tilde{P}_2(t)$ and $\tilde{R} = \tilde{R}_2(t)$. Since $\Delta_2(t) > 0$, the sign of the above expression is the same as that of

$$\tilde{P}g(\tilde{R})\Delta_2(t) + \tilde{P}g'(\tilde{R}) - \frac{\pi^p g(\tilde{R})}{\pi^r}$$

$$= \tilde{P}g(\tilde{R})\left(\frac{\pi^r t\tilde{P}g''(\tilde{R})}{\pi^p} - 2tg'(\tilde{R})\right) + \tilde{P}g'(\tilde{R}) - \frac{\pi^p g(\tilde{R})}{\pi^r}$$

$$= \frac{\pi^p}{\pi^r tg'(\tilde{R})^2}\left((1 - tg(\tilde{R}))^2\left(g(\tilde{R})g''(\tilde{R}) - g'(\tilde{R})^2\right) - \left(tg(\tilde{R}g'(\tilde{R}))\right)^2\right)$$

$$< \frac{\pi^p(1 - tg(\tilde{R}))^2\left(g(\tilde{R})g''(\tilde{R}) - g'(\tilde{R})^2\right)}{\pi^r tg'(\tilde{R})^2}$$

$$\leq 0.$$

Therefore, $u_G^*(\hat{t}_0) \geq u_G^*(\hat{t}_1) > u_G^*(t)$ for all $t > \hat{t}_1$.

Combining these findings, $u_G^*(\hat{t}_0) \geq u_G^*(t)$ for all $t \in [0,1] \setminus \{\hat{t}_0\}$, strictly so if g or ϕ is

strictly log-concave. Therefore, there is an equilibrium in which $t = \hat{t}_0$, and every equilibrium has this tax rate if g or ϕ is strictly log-concave. $\qquad\square$

A.5 Proof of Remarks 1 and 2

The comparative statics in both remarks come from the same system of equations, so I prove them jointly.

Remark 1. *Total production in the baseline equilibrium, $\bar{P}_0$, is strictly decreasing in the number of factions, N.*

Remark 2. *Total production in the baseline equilibrium, $\bar{P}_0$, strictly decreases with a marginal increase in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

Proof. I will treat N as if it were continuous in order to obtain comparative statics by implicit differentiation. Throughout the proof I write $\bar{P}_0$ and $\bar{c}_0$ as functions of (N, π^c).

Recall that $(\bar{P}_0(N, \pi^c), \bar{c}_0(N, \pi^c))$ is defined as the solution to (20) and (21). To reduce clutter in what follows, I omit the evaluation point $(\bar{P}_0(N, \pi^c), 0, \bar{c}_0(N, \pi^c); t, \pi)$ from all partial derivative expressions. The Jacobian of the system is

$$\mathbf{J}_0 = \begin{bmatrix} \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial C \\ \partial Q^b/\partial P & \partial Q^b/\partial C \end{bmatrix} = \begin{bmatrix} -\pi^p/\bar{P}_0(N, \pi^c)^2 & -(N-1)\pi^c\hat{\phi}''(\bar{c}_0(N, \pi^c))/N \\ -1/\pi^p & -N/\pi^c \end{bmatrix},$$

with determinant

$$|\mathbf{J}_0| = \frac{N\pi^p}{\pi^c\bar{P}_0(N, \pi^c)^2} - \frac{N-1}{N}\frac{\pi^c}{\pi^p}\hat{\phi}''(\bar{c}_0(N, \pi^c)) > 0.$$

By the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{\partial \bar{P}_0(N, \pi^c)}{\partial N} &= \frac{\begin{vmatrix} -\partial Q^{pc}/\partial N & \partial Q^{pc}/\partial C \\ -\partial Q^b/\partial N & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_0|} \\
&= \frac{\begin{vmatrix} \pi^c\hat{\phi}'(\bar{c}_0(N, \pi^c))/N^2 & -(N-1)\pi^c\hat{\phi}''(\bar{c}_0(N, \pi^c))/N \\ \bar{c}_0(N, \pi^c)/\pi^c & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_0|} \\
&= \frac{1}{|\mathbf{J}_0|}\left(\frac{N-1}{N}\bar{c}_0(N, \pi^c)\hat{\phi}''(\bar{c}_0(N, \pi^c)) - \frac{\hat{\phi}'(\bar{c}_0(N, \pi^c))}{N}\right) \\
&< 0,
\end{aligned}$$

