Campaign Spending and Hidden Policy Intentions*

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Abstract

I develop a model of electoral competition in which candidates' ability to raise money is related to their private information about the policy they will implement if elected. I use the model to analyze how politicians' fundraising decisions are affected by concerns about signaling their policy intentions, and to query whether campaign finance reform can reduce the possibility of electing well-funded candidates with policy intentions far from the median voter. I find an equilibrium of the model and show that it is unique among equilibria that satisfy the D1 refinement. If centrist candidates can raise money more easily than others, they always exploit this advantage in equilibrium, using high spending as a signal of their proximity to the median voter. The reverse is not true when non-centrists have the fundraising advantage. In this case, candidates who spend highly are perceived as having extreme policy intentions, offsetting the otherwise positive electoral effect of spending. The candidates who face this dilemma resolve it either by imitating the strategy of their centrist peers or by spending enough to compensate for their views being revealed. The electoral consequences of campaign finance reform are also asymmetric. When non-centrists can raise money more easily, a marginal decrease in the size of their advantage may increase the chance that a centrist is elected; however, in the reverse case, such measures have no effect on the chance of electing a non-centrist.

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1 Introduction

Fundraising from special interest groups is often seen as a corrupt influence on the political process, distorting electoral outcomes in favor of candidates who can raise money easily rather than those who will enact policies representative of public opinion as a whole. For example, a recent editorial in the *New York Times* stated that "the corrupting influence of money is not limited to bribery—the broader problem is the ability of moneyed interests to put into office those who support their political agendas or financial interests."[1] Such denunciations fail to consider how voters might rationally respond if high campaign spending is driven by candidates' connections to special interest groups. If candidates can only raise large sums by privately promising to enact policies that run counter to the public interest, high spending sends a negative signal to voters. Upon observing that a candidate has raised a great deal of money, voters can infer that he or she will enact policies that favor special interests. Therefore, if such candidates are electorally successful, it is in spite of the electorate having learned their policy intentions, not because they have been kept hidden.

This paper uses a formal model to analyze electoral competition when a candidate's ability to raise money is related to his privately known policy intentions. In the model, each candidate has private information about the kind of policy he will enact once elected—specifically, whether it is close to the median voter's ideal point or more extreme. Candidates first choose how much to spend on campaign advertising, and then voters make their selections. Spending is costly for candidates, which represents the fact that candidates must devote time and effort to soliciting donations. Campaign spending has two effects on a candidate's chance of winning the election. The direct effect is to raise voters' perceptions of a candidate's valence, or non-policy quality. The indirect effect is that spending sends a signal of what kind of policy the candidate will implement if put in office. In particular, if candidates with one kind of policy intention (centrist or non-centrist) can raise money more easily—i.e., have a lower marginal cost per dollar raised—high spending signals that a candidate is of this type. If candidates who have made policy promises to extreme special interests are advantaged in fundraising, the direct and indirect effects of campaign spending run counter to one another. I find an equilibrium of the model and show that it is unique among those that satisfy the D1 refinement for off-the-path beliefs.

[1] "When Other Voices Are Drowned Out." *New York Times*, 25 March 2012. http://www.nytimes.com/2012/03/26/opinion/when-other-voices-are-drowned-out.html.

The main results of the model follow from the fact that it is costly for a candidate to raise (and thus spend) campaign funds. For example, a candidate never chooses to spend a particular amount if he could spend less and have at least as great a chance of winning the election. Candidates do not reveal their type to voters via high spending unless there is an electoral benefit to doing so. Although there are counterfactual scenarios in which additional spending would reduce a candidate's chance of winning the election, such as if it would cause voters to infer that he would enact extreme policies, these are not observed along the equilibrium path. A closely related result is that if one type of candidate has a lower marginal cost of fundraising, then it is always at least as well off in equilibrium as the other type. If a candidate with a lower cost of fundraising were to mimic the strategy of one with a higher cost, he would have the same chance of winning the election at no greater cost. This result demonstrates the strategic advantage held by candidates who can raise money relatively easily. They can choose whichever option best suits their electoral fortunes: spending enough that their type is revealed or preserving ambiguity by not spending much.

I first derive the form of the equilibrium when candidates who will enact centrist policies can raise campaign funds relatively easily. Good things go together here: high spending has a positive effect on perceptions of a candidate's valence and signals that he will implement representative policies. In equilibrium, centrist candidates fully exploit their fundraising advantage. They spend enough to deter any non-centrist from mimicking their strategy, resulting in a fully separating equilibrium in which non-centrists never defeat a centrist opponent. This holds even if centrists' marginal cost of effort per dollar raised is only slightly less than that of non-centrists.

I then examine the opposite case, in which candidates who will enact relatively extreme policies can raise money more easily than centrists. This is the scenario in which campaign spending is usually thought to have a corrupt influence. The form of the equilibrium here is not a mirror image of the previous case. The candidates with a fundraising advantage now face a dilemma, in that the direct benefit of campaign spending may be offset by its negative effect on voters' beliefs about a candidate's policy intentions. If voters know that a candidate will enact extreme policies in office, he must spend an additional amount on valence in order to defeat centrists who spend less. The form of the equilibrium depends on whether the electoral gain of doing so outweighs the cost of the additional fundraising. If so, then there is separation in equilibrium, with non-centrists spending substantially more than centrists in order to maintain an electoral advantage over them.

Otherwise, if this strategy would be too costly, non-centrists conceal their type from voters by using the same strategy as centrists (namely, spending nothing). The equilibrium therefore has a somewhat paradoxical feature: whenever voters know for sure that a candidate will enact extreme policies, that candidate defeats any centrist opponent. This holds because information revelation is not random. Non-centrists only employ their fundraising advantage—and thus reveal their type—when it can be electorally profitable to do so.