as claimed. Similarly,

$$\frac{\partial \bar{P}_0(N, \pi^c)}{\partial \pi^c} = \frac{\begin{vmatrix} -\partial Q^{pc}/\partial \pi^c & \partial Q^{pc}/\partial C \\ -\partial Q^b/\partial \pi^c & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_0|}$$

$$= \frac{\begin{vmatrix} (N-1)\hat{\phi}'(\bar{c}_0(N, \pi^c))/N & -(N-1)\pi^c\hat{\phi}''(\bar{c}_0(N, \pi^c))/N \\ -N\bar{c}_0(N, \pi^c)/(\pi^c)^2 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_0|}$$

$$= -\frac{N-1}{\pi^c|\mathbf{J}_0|}\left(\hat{\phi}'(\bar{c}_0(N, \pi^c)) + \bar{c}_0(N, \pi^c)\hat{\phi}''(\bar{c}_0(N, \pi^c))\right),$$

which is negative if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0(N, \pi^c)$. $\square$

I also prove the claim in footnote 11.

Lemma 12. *Let $\theta, \lambda > 0$. If $\phi(C) = \theta \exp(\lambda C)$, then the incentive effect outweighs the labor-saving effect for all $C \geq 0$. If $\phi(C) = \theta C^\lambda$, then the incentive and labor-saving effects are exactly offsetting for all $C > 0$.*

Proof. First consider the difference contest success function, $\phi(C) = \theta \exp(\lambda C)$. Then $\hat{\phi}(C) = \log \theta + \lambda C$, $\hat{\phi}'(C) = \lambda$, and $\hat{\phi}''(C) = 0$ for all $C \geq 0$. Therefore,

$$\hat{\phi}'(C) + C\hat{\phi}''(C) = \lambda > 0.$$

Now consider the ratio contest success function, $\phi(C) = \theta C^\lambda$. Then $\hat{\phi}(C) = \log \theta + \lambda \log C$, $\hat{\phi}'(C) = \lambda/C$, and $\hat{\phi}''(C) = -\lambda/C^2$. Therefore,

$$\hat{\phi}'(C) + C\hat{\phi}''(C) = -\frac{\lambda}{C^2} + C\frac{\lambda}{C} = 0. \qquad \square$$

A.6 Proof of Proposition 4

Proposition 4. *A labor-financed government's equilibrium payoff is strictly decreasing in the number of factions, N. It strictly decreases with a marginal increase in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

Proof. By Proposition 3, the government's equilibrium payoff is

$$\hat{t}_0 \bar{P}_0 = \frac{\pi^p \bar{P}_0}{\pi^p - \pi^r \bar{P}_0 g'(0)} = \frac{\pi^p}{(\pi^p/\bar{P}_0) - \pi^r g'(0)}.$$

This expression is strictly increasing in $\bar{P}_0$. Since N and π^c only enter through the equilibrium value of $\bar{P}_0$, the claim follows from Remark 1 and Remark 2. $\square$

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A.7 Proof of Proposition 5

Let $\Gamma_X(t)$ denote the labor allocation subgame with tax rate t in the model with a capital-financed government. The quantities defined in Proposition 5 are as follows. $(\tilde{R}_X(t), \tilde{c}_X(t))$ is the solution to the system

$$Q^{pr}(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi) = \frac{\pi^r t g'(\tilde{R}_X(t))}{1 - t g(\tilde{R}_X(t))} + \frac{N-1}{N} \pi^c \hat{\phi}'(\tilde{c}_X(t)) = 0, \tag{38}$$

$$Q^b(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi) = L - \frac{\tilde{R}_X(t)}{\pi^r} - \frac{N\tilde{c}_X(t)}{\pi^c} = 0. \tag{39}$$

The cutpoint tax rates are

$$\hat{t}_0^X = \frac{\eta \pi^c \hat{\phi}'(\pi^c L/N)}{\eta \pi^c \hat{\phi}'(\pi^c L/N) - \pi^r g'(0)}, \tag{40}$$

$$\hat{t}_1^X = \frac{\eta \pi^c \hat{\phi}'(0)}{\eta \pi^c g(\pi^r L)\hat{\phi}'(0) - \pi^r g'(\pi^r L)}, \tag{41}$$

where $\eta = (N-1)/N$. Similar to the cutpoints in the original model,

$$\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} = \eta \pi^c \hat{\phi}'(\pi^c L/N) = \pi^c \frac{\partial \log \omega_i((\pi^c L/N)\mathbf{1}_N)}{\partial c_i},$$

$$\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1^X, (\pi^r L/N)\mathbf{1}_N)}{\partial r_i} = \eta \pi^c \hat{\phi}'(0) = \pi^c \frac{\partial \log \omega_i(0)}{\partial c_i}$$

for each $i \in \mathcal{N}$.

Proposition 5. *If the government is capital-financed, every labor allocation subgame has a unique equilibrium. There exists a tax rate $\hat{t}_0^X \in (0,1)$ such that each $r_i = 0$ in equilibrium if and only if $t \leq \hat{t}_0^X$. There exists $\hat{t}_1^X > \hat{t}_0^X$ such that each $c_i = 0$ in equilibrium if and only if $t \geq \hat{t}_1^X$. For $t \in (\hat{t}_0^X, \hat{t}_1^X)$, in equilibrium each $r_i = \tilde{R}_X(t)/N > 0$ (strictly increasing in t) and each $c_i = \tilde{c}_X(t) > 0$ (strictly decreasing).*

Proof. The existence and essential uniqueness results from the original game, Proposition 10 and Proposition 11, carry over to the capital-financed setting. So does Lemma 7, showing that all individual allocations toward conflict are equal under symmetry. Therefore, $\Gamma_X(t)$ has an equilibrium, and there exists $c_X^*(t)$ such that each $c_i = c_X^*(t)$ in every equilibrium of $\Gamma_X(t)$. The budget constraint then implies each $r_i = \pi^r(L/N - c_X^*(t)/\pi^c)$ in every equilibrium of $\Gamma_X(t)$, so the equilibrium is unique.

Let (r, c) be the equilibrium of $\Gamma_X(t)$. Let $R = \sum_i r_i$ and $C = c_1$, so by Lemma 7 each $c_i = C$. If $t \leq \hat{t}_0^X$ and $R > 0$, then the first-order conditions give

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} = \pi^c \frac{\partial \log \omega_i((\pi^c L/N)\mathbf{1}_N)}{\partial c_i} \leq \pi^c \frac{\partial \log \omega_i(c)}{\partial c_i}$$

for each $i \in \mathcal{N}$. But this implies each $r_i = 0$, contradicting $R > 0$. Therefore, if $t \leq \hat{t}_0^X$, then

$R = 0$. Similarly, if $t > \hat{t}_0^X$ and $R = 0$, then each $c_i = \pi^c L/N$ and thus

$$\pi^c \frac{\partial \log \omega_i(c)}{\partial c_i} = \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i}.$$

But this implies each $c_i = 0$, a contradiction. Therefore, if $t > \hat{t}_0^X$, then $R > 0$. The proof that $C > 0$ if and only if $t < \hat{t}_1^X$ is analogous.

For $t \in (\hat{t}_0^X, \hat{t}_1^X)$, the first-order conditions imply that R and C solve $Q^{rc}(0, R, C; t, \pi) = Q^b(0, R, C; t, \pi) = 0$; therefore, $R = \tilde{R}_X(t)$ and $C = \tilde{c}_X(t)$. To reduce clutter in what follows, I omit the evaluation point $(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi)$ from all partial derivative expressions. The Jacobian of the system defining $(\tilde{R}_X(t), \tilde{c}_X(t))$ is

$$\mathbf{J}_X = \begin{bmatrix} \partial Q^{rc}/\partial R & \partial Q^{rc}/\partial C \\ \partial Q^b/\partial R & \partial Q^b/\partial C \end{bmatrix}$$

$$= \begin{bmatrix} \pi^r t \dfrac{g''(\tilde{R}_X(t)) - tg(\tilde{R}_X(t))^2 \hat{g}''(\tilde{R}_X(t))}{(1 - tg(\tilde{R}_X(t)))^2} & \eta \pi^c \hat{\phi}''(\tilde{c}_X(t)) \\ -1/\pi^r & -N/\pi^c \end{bmatrix}$$

where $\eta = (N-1)/N$ and $\hat{g} = \log g$. Its determinant is

$$|\mathbf{J}_X| = \frac{\pi^c}{\pi^r} \left(\eta \hat{\phi}''(\tilde{c}_X(t)) - N\pi^r t \frac{g''(\tilde{R}_X(t)) - tg(\tilde{R}_X(t))^2 \hat{g}''(\tilde{R}_X(t))}{(1 - tg(\tilde{R}_X(t)))^2} \right) < 0.$$