After characterizing the equilibrium of the model, I analyze the potential effects of campaign finance reform on candidate and voter behavior by performing comparative statics analysis. Specifically, I investigate policies that seek to level the playing field by increasing the marginal cost of fundraising for whichever type can raise money more easily. An example of this type of policy would be individual contribution limits, which prevent candidates from raising a large sum of money from a concentrated set of contributors. When non-centrists can raise money more easily, I find that marginal reductions in their advantage can decrease the *ex ante* probability that a non-centrist candidate wins the election. However, the reverse is not true when centrists have a lower cost of fundraising. In this case, marginal changes in the size of centrists' advantage have no effect on the probability that a centrist wins the election. Therefore, this type of reform is appealing from the standpoint of maximizing voter welfare, as it only reduces the chance of victory for candidates who would enact unrepresentative policies once elected.

The substantive concerns in this paper are similar to those in the political economy literature on the connections between interest groups, campaign funding, and public opinion. Previous research in this area suggests that campaign advertisements funded by interest-group donations serve as a signal of underlying candidate quality. For example, Prat (2002*a*) and Wittman (2007) each develop models in which an interest group directly observes candidate quality prior to the election, but voters do not. Both find that it is often worthwhile for high-quality candidates to make policy promises to the group in exchange for its support. The high-quality candidate gains because the group's willingness to donate to him signals his quality to the electorate; the interest group gains because it can extract policy promises from the candidate in exchange for endorsing him. Prat (2002*b*) extends the model to include multiple lobbies and finds similar results. Coate (2004) and Ashworth (2006) develop models that are similar, except that pressure groups no longer directly observe a candidate's underlying quality. Instead, these models assume that advertisements cannot transmit false information, so only high-quality candidates would want to raise funds and run ads. In these models as in the previous ones, high-quality candidates have an incentive to give policy favors to interest groups—even though doing so reduces their standing among the broader electorate—in order to be able to signal their quality via advertising.

The model in the present paper differs from the previous political economy literature in its treatment of candidate quality. I do not assume that quality (or valence) is a fixed, immutable attribute of a candidate. Instead, I examine a scenario in which candidates exert costly effort to raise money that can be used on advertisements and other means of raising public perception of their quality. In this sense, the modeling approach here is drawn from other recent models of endogenous valence, in which public perception of a candidate's quality arises from the candidate's own choices instead of being fixed in advance. The most basic model of endogenous valence is that of Meirowitz (2008), who considers valence competition with a potential incumbency advantage. He finds that disadvantaged candidates tend to exert more effort in equilibrium, which means spending caps typically work to the benefit of incumbents. The model in the present paper uses the same all-pay auction structure as Meirowitz's. Other models of endogenous valence have included a stage in which candidates make Downsian policy announcements, in order to investigate how valence competition affects incentives for convergence (Zakharov 2008; Ashworth and Bueno de Mesquita 2009; Serra 2010). The present paper contributes to the endogenous valence literature by allowing a candidate's marginal cost of effort to be related to his privately known policy intentions. The addition of this signaling component has a dramatic effect on the results. For example, unlike in typical endogenous valence competitions (or all-pay auctions), there can exist an equilibrium in which both candidates refrain from spending so as not to send a negative signal to the electorate.

The remainder of the paper proceeds as follows. The model is presented formally in Section 2. I solve for an equilibrium of the model and show that it is unique under the D1 refinement in Section 3. Section 4 contains the comparative statics analysis of how campaign finance reform affects voter welfare. Concluding remarks are given in Section 5. The Appendix contains proofs of all results given in the paper, as well as intermediate results not stated in the main text.

2 A Model of Campaign Spending with Private Policy Intentions

There is an election in which two candidates, labeled 1 and 2, are competing for support from a set of n voters (n odd). Generic candidates are labeled i and take male pronouns; generic voters are labeled j and take female pronouns. The policy space is the real line, and each voter has a commonly known ideal point $x_j \in \Re$. Without loss of generality, the median voter m is assumed to have ideal point $x_m = 0$. In the game, the candidates simultaneously select levels of spending $v_i \in \Re_+$, and then the election is held.

Each candidate has a type $t_i \in T = \{C, E\}$, where each type $t \in T$ is associated with a pair (b_t, x_t). The first element, b_t, is the candidate's marginal cost of fundraising: for each unit he spends, he incurs a cost of b_t. The higher b_t is, the more effort a candidate of type t must exert to raise the same amount of money. The second element, x_t, is the policy that a candidate of type t will implement if elected. Candidate types are drawn independently from each other, with $p_E \in (0, 1)$ denoting the prior probability that a candidate is type E, and $p_C = 1 - p_E$ denoting the prior probability of type C. Each candidate's type is private information, unknown to the other candidate and to the electorate. Therefore, a candidate cannot base his spending decisions on the other candidate's type, and voters must infer candidates' policy intentions from their spending decisions. A candidate's expected utility from spending v_i, informally expressed, is

$$Eu_i(v_i \,|\, t_i) = \Pr(i \text{ wins} \,|\, v_i) - b_{t_i} v_i. \tag{1}$$

It is important to note that although candidates possess policy *intentions*, they are not "policy-motivated" in Calvert's (1985) sense. That is, candidates' utility depends on their own probability of winning, not the expected policy choice of the winner.[2]

The two types of candidate, C and E, respectively represent those who are centrists and those who are more extreme (or non-centrist). Centrists are further from the median voter in their policy intentions than non-centrist types are. The respective policy positions of each type are $x_C = x_m = 0$ and $x_E = x > 0$.[3] The two types may also differ in their marginal cost of fundraising,

[2] Banks (1990) and Callander and Wilkie (2007) also use signaling models in which candidates have privately known policy intentions but are office-motivated.

[3] The assumption that C's ideal point is the same as the median voter's does not substantively affect the results. If we set $x_C \neq 0$ with $|x_C| < |x_E|$, then all of the results given in the paper would hold, replacing x with $|x_E| - |x_C|$.

which is normalized to $b_C = 1$ for centrists and is $b_E = b > 0$ for non-centrists. Values of $b < 1$ represent a scenario in which candidates with more extreme policy preferences can more easily raise money than centrists. Conversely, if $b > 1$, then it is relatively difficult for non-centrists to raise money. This relationship between policy intentions and cost of fundraising may be thought of as a reduced form representation of the interaction between candidates and interest groups. For example, interest groups may have better information than the electorate about what each candidate would implement if elected. If groups with centrist views are better-funded than others and candidates mainly raise money from sympathetic groups, this would mean centrist candidates can raise campaign funds more easily. On the other hand, if more extreme interest groups have more money, then centrist candidates would be at a fundraising disadvantage.