By the implicit function theorem and Cramer's rule,

$$\frac{d\tilde{R}_X(t)}{dt} = \frac{\begin{vmatrix} -\partial Q^{rc}/\partial t & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial t & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X|}$$

$$= \frac{\begin{vmatrix} -\pi^r g'(\tilde{R}_X(t))/(1 - tg(\tilde{R}_X(t)))^2 & \eta \pi^c \hat{\phi}''(\tilde{c}_X(t)) \\ 0 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X|}$$

$$= \frac{N\pi^r g'(\tilde{R}_X(t))}{\pi^c |\mathbf{J}_X|}$$

$$> 0,$$

as claimed. The budget constraint then implies $d\tilde{c}_X(t)/dt < 0$, as claimed. $\square$

A.8 Proof of Proposition 6

Before proving the proposition, I separately derive the comparative statics of $\hat{t}_0^X$ and $\tilde{R}_X(t)$ in N and π^c.

Lemma 13. *The lower cutpoint $\hat{t}_0^X$ is strictly increasing in the number of factions, N. It is*

locally decreasing in the effectiveness of conflict, π^c, if and only if

$$\hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N}\hat{\phi}''\left(\frac{\pi^c L}{N}\right) \geq 0.$$

Proof. Recall that

$$\hat{t}_0^X = \frac{((N-1)/N)\pi^c\hat{\phi}'(\pi^c L/N)}{((N-1)/N)\pi^c\hat{\phi}'(\pi^c L/N) - \pi^r g'(0)}.$$

Since $g'(0) < 0$, $(N-1)/N$ is strictly increasing in N, and $\hat{\phi}'(\pi^c L/N)$ is weakly increasing in N, $\hat{t}_0^X$ is strictly increasing in N. Notice that

$$\frac{\partial}{\partial \pi^c}\left[\pi^c\hat{\phi}'\left(\frac{\pi^c L}{N}\right)\right] = \hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N}\hat{\phi}''\left(\frac{\pi^c L}{N}\right),$$

so $\hat{t}_0^X$ is locally increasing in π^c if and only if the above expression is positive. $\square$

Lemma 14. *For fixed $t \in (\hat{t}_0^X, \hat{t}_1^X)$, total resistance, $\tilde{R}_X(t)$, is strictly decreasing in the number of factions, N. It is locally decreasing in the effectiveness of conflict, π^c, if and only if*

$$\hat{\phi}'(\tilde{c}_X(t)) + \tilde{c}_X(t)\hat{\phi}''(\tilde{c}_X(t)) \geq 0.$$

Proof. As in the proof of Remark 1, I will treat N as if it were continuous in order to obtain comparative statics by implicit differentiation. Throughout the proof I write $\tilde{R}_X(t)$ and $\tilde{c}_X(t)$ as functions of (N, π^c).

I first consider comparative statics in N. To reduce clutter in what follows, I omit the evaluation point $(0, \tilde{R}_X(t; N, \pi^c), \tilde{c}_X(t; N, \pi^c); t, \pi)$ from all partial derivative expressions. By the implicit function theorem and Cramer's rule,

$$\frac{\partial\tilde{R}_X(t; N, \pi^c)}{\partial N} = \frac{\begin{vmatrix} -\partial Q^{rc}/\partial N & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial N & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|}$$

$$= \frac{\begin{vmatrix} -\pi^c\hat{\phi}'(\tilde{c}_X(t; N, \pi^c))/N^2 & ((N-1)/N)\pi^c\hat{\phi}''(\tilde{c}_X(t; N, \pi^c)) \\ \tilde{c}_X(t; N, \pi^c)/\pi^c & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|}$$

$$= \frac{\hat{\phi}'(\tilde{c}_X(t; N, \pi^c)) - (N-1)\tilde{c}_X(t; N, \pi^c)\hat{\phi}''(\tilde{c}_X(t; N, \pi^c))}{N|\mathbf{J}_X(t; N, \pi^c)|}$$

$$< 0,$$

as claimed, where $|\mathbf{J}_X(t; N, \pi^c)| < 0$ is defined as in the proof of Proposition 5.

I now consider comparative statics in π^c. Again by the implicit function theorem and

Cramer's rule,

$$
\begin{aligned}
\frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial \pi^c} &= \frac{\begin{vmatrix} -\partial Q^{rc}/\partial \pi^c & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial \pi^c & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|} \\
&= \frac{\begin{vmatrix} -((N-1)/N)\hat{\phi}'(\tilde{c}_X(t; N, \pi^c)) & ((N-1)/N)\pi^c\hat{\phi}''(\tilde{c}_X(t; N, \pi^c)) \\ -N\tilde{c}_X(t; N, \pi^c)/(\pi^c)^2 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|} \\
&= \frac{(N-1)\left(\hat{\phi}'(\tilde{c}_X(t; N, \pi^c)) + \tilde{c}_X(t; N, \pi^c)\hat{\phi}''(\tilde{c}_X(t; N, \pi^c)) \right)}{\pi^c |\mathbf{J}_X(t; N, \pi^c)|}.
\end{aligned}
$$

Therefore, $\partial \tilde{R}_X(t; N, \pi^c)/\partial \pi^c \leq 0$ if and only if

$$
\hat{\phi}'(\tilde{c}_X(t; N, \pi^c)) + \tilde{c}_X(t; N, \pi^c)\hat{\phi}''(\tilde{c}_X(t; N, \pi^c)) \geq 0,
$$

as claimed. $\qquad \square$

The proof of Proposition 6 follows mainly from these lemmas.

Proposition 6. *A capital-financed government's equilibrium payoff is increasing in the number of factions, N. If there is a unique equilibrium tax rate t^*, the government's equilibrium payoff is locally increasing in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at the corresponding equilibrium level of internal conflict.*

Proof. Throughout the proof I write various equilibrium quantities, including the cutpoints $\hat{t}_0^X$ and $\hat{t}_1^X$, as functions of (N, π^c). Let the government's equilibrium payoff as a function of these parameters be

$$
u_G^*(N, \pi^c) = \max_{t \in [0,1]} t \times g(R^*(t; N, \pi^c)) \times X.
$$

I begin with the comparative statics on N. First, suppose $t = \hat{t}_0^X(N', \pi^c)$ is an equilibrium for all N' in a neighborhood of N.[19] Then $u_G^*(N', \pi^c) = \hat{t}_0^X(N', \pi^c) \times X$ in a neighborhood of N', which by Lemma 13 is strictly increasing in N'. Next, suppose there is an equilibrium with $t \in (\hat{t}_0^X(N', \pi^c), \hat{t}_1^X(N', \pi^c))$ for all N' in a neighborhood of N. Then, by the envelope theorem,

$$
\frac{\partial u_G^*(N, \pi^c)}{\partial N} = g'(\tilde{R}_X(t; N, \pi^c))\frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial N} \times X > 0,
$$

where the inequality follows from Lemma 14. Finally, suppose $t = 1$ is an equilibrium for all N' in a neighborhood of N.[20] Then $u_G^*(N, \pi^c) = g(\pi^r L) \times X$ is locally constant in N, and thus weakly increasing.

I now consider the comparative statics on π^c. First, suppose $t = \hat{t}_0^X(N, \pi^{c'})$ is an equilibrium for all $\pi^{c'}$ in a neighborhood of π^c. Then $u_G^*(N, \pi^{c'}) = \hat{t}_0^X(N, \pi^{c'}) \times X$ in a neighborhood

---

[19]As in the original game, there cannot be an equilibrium tax rate $t < \hat{t}_0^X$.