Because each candidate $i \in \{1, 2\}$ has the same type space and utility function, the stage where candidates select spending levels is a symmetric game. I restrict attention to symmetric strategy profiles, in which the strategy of type t of candidate 1 is the same as that of type t of candidate 2. A strategy profile is thus a set of two strategies, one for each type $t \in \{C, E\}$. A pure strategy profile is denoted by $(v_C, v_E) \in \Re^2_+$, where each element represents the spending level of the corresponding type. I focus mainly on mixed strategy profiles, denoted by (σ_C, σ_E), where each σ_t is the probability measure of the corresponding strategy. I use F_t to denote the cumulative distribution function induced by a probability measure σ_t. The notation $\operatorname{supp} \sigma_t$ denotes the support of σ_t, which, loosely speaking, is the set of spending choices included in type t's mixed strategy. Technically, the support is the smallest closed set whose complement has probability 0 under the given mixed strategy (Cohn 1994, 226).

Voters' utility depends on the winning candidate's spending and policy intentions. First, each voter prefers that the winner's policy intentions be close to her own (the voter's) ideal point. I capture this with single-peaked, tent-shaped preferences: the policy component of a voter j's utility for candidate i winning is $-|x_j - x_{t_i}|$. Second, as a candidate spends more on the campaign, voters perceive him to be of higher quality. As in similar models of "endogenous valence" (e.g., Meirowitz 2008; Ashworth and Bueno de Mesquita 2009), I assume that voter utility is additively separable in policy and valence. That is, the effect of an increase in valence on voter utility is independent of that voter's policy preferences. Moreover, for simplicity, I assume that a voter's perception of candidate valence is linear in the amount that the candidate spends, and that these perceptions are common across the electorate. A voter's utility from candidate i winning the election can thus be written as

$$u_j(i \,|\, v_i) = v_i - |x_j - x_{t_i}|. \tag{2}$$

However, because candidate's types are private information, this value is unknown to voters. Instead, voters must form expectations about a candidate's type based on the observable information—i.e., how much the candidate has chosen to spend. After observing the spending decisions, voters update their beliefs about each candidate's type. Since the candidates do not observe each other's type, a voter makes inferences about each candidate based solely on that candidate's level of spending. Because I consider only symmetric strategies, a voter's inference from a particular level of spending must be the same for either candidate. Therefore, the belief function can be defined as $\mu : \Re_+ \to [0,1]$, where $\mu(v)$ denotes the probability that a candidate who spends v is of type E. I assume that all voters use the same rules for making inferences, even for spending decisions that are off the equilibrium path. Using (2), we can thus write voter j's expected utility from victory by a candidate who spends v as

$$Eu_j(v;\mu) = v - \mu(v)|x_j - x| - (1 - \mu(v))|x_j|. \tag{3}$$

To derive the relationship between spending and electoral success, I make two assumptions about voter behavior that are common in models of candidate competition. The first is that voters do not play weakly dominated strategies, so voting is sincere: each individual votes for the candidate from whom their expected utility is greater. The other is that voters randomize uniformly when indifferent between two candidates. These assumptions, combined with single-peaked policy preferences and agreement in beliefs, allow us to consider only the median voter's expected utility to determine a candidate's probability of winning. Let $r : \Re_+^2 \to \{0, \frac{1}{2}, 1\}$ denote the median voter's probability of voting for candidate 1 under a given set of beliefs, where

$$r(v_1, v_2; \mu) = \begin{cases} 0 & \text{if } Eu_m(v_1;\mu) < Eu_m(v_2;\mu); \\ \frac{1}{2} & \text{if } Eu_m(v_1;\mu) = Eu_m(v_2;\mu); \\ 1 & \text{if } Eu_m(v_1;\mu) > Eu_m(v_2;\mu). \end{cases} \tag{4}$$

The probability that a candidate who spends v will defeat one who spends v' is therefore $r(v, v'; \mu)$.

With the election probability now defined, it is possible to derive the candidates' utilities under any given mixed strategy profile. The *ex ante* probability of victory by a candidate who spends v is

$$\lambda(v; \sigma, \mu) = p_C \int_{-\infty}^{\infty} r(v, v'; \mu)\, dF_C(v') + p_E \int_{-\infty}^{\infty} r(v, v'; \mu)\, dF_E(v'). \tag{5}$$

(These are Riemann-Stieltjes integrals, as are all integrals hereafter.) It will be more convenient to work with an alternative form of this expression. From the standpoint of candidate i, the spending level of his opponent is a random variable v_{-i} drawn from a distribution whose cumulative distribution function is $p_C F_C + p_E F_E$. The median voter's expected utility from his opponent, $Eu_m(v_{-i}; \mu)$, is also a random variable. This gives the following expression, which is equivalent to (5):

$$\begin{aligned} \lambda(v; \sigma, \mu) = \Pr(&Eu_m(v_{-i}; \mu) < Eu_m(v; \mu)) \\ &+ \frac{1}{2} \Pr(Eu_m(v_{-i}; \mu) = Eu_m(v; \mu)). \end{aligned} \tag{6}$$

Using (1), the expected utility from spending v for a candidate of type $t \in T$, given the mixed strategy profile σ and voters' beliefs μ, is

$$Eu_t(v; \sigma, \mu) = \lambda(v; \sigma, \mu) - b_t v. \tag{7}$$

Accordingly, the expected utility to a candidate of type t under the given mixed strategy profile is

$$U_t(\sigma, \mu) = \int_{-\infty}^{\infty} [\lambda(v; \sigma, \mu) - b_t v]\, dF_t(v). \tag{8}$$

This is a two-stage game of incomplete information, so the appropriate solution concept is weak perfect Bayesian equilibrium. The equilibrium concept places requirements on both the beliefs of voters and the candidates' strategies. First, voters must update their beliefs in accordance with Bayes' rule whenever possible. This implies that $\mu(v)$ is uniquely defined by the strategies for all $v \in \operatorname{supp} \sigma_C \cup \operatorname{supp} \sigma_E$,[4] though it is unrestricted outside this set. Second, each candidate's strategy

[4]If v is a mass point of σ_C or σ_E, Bayes' rule gives

$$\mu(v) = \frac{p_E \sigma_E(\{v\})}{p_C \sigma_C(\{v\}) + p_E \sigma_E(\{v\})}$$

must maximize his expected utility, given the other candidate's strategy and voters' beliefs. This requires that $U_t(\sigma, \mu) \geq Eu_t(v; \sigma, \mu)$ for each type $t \in T$ and potential spending choice $v \in \Re_+$. An implication of this requirement is the indifference condition of mixed-strategy equilibrium, which, loosely speaking, is that a candidate of type t be indifferent across all spending choices employed in his mixed strategy.[5] An assessment (σ, μ) is an equilibrium if it satisfies both consistency of beliefs and optimality of strategies.