[20]$t = 1$ is the only $t \geq \hat{t}_1^X$ that can be an equilibrium, since resistance is constant above $\hat{t}_1^X$.

of $\pi^{c\prime}$, which by Lemma 13 is locally increasing at π^c if and only if

$$\hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N}\hat{\phi}''\left(\frac{\pi^c L}{N}\right) \geq 0.$$

Next, suppose there is an equilibrium with $t \in (\hat{t}_0^X(N, \pi^{c\prime}), \hat{t}_1^X(N, \pi^{c\prime}))$ for all $\pi^{c\prime}$ in a neighborhood of π^c. Then, by the envelope theorem,

$$\frac{\partial u_G^*(N, \pi^c)}{\partial \pi^c} = g'(\tilde{R}_X(t; N, \pi^c))\frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial \pi^c} \times X.$$

This is positive if and only if $\hat{\phi}'(\tilde{c}_X(t)) + \tilde{c}_X(t)\hat{\phi}''(\tilde{c}_X(t)) \geq 0$, per Lemma 14. Finally, suppose $t = 1$ is an equilibrium for all $\pi^{c\prime}$ in a neighborhood of π^c. Then $u_G^*(N, \pi^c) = g(\pi^r L) \times X$ is locally constant in π^c, and thus weakly increasing. $\square$

A.9 Proof of Propositions 7 and 8

Similar to above, let $\Gamma(t_1, t_2)$ denote the labor allocation subgame following the government's choice of the given tax rates. Notice that in the model with asymmetric taxation we have

$$\frac{\partial u_i(t, p, r, c)}{\partial p_i} = \frac{\phi(c_i)}{\phi(c_i) + \phi(c_j)}(1 - t_i g(r_i + r_j)),$$

$$\frac{\partial u_i(t, p, r, c)}{\partial r_i} = \frac{\phi(c_i)}{\phi(c_i) + \phi(c_j)}(-g'(r_i + r_j))(t_i p_i + t_j p_j),$$

$$\frac{\partial u_i(t, p, r, c)}{\partial c_i} = \frac{\phi'(c_i)\phi(c_j)}{(\phi(c_i) + \phi(c_j))^2}\left[(1 - t_i g(r_i + r_j))p_i + (1 - t_j g(r_i + r_j))p_j\right].$$

Proposition 7. *In the game with asymmetric taxation, if the government chooses $t_1 > t_2$, then $p_1 \leq p_2$, $r_1 \geq r_2$, and $c_1 \geq c_2$ in any equilibrium of the subsequent labor allocation subgame.*

Proof. First I will prove $c_1 \geq c_2$. To this end, suppose $c_i > c_j$; I will show this implies $t_i > t_j$ and thus $i = 1$. Log-concavity of ϕ implies $\phi'(c_i)\phi(c_j) \leq \phi'(c_j)\phi(c_i)$, so we have

$$\pi^c\frac{\partial u_i(t, p, r, c)}{\partial c_i} \leq \pi^c\frac{\partial u_j(t, p, r, c)}{\partial c_j}.$$

The first-order conditions of equilibrium then imply

$$\max\left\{\pi^p\frac{\partial u_i(t, p, r, c)}{\partial p_i}, \pi^r\frac{\partial u_i(t, p, r, c)}{\partial r_i}\right\} \leq \max\left\{\pi^p\frac{\partial u_j(t, p, r, c)}{\partial p_j}, \pi^r\frac{\partial u_j(t, p, r, c)}{\partial r_j}\right\}.$$

As $\phi(c_i) > \phi(c_j)$, this can hold only if $1 - t_i g(r_i + r_j) < 1 - t_j g(r_i + r_j)$; i.e., $t_i > t_j$.

I now prove $r_1 \geq r_2$. First suppose $c_1 > 0$. The first-order conditions of equilibrium,

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combined with the fact that $c_1 \geq c_2$, imply

$$\pi^c \frac{\partial u_2(t,p,r,c)}{\partial c_2} \geq \pi^c \frac{\partial u_1(t,p,r,c)}{\partial c_1} \geq \pi^r \frac{\partial u_1(t,p,r,c)}{\partial r_1} > \pi^r \frac{\partial u_2(t,p,r,c)}{\partial r_2}.$$

It then follows from the first-order conditions that $r_2 = 0$, which implies $r_1 \geq r_2$. On the other hand, suppose $c_1 = 0$, in which case $c_2 = 0$ per above. It is then immediate from the budget constraint that $r_1 \geq r_2$ if $p_1 = 0$, so suppose $p_1 > 0$. The first-order conditions and $t_1 > t_2$ imply

$$\pi^p \frac{\partial u_2(t,p,r,c)}{\partial p_2} > \pi^p \frac{\partial u_1(t,p,r,c)}{\partial p_1} \geq \pi^r \frac{\partial u_1(t,p,r,c)}{\partial r_1} = \pi^r \frac{\partial u_2(t,p,r,c)}{\partial r_2}.$$

It then follows from the first-order conditions that $r_2 = 0$, which implies $r_1 \geq r_2$.

Finally, under the budget constraint, $c_1 \geq c_2$ and $r_1 \geq r_2$ imply $p_1 \leq p_2$. $\qquad \square$

Proposition 8. *Asymmetric taxation does not raise the equilibrium payoff of a labor-financed government.*

Proof. Assume $t_1 > t_2$, and let (p,r,c) be an equilibrium of $\Gamma(t_1,t_2)$. Let $\hat{t}_0$, $\bar{P}_0$, and $\bar{c}_0$ be defined as in Proposition 1. In addition, let $P = p_1 + p_2$, $R = r_1 + r_2$, and $C = c_1 + c_2$.

My first task is to prove $P \leq \bar{P}_0$. As any equilibrium entails $P > 0$, it follows from Proposition 7 that $p_2 > 0$. If $p_1 = 0$, in which case $P = p_2$, the first-order conditions for equilibrium imply

$$\pi^p \phi(c_2) \geq \frac{\pi^c \phi'(c_2)\phi(c_1)}{\phi(c_1) + \phi(c_2)} P.$$

Rearranging terms and applying the fact that $\phi(c_1) \geq \phi(c_2)$ (per Proposition 7) gives

$$P \leq \frac{\pi^p (\phi(c_1) + \phi(c_2))\phi(c_2)}{\pi^c \phi(c_1)\phi'(c_2)} \leq \frac{2\pi^p \phi(c_2)}{\pi^c \phi'(c_2)}.$$

Under this inequality, $P > \bar{P}_0$ would imply $c_2 > \bar{c}_0$, violating the budget constraint. Therefore, $P \leq \bar{P}_0$. Next, suppose $p_1 > 0$, so the first-order conditions for equilibrium imply

$$\begin{aligned} \pi^p(1 - t_1 g(R)) &\geq \frac{\pi^c \phi'(c_1)\phi(c_2)}{\phi(c_1)(\phi(c_1) + \pi(c_2))} \left[P - (t \cdot p)g(R)\right], \\ \pi^p(1 - t_2 g(R)) &\geq \frac{\pi^c \phi'(c_2)\phi(c_1)}{\phi(c_2)(\phi(c_1) + \pi(c_2))} \left[P - (t \cdot p)g(R)\right]. \end{aligned} \tag{42}$$

As $\log \phi$ is concave and its derivative is convex, $c_1 \geq c_2$ (per Proposition 7) implies $\phi'(c_1)/\phi(c_1) \leq \phi'(c_2)/\phi(c_2)$ and

$$\frac{1}{2}\left(\frac{\phi'(c_1)}{\phi(c_1)} + \frac{\phi'(c_2)}{\phi(c_2)}\right) \geq \frac{\phi'(C/2)}{\phi(C/2)}.$$

In addition, $p_1 \leq p_2$ and $t_1 > t_2$ imply

$$\left(\frac{t_1 + t_2}{2}\right) P \geq t_1 p_1 + t_2 p_2.$$

Summing the conditions in (42) and applying these inequalities gives

$$2\pi^p \left(1 - \frac{t_1 + t_2}{2} g(R)\right)$$

$$\geq \pi^c \left[P - (t \cdot p)g(R)\right] \left[\left(\frac{\phi(c_2)}{\phi(c_1) + \phi(c_2)}\right) \frac{\phi'(c_1)}{\phi(c_1)} + \left(\frac{\phi(c_1)}{\phi(c_1) + \phi(c_2)}\right) \frac{\phi'(c_2)}{\phi(c_2)}\right]$$

$$\geq \pi^c \left[P - (t \cdot p)g(R)\right] \left[\frac{1}{2} \left(\frac{\phi'(c_1)}{\phi(c_1)} + \frac{\phi'(c_2)}{\phi(c_2)}\right)\right]$$

$$\geq \pi^c \left[P - (t \cdot p)g(R)\right] \left(\frac{\phi'(C/2)}{\phi(C/2)}\right)$$

$$\geq \pi^c P \left(1 - \frac{t_1 + t_2}{2} g(R)\right) \left(\frac{\phi'(C/2)}{\phi(C/2)}\right)$$

Simplifying and rearranging terms gives

$$P \leq \frac{2\pi^p \phi'(C/2)}{\pi^c \phi(C/2)}.$$

As in the previous case, the budget constraint then implies $P \leq \bar{P}_0$.