3 Results

In this section, I characterize an equilibrium of the model and show that it is the only equilibrium that satisfies the D1 refinement for off-the-path beliefs. The exact nature of the equilibrium strategies depends on the policy distance between centrist and non-centrist candidates, as well as on which type of candidate can raise money more easily. If centrists have the edge in fundraising, they use this advantage to outpace any non-centrist opponent, as high spending both increases voters' perceptions of a candidate's valence and reveals that his policy intentions are close to the median. The strategic calculus is more complex if relatively extreme candidates are advantaged in fundraising. In this case, a candidate who outs himself as a non-centrist must spend extra to increase perceptions of his quality enough to make up for his policy distance from the median voter. Whether it is profitable to do so depends on the magnitude of that distance relative to the cost of fundraising.

3.1 Basic Characterization

I begin by establishing that there is an upper bound on the amount that each type of candidate will spend in equilibrium: the equilibrium strategy for a candidate of type t may not place positive probability on spending levels greater than $\frac{1}{b_t}$.[6]

Lemma 1 (Strict dominance)**.** *In any equilibrium,* $\operatorname{supp} \sigma_t \subseteq [0, \frac{1}{b_t}]$ *for each type* $t \in T$.

[5]Formally, the condition is that

$$\frac{\int_{-\infty}^{\infty} I_S(v)[\lambda(v; \sigma, \mu) - b_t v] \, dF_t(v)}{\sigma_t(S)} = U_t(\sigma, \mu)$$

for any set $S \subseteq \Re$ such that $\sigma_t(S) > 0$.

[6]For ease of exposition in the remainder of the paper, the dependence of $Eu_m(\cdot)$, $r(\cdot, \cdot)$, $\lambda(\cdot)$, $Eu_t(\cdot)$, and U_t on the assessment (σ, μ) is suppressed except when necessary to avoid confusion. All proofs are in the Appendix.

This simply means a candidate would never spend so much that the cost of raising the money exceeds the benefit of winning the election. Recall from (1) that a candidate's utility gain from winning the election is 1. Therefore, a candidate would be better off spending nothing and losing the election than winning while incurring costs of greater than 1, which corresponds to $v > \frac{1}{b_t}$. An important implication of this result is that the support of each type's mixed strategy is bounded and thus has a well-defined minimum and maximum.

An interesting feature of the model is that additional spending can actually decrease a candidate's chance of winning the election. In particular, if greater spending increases voters' beliefs that the candidate has non-centrist policy intentions, it could be associated with a lower probability of victory. However, as the next result shows, this kind of behavior is not observed along the equilibrium path. If the mixed strategy of either type of candidate includes v, then any amount of spending less than v must give a strictly worse chance of winning the election.[7]

Lemma 2. *Consider an equilibrium assessment (σ, μ), a type $t \in T$, and $v \in \operatorname{supp} \sigma_t$ such that $U_t = Eu_t(v) = \lambda(v) - b_t v$. For all $v' < v$, $\lambda(v) > \lambda(v')$ and $Eu_m(v) > Eu_m(v')$. This holds for almost all $v \in \operatorname{supp} \sigma_t$, and for all $v \in \operatorname{supp} \sigma_t$ such that $\lambda(\cdot)$ is continuous at v or $\sigma_t(\{v\}) > 0$.*

This result holds because spending is costly to the candidates. If two potential levels of spending would give a candidate the same likelihood of winning, there is no reason to choose the higher one, as it brings the same benefit at a greater cost. The scenarios where a candidate could jeopardize his chance of victory by spending more are unobserved counterfactuals. Within the set of spending levels that are chosen by at least one type of candidate, greater amounts must correspond to a better chance of winning the election.

The next set of results concern the consequences of one type of candidate having an advantage in fundraising. Recall that the marginal cost of campaign spending is 1 for centrists and b for non-centrists, so $b > 1$ corresponds to centrists having the advantage (and vice versa for $b < 1$). The most basic result here is that the equilibrium utility of the advantaged type must be no less than that of the disadvantaged type. If a candidate of the advantaged type simply mimicked the other type's strategy, he would have the same chance of winning at no greater cost, giving him at

[7] To be more precise, this holds for *almost* all amounts included in a type's mixed strategy. That is, if there is any set S of spending amounts for which the claim does not hold, each type's mixed strategy must place probability 0 on S. This means that such amounts may be included in the support of a candidate's mixed strategy, but they occur with probability 0.

least as high of a utility. Therefore, there cannot be an equilibrium where one type of candidate can raise funds at a relatively low cost but is worse off in terms of utility. This is summarized in the following lemma.

Lemma 3. *Let (σ, μ) be an equilibrium. If $b \geq 1$, then $U_C \geq U_E$. Conversely, if $b \leq 1$, then $U_C \leq U_E$.*

Similar logic can be used to derive the following result, which is that candidates who have a fundraising advantage always spend at least as much as those who do not. More precisely, the lowest amount included in the advantaged type's mixed strategy must be no less than the greatest amount in that of the disadvantaged type.

Lemma 4. *Let (σ, μ) be an equilibrium and define*

$$\hat{a} \equiv \frac{U_E - U_C}{1 - b}. \tag{9}$$

If $b < 1$, then $\max \operatorname{supp} \sigma_C \leq \hat{a} \leq \min \operatorname{supp} \sigma_E$. *If $b > 1$, then* $\max \operatorname{supp} \sigma_E \leq \hat{a} \leq \min \operatorname{supp} \sigma_C$.