If $t_i g(R) \leq \hat{t}_0$ for each $i = 1, 2$ such that $p_i > 0$, then $P \leq \bar{P}_0$ implies $u_G(t, c, p, r) \leq \hat{t}_0 \bar{P}_0$, as claimed. So suppose there is a group i such that $p_i > 0$ and $t_i g(R) > \hat{t}_0$. The first-order conditions for equilibrium imply

$$\pi^p (1 - t_i g(R)) \geq \pi^r (-g'(R))(t_i p_i + t_j p_j).$$

This inequality, combined with log-concavity of g and the assumption that $t_i g(R) > \hat{t}_0$, gives

$$u_G(t, p, c, r) = (t_i p_i + t_j p_j) g(R)$$

$$\leq -\left(\frac{g(R)}{g'(R)}\right) \frac{\pi^p (1 - t_i g(R))}{\pi^r}$$

$$< -\left(\frac{1}{g'(0)}\right) \frac{\pi^p (1 - \hat{t}_0)}{\pi^r}$$

$$= \left(\frac{\pi^p}{\pi^p - \pi^r \bar{P}_0 g'(0)}\right) \bar{P}_0$$

$$= \hat{t}_0 \bar{P}_0,$$

as claimed. $\qquad \square$

A.10 Proof of Proposition 9

I begin by characterizing the equilibrium of the conquest game. Throughout the proofs, let $\hat{\chi} = \log \chi$ and $\hat{\psi} = \log \psi$. I will characterize equilibria in terms of the criterion function

$$Q^{ds}(S; N) = \frac{N-1}{N}(\psi(S) + \bar{s}_O)\hat{\chi}'\left(\frac{L-S}{N}\right) - \hat{\psi}'(S)\bar{s}_O, \tag{43}$$

which is strictly increasing in both S and N.

Lemma 15. *The conquest game has a unique equilibrium in which each*

$$s_i = \begin{cases} 0 & Q^{ds}(0; N) \geq 0, \\ \tilde{S}(N)/N & Q^{ds}(0; N) < 0, Q^{ds}(L; N) > 0, \\ L/N & Q^{ds}(L; N) \leq 0, \end{cases}$$

and each $d_i = L/N - s_i$, where $\tilde{S}(N)$ is the unique solution to $Q^{ds}(\tilde{S}(N); N) = 0$.

Proof. Like the original game, the conquest game is log-concave, so a pure-strategy equilibrium exists can be characterized by first-order conditions. In addition, the proof of Lemma 7 carries over to the conquest game, so in equilibrium each $d_i = d_j$ for $i, j \in \mathcal{N}$. The claim then follows from the first-order conditions for maximization of each faction's utility. □

The proof of Proposition 9 follows from this equilibrium characterization.

Proposition 9. *In the conquest model, the probability that the outsider wins is increasing in the number of factions, N.*

Proof. I will prove that the equilibrium value of $\sum_i s_i$ decreases with the number of factions. Let (d, s) and (d', s') be the equilibria at N and N' respectively, where $N' > N$, and let $S = \sum_i s_i$ and $S' = \sum_i s_i'$. If $S = 0$, then $Q^{ds}(0; N) \geq 0$ and thus $Q^{ds}(0; N') \geq 0$, so $S' = 0$ as well. If $S \in (0, L)$, then $S = \tilde{S}(N)$, which implies $Q^{ds}(L; N) > 0$ and thus $Q^{ds}(L; N') > 0$. This in turn implies either $S' = \tilde{S}(N') < \tilde{S}(N)$ or $S' = 0 < \tilde{S}(N)$. Finally, if $S = L$, then it is trivial that $S' \leq S$. □