The logic of this result can be illustrated as follows. Suppose the equilibrium strategy of the disadvantaged type—the one with a higher marginal cost of raising money—places positive probability on some amount v. Compared to any alternative amount $v' < v$, the increase in the chance of winning by spending v must be no greater than the additional cost it incurs. Now consider the advantaged type's choice between v and v'. His increase in cost between v and v' is strictly less than the disadvantaged type's (as his marginal cost of fundraising is lower), and thus it is strictly less than the increase in the probability of victory. Therefore, the advantaged type's equilibrium strategy cannot place positive probability on v'. More generally speaking, the advantaged type's mixed strategy cannot include any amount less than the greatest level of spending included in the disadvantaged type's mixed strategy.

Before turning to the full characterization of the equilibrium, it is instructive to examine one of the forms that it *cannot* take: a fully separating equilibrium in pure strategies. That is, there is no equilibrium in which centrist types always spend v_C and non-centrists always spend v_E ($\neq v_C$), which is stated formally in the next proposition.

Proposition 1. *There is no equilibrium in which* $\operatorname{supp}\sigma_C = \{v_C\}$ *and* $\operatorname{supp}\sigma_E = \{v_E\}$ *with* $v_C \neq v_E$.

This result follows from the game's information structure and its similarity to an all-pay auction. Suppose there is a strategy profile where centrists and non-centrists spend v_C and v_E respectively, with $v_C \neq v_E$. In equilibrium, a candidate of type E faces another non-centrist with probability p_E; this will cause the election to result in a tie, as both candidates spend v_E. If a candidate of type E were instead to spend infinitesimally more than v_E, he would defeat other non-centrists for certain. The deviation would increase voters' valence utility, and it could not decrease the median voter's policy utility, since the voter already infers with certainty that a candidate spending v_E has extreme policy intentions. This means that by deviating, a candidate of type E would defeat other non-centrist candidates rather than tying with them. As such, the deviant would increase his chance of winning the election by $\frac{p_E}{2}$ at arbitrarily low cost, meaning the strategy profile cannot be an equilibrium.

I now examine the baseline case of parity between centrists and non-centrists. This occurs when $b = 1$, meaning both types of candidates have the same marginal cost of fundraising. In this case, both types receive the same utility from any particular spending decision, so it is natural to restrict attention to strategy profiles in which centrists and non-centrists employ the same mixed strategy. Moreover, since neither type of candidate could possibly have more incentive than the other to make any particular off-the-path deviation, it is reasonable to assume off-the-path beliefs are the same as the prior, $\mu(v) = p_E$. Under these assumptions, the model is essentially the same as the symmetric case in Meirowitz (2008). The following proposition is then immediate from Meirowitz's Proposition 2.

Proposition 2. *Suppose* $b = 1$. *There exists an equilibrium* (σ^*, μ^*) *in which both types' strategy is a uniform mixture over* $[0, 1]$ *and* $\mu^*(v) = p_E$ *for all* v. *There is no other equilibrium in which* $\sigma_C = \sigma_E$ *and* $\mu^*(v) = p_E$ *for all off-the-path* v.

In the remainder of the paper, I consider the form of the equilibrium when centrists and non-centrists have different marginal costs for campaign spending. As in most models with a signaling component, it is possible to configure off-the-path beliefs so as to support numerous different equilibria. In order to rule out equilibria that depend on unreasonable configurations of off-the-path beliefs, I characterize only those that satisfy the D1 refinement (see Cho and Kreps 1987). D1 requires, in essence, that voters ascribe any off-the-path spending choice to the type of candidate that could potentially benefit most from making it. This is similar to the refinements used in other models where candidates' policy intentions are private information (e.g., Banks 1990; Callander and Wilkie 2007). The next result shows that, under D1, there is a cutpoint defining voters' beliefs for off-the-path spending choices: those above it are ascribed to the type with a fundraising advantage, those below it to the disadvantaged type.

Lemma 5. *Suppose $b \neq 1$. Let (σ, μ) be an equilibrium and let $\hat{a}$ be defined as in (9). The equilibrium survives the D1 refinement if and only if the following holds for all off-the-path $v \leq \max\{1 - U_C, (1 - U_E)/b\}$:*

- *If $b < 1$, $v < \hat{a}$ implies $\mu(v) = 0$ and $v > \hat{a}$ implies $\mu(v) = 1$.*
- *If $b > 1$, $v < \hat{a}$ implies $\mu(v) = 1$ and $v > \hat{a}$ implies $\mu(v) = 0$.*

This result comports with Lemma 4, which showed that the advantaged type must spend weakly more than the disadvantaged type in equilibrium. In fact, $\hat{a}$, the cutpoint for beliefs under D1, is the same as the cutpoint that divides the potential support of each respective type's mixed strategy.

3.2 Equilibrium When Centrists Are Advantaged

I now consider the case where raising money is easier for candidates who will enact policy relatively close to the median voter's ideal point. Since the marginal cost of fundraising is 1 for centrists and b for non-centrists, a centrist advantage holds when $b > 1$. Because candidates' spending decisions act as signals of their policy intentions, the fundraising advantage for centrists is doubly potent in this case. High spending has both the direct effect of inducing better perceptions of the candidate's quality and the indirect effect of signaling that the candidate will implement centrist policies if elected. In equilibrium, these effects work in tandem to benefit centrists' electoral fortunes.

The next proposition characterizes the unique (under the D1 refinement) equilibrium of the game when centrists have a fundraising advantage. Both types of candidate employ mixed strategies in equilibrium. Non-centrists' equilibrium strategy is to select a spending amount from a uniform distribution over $[0, \frac{p_E}{b}]$; centrists spend more, employing a uniform distribution over $[\frac{p_E}{b}, \frac{p_E}{b} + p_C]$.

Proposition 3. *Suppose $b > 1$. There exists an equilibrium (σ^*, μ^*) that satisfies D1 in which*

$$F_E^*(v) = \begin{cases} 0 & v < 0, \\ \frac{bv}{p_E} & 0 \leq v \leq \bar{v}_E^*, \\ 1 & v > \bar{v}_E^*, \end{cases}$$

and

$$F_C^*(v) = \begin{cases} 0 & v < \bar{v}_E^*, \\ \frac{1}{p_C}(v - \frac{p_E}{b}) & \bar{v}_E^* \leq v \leq \bar{v}_C^* \\ 1 & v > \bar{v}_C^*, \end{cases}$$

where $\bar{v}_E^* = \frac{p_E}{b}$ and $\bar{v}_C^* = \frac{p_E}{b} + p_C$. Beliefs are $\mu^*(v) = 1$ for all $v < \bar{v}_E^*$, $\mu^*(\bar{v}_E^*) = p_E$, and $\mu^*(v) = 1$ for all $v > \bar{v}_E^*$. In any other equilibrium (σ, μ) that satisfies D1, $\sigma = \sigma^*$.

One striking feature of the equilibrium is that a centrist candidate is guaranteed electoral victory if his opponent is a non-centrist.[8] The electoral advantage of centrists in this scenario is a straightforward consequence of the results established previously. Because centrists' cost of fundraising is lower, in equilibrium they must spend at least as much as non-centrists do (Lemma 4). Then, since the probability of victory strictly increases with spending on the equilibrium path (Lemma 2), the only way for a centrist not to defeat a non-centrist would be in a tie. For a tie between a centrist and non-centrist to have a chance of happening, there must be a spending level v that both types choose with positive probability. However, this possibility can be ruled out with a line of reasoning similar to the proof that there is no fully separating equilibrium in pure strategies (Proposition 1). In any equilibrium that satisfies the D1 refinement, a deviation to an amount infinitesimally greater than v would be ascribed to a centrist candidate (Lemma 5). The median voter would therefore prefer the deviant over a candidate who spends v. Since a candidate would be strictly better off by spending a bit more than v and winning than by spending v and tying, this cannot be an equilibrium. This demonstrates that a tie between a centrist and a non-centrist cannot occur with positive probability in equilibrium, and thus that a centrist candidate always defeats a non-centrist opponent.

[8] The only instance in which a tie between a centrist and a non-centrist is possible, in which both spend $\frac{p_E}{b}$, occurs with probability 0.

Given the electoral advantage that centrists possess in this case, it is worth exploring why non-centrists do not have an incentive to deviate and imitate the centrist strategy. This may appear puzzling because there is no gap between the supports of the two types' mixed strategies: why would non-centrists not simply spend a bit more to reap (some of) the electoral rewards that centrists enjoy? The answer is a consequence of the equilibrium being in mixed strategies: there is no discrete jump in the probability of victory to be gained from additional spending. Instead, the relevant calculation is whether the marginal increase in the chance of victory from more spending is greater than the marginal cost that it incurs. Under the given equilibrium strategies and beliefs, a candidate who spends v defeats one who spends v' if and only if $v > v'$. The *ex ante* probability of victory for a candidate who spends v is therefore $\lambda(v) = p_C F_C^*(v) + p_E F_E^*(v)$. Using this along with the expressions given in Proposition 3, we see that the marginal effect of spending on the probability of victory decreases from b to 1 at $v = \frac{p_E}{b}$, the lowest point in the centrist's mixed strategy. This marginal benefit is less than the non-centrist's marginal fundraising cost of b, meaning that such a candidate would experience a net loss if he were to imitate a centrist and spend more than $\frac{p_E}{b}$.

3.3 Equilibrium When Centrists Are Disadvantaged

The other case to examine is when relatively extreme candidates can raise money more easily than centrists, which holds when $b < 1$. The form of the equilibrium here is not simply the reverse of when centrists are advantaged; on the contrary, it is considerably different. This difference is driven by the new strategic tradeoffs that emerge when non-centrists have an advantage in fundraising. In the previous case, when centrists were advantaged, high spending by a candidate both increased his valence and signaled to voters that he would enact centrist policies. As such, additional spending could never cause an immediate, discrete drop in a candidate's chance of victory—the only question was whether the marginal cost outweighed the marginal gain in electoral fortunes. But when non-centrists can raise money easily, meaning high spending is a signal of relatively extreme policy intentions, the indirect effect of additional spending (the policy signal) is counter to the direct effect (increasing valence). For a candidate who has revealed himself as a non-centrist to maintain electoral viability, he must spend much more than centrist types do.

The form of the equilibrium in this case depends on the size of the non-centrist type's fundraising advantage, which is inversely proportional to b, relative to this type's policy distance from the median voter, given by x. In general, as the marginal cost of fundraising for non-centrists decreases, it becomes more likely in equilibrium that they will win the election. When the non-centrist's advantage is slight, meaning x is high or b is close to 1, neither type of candidate ever spends in equilibrium. Under this pooling strategy, voters learn nothing about candidates' policy intentions, and the *ex ante* chance of victory for both types of candidate is $\frac{1}{2}$. At the other end of the spectrum, when x and b are sufficiently small, the equilibrium is fully separating: non-centrists always spend more than any centrist, which means voters learn each candidate's type perfectly from his choice of spending. Despite revealing their extreme policy intentions, non-centrist candidates always defeat centrist opponents when the equilibrium takes this form. This illustrates the paradox that arises when non-centrists are advantaged: whenever voters know that a candidate has extreme policy intentions, he is sure to defeat any centrist opponent. Voters only learn that a candidate is a non-centrist in equilibrium if he can spend enough to overcome the negative electoral consequences of revealing his policy intentions. Otherwise, non-centrists simply mimic the spending strategy of centrists, keeping voters uninformed and assuring themselves at least as great a chance of winning the election. Because it is more costly for centrists to raise money, they have no means of separating themselves and gaining an electoral advantage; anything they do can be mimicked more cheaply by non-centrists.

I first consider the form of the equilibrium when the policy distance between centrists and non-centrists is highest relative to the size of non-centrists' fundraising advantage. It is a fully pooling equilibrium where no candidate spends at all. The following proposition gives a formal statement of the equilibrium and the conditions for it to obtain.

Proposition 4. *Suppose $b < 1$ and $x \geq \frac{1}{2bp_C}$. There exists an equilibrium (σ^*, μ^*) that satisfies D1 in which $\operatorname{supp} \sigma_C^* = \operatorname{supp} \sigma_E^* = \{0\}$, $\mu^*(0) = p_E$, and $\mu^*(v) = 1$ for all $v > 0$. In any other equilibrium (σ, μ) that satisfies D1, $\sigma = \sigma^*$.*

In this case, there is always a tie, meaning the probability of victory (and expected utility) for all candidates is $\frac{1}{2}$. Since this is a pooling equilibrium, voters' beliefs do not change from the prior upon observing a candidate who spends nothing. However, if any candidate were to deviate to a positive amount of spending, voters would infer with certainty that he is a non-centrist. Therefore, in order to win the election instead of tying, a deviating candidate would need to spend enough to make up for this unfavorable shift in voters' beliefs. The condition $x \geq \frac{1}{2bp_C}$ ensures that the cost of this additional spending meets or exceeds the increase in the probability of victory.

It is worth noting why a tie always occurs in equilibrium in this case, whereas it is impossible when the fundraising advantage belongs to centrists. In that situation, under any strategy profile where centrists and non-centrists both spend v with positive probability, it would be profitable for either to deviate to slightly more than v, as shown in the discussion of Proposition 3. The crucial factor here is that voters would infer that the deviating candidate is a centrist, because under the D1 refinement, deviations to higher amounts are ascribed to the type with a fundraising advantage (see Lemma 5). The same logic no longer applies when non-centrists can raise money more easily. Voters would now ascribe the deviation to a non-centrist, meaning that the small increase in valence utility would be more than offset by the median voter's loss in policy utility. For example, under the strategy profile given in Proposition 4, the net effect of a deviation to $\epsilon > 0$ on the median voter's expected utility would be $\epsilon - xp_C$, which is negative for all sufficiently small ϵ. Because of this adverse shift in beliefs, the usual reason for nonexistence of a pure strategy equilibrium in an all-pay auction model—the profitability of deviating to a slightly greater amount—no longer applies.

I now turn to an intermediate case, in which the policy distance between centrist and non-centrist types occupies a middle range and the equilibrium is partially pooling. The form of the equilibrium in this case is that centrist candidates always spend 0, while non-centrists' mixed strategy includes both 0 and a range of positive amounts. The probability that non-centrists spend 0 is increasing in both their policy distance from centrists and their marginal cost of fundraising. In this sense, the less advantaged non-centrists are, the more likely they are to mimic centrists by refraining from spending. The exact formulation of the equilibrium and the condition for it to hold are given in the next result.

Proposition 5. *Suppose $b < 1$ and $\frac{p_C}{2b} < x < \frac{1}{2bp_C}$. There exists an equilibrium (σ^*, μ^*) that* *satisfies D1 in which* $\operatorname{supp}\sigma_C^* = \{0\}$ *and*

$$F_E^*(v) = \begin{cases} 0 & v < 0, \\ \pi & 0 \leq v \leq \tilde{v}_E^*, \\ \frac{\pi}{2} + \frac{1}{p_E}(bv - \frac{p_C}{2}) & \tilde{v}_E^* < v < \bar{v}_E^*, \\ 1 & v \geq \bar{v}_E^*, \end{cases}$$

where $\pi = \frac{\sqrt{2bxp_C} - p_C}{p_E}$, $\tilde{v}_E^* = \frac{1}{2b}(p_C + \pi p_E)$, *and* $\bar{v}_E^* = \frac{1}{b} - \tilde{v}_E^*$. *Beliefs are* $\mu(0) = \frac{\pi p_E}{p_C + \pi p_E}$ *and* $\mu(v) = 1$ *for all* $v > 0$. *In any other equilibrium* (σ, μ) *that satisfies D1,* $\sigma = \sigma^*$.

To see why there is partial pooling in equilibrium, recall that voters attribute all positive amounts of spending to non-centrists if centrists spend 0 for certain. The cutpoint $\tilde{v}_E^*$ is the amount of spending that gives the median voter the same expected utility as a candidate who spends 0, accounting for this unfavorable shift in voters' beliefs. Because the median voter prefers a centrist, that utility value decreases with the probability that a non-centrist spends 0, and thus so does $\tilde{v}_E^*$. Under a fully pooling strategy profile, as in Proposition 4, a candidate could assure himself victory instead of a tie by spending just more than $\tilde{v}_E^*$. The condition $x < \frac{1}{2bp_C}$ ensures that $\tilde{v}_E^*$ is small enough for such a deviation to be profitable. On the other hand, under a fully separating strategy profile, non-centrists would need to spend at least x more than non-centrists in order to have a greater chance of victory, yielding an additional cost of bx. The greatest possible increase in the probability of victory from doing so is $\frac{p_C}{2}$, which occurs if all centrists spend the same amount. But because $x > \frac{p_C}{2b}$, a non-centrist candidate would strictly prefer to mimic a centrist than to pay the additional cost, so this is not an equilibrium either. Since neither full pooling nor full separation can be sustained in equilibrium in this case, we yield partial pooling.

Finally, when the policy distance between non-centrists and the median voter is sufficiently low, the equilibrium is fully separating. Both types of candidate employ mixed strategies, with non-centrists spending strictly more than centrists; because the types' strategies do not overlap, voters can infer a candidate's type with certainty from the amount that he spends. In equilibrium, non-centrist candidates always defeat centrist opponents. The precise forms of these strategies are given in the following proposition.

Proposition 6. *Suppose $b < 1$ and $x \leq \frac{p_C}{2b}$. There exists an equilibrium (σ^*, μ^*) that satisfies D1 in which*

$$F_C^*(v) = \begin{cases} 0 & v < 0, \\ \frac{v}{p_C} & 0 \leq v \leq \tilde{v}_C^*, \\ 1 - \frac{2bx}{p_C} & \tilde{v}_C^* < v < \bar{v}_C^*, \\ 1 & v \geq \bar{v}_C^*, \end{cases}$$

and

$$F_E^*(v) = \begin{cases} 0 & v < \underline{v}_E^*, \\ \frac{b}{p_E}(v - p_C - (1-b)x) & \underline{v}_E^* \leq v \leq \bar{v}_E^*, \\ 1 & v > \bar{v}_E^*, \end{cases}$$

where $\tilde{v}_C^* = p_C - 2bx$, $\bar{v}_C^* = p_C - bx$, $\underline{v}_E^* = p_C + (1-b)x$, and $\bar{v}_E^* = \frac{1}{b}(1 - (1-b)(p_C - bx))$. Beliefs are $\mu(v) = 0$ for all $v \leq \bar{v}_C^*$ and $\mu(v) = 1$ for all $v > \bar{v}_C^*$. If $x < \frac{p_C}{2b}$, then in any other equilibrium (σ, μ) that satisfies D1, $\sigma = \sigma^*$.

The form of the equilibrium here is notably different than in Propositions 4 and 5. In those scenarios, centrist types always spent 0 and non-centrist types had a positive probability of doing so. In the present case, both types always spend in equilibrium. The cumulative distribution functions that correspond to their mixed strategies are plotted in Figure 1. Centrists employ a mixed strategy similar to the one used by non-centrists under Proposition 5. With probability $\frac{2bx}{p_C}$, a centrist spends exactly $\bar{v}_C^*$; otherwise, with probability $1 - \frac{2bx}{p_C}$, he selects v from a uniform distribution on $[0, \tilde{v}_C^*]$. Non-centrists' mixed strategy is simply a uniform distribution on $[\underline{v}_E^*, \bar{v}_E^*]$. Once again, the crucial feature of the equilibrium is that voters' beliefs shift immediately above the highest amount spent by centrists: any deviation to $v > \bar{v}_C^*$ is attributed to a non-centrist. If this were not the case, then it would be profitable for centrists to avert the chance of a tie by spending infinitesimally more than $\bar{v}_C^*$.

The non-centrist type's low marginal cost of fundraising is what causes *both* types to spend more when the equilibrium takes this form than in previous cases. The reasoning for why non-centrists spend more is straightforward. As noted in the discussion of Proposition 5, a non-centrist who

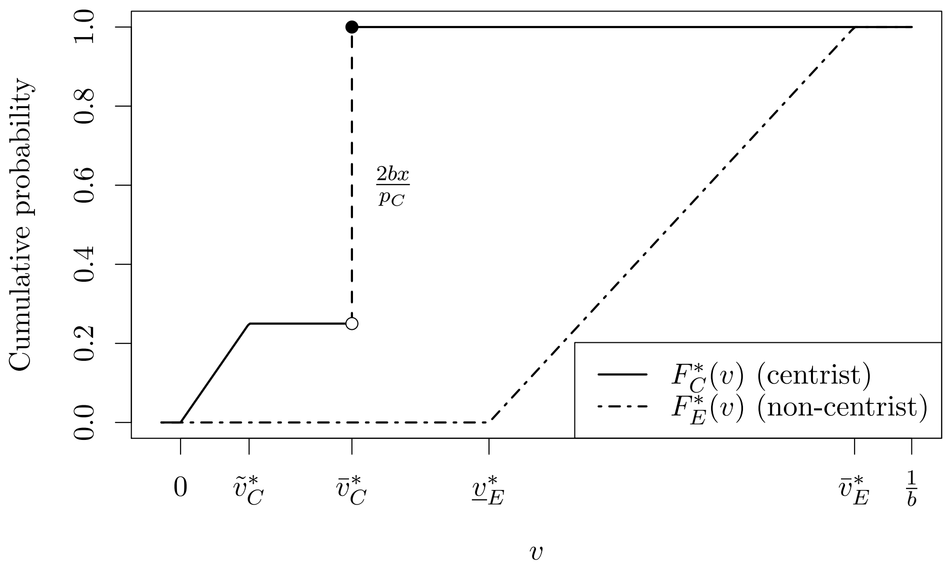

Figure 1: Cumulative distribution functions for the mixed strategy equilibrium in Proposition 6.

reveals his type must spend x more than the highest centrist in order to have a greater chance of victory. The condition $x \leq \frac{p_C}{2b}$ ensures that this is indeed profitable, so non-centrists no longer have an incentive to mimic centrists. This separation is indirectly responsible for why centrists no longer spend 0 in equilibrium, as shown formally in the proof of Proposition 6 (in the Appendix). Under a strategy profile in which centrists never spend, the best response by non-centrists yields an expected utility strictly greater than that of centrists. Recall from Lemma 5 that $\hat{a}$, the cutpoint defining off-the-path beliefs under D1, is a function of $U_E - U_C$, the difference in the types' equilibrium expected utilities. Because $U_E > U_C$, the cutpoint is no longer 0, meaning that deviations to amounts slightly above 0 would be ascribed to a centrist type. Since a deviation would no longer cause an unfavorable shift in the median voter's beliefs, the normal logic of all-pay auctions comes into effect: centrists would be better off spending $\epsilon > 0$ and defeating other centrists than spending 0 and tying with them. Therefore, there cannot be an equilibrium that satisfies D1 in which centrists do not spend.

To summarize, I have shown how non-centrist candidates can use a fundraising advantage to neutralize the policy advantage held by centrists. When non-centrists' fundraising advantage is large, as in Proposition 6, the mechanism is obvious: they can, at a relatively low cost, spend enough money increasing perceptions of their valence to overcome the negative electoral effect of their policy intentions. However, there is a significant effect even when the difference in the types' marginal cost of spending is slight. In these cases, represented by Propositions 4 and 5, the threat of being perceived by voters as an extremist on policy prevents centrist candidates from spending on the election. This, in turn, assures any non-centrist candidate of at least a 50% chance of winning the election: his *ex ante* chance of defeating another non-centrist is 50% under a symmetric equilibrium, and at worst he can assure a tie with centrists by spending nothing. Because non-centrists can choose not to reveal their type via high spending unless it is electorally beneficial to do so, we observe the apparent paradox of voter information: non-centrists are *more* likely to win in the forms of the equilibrium in which voters know their policy intentions. The same would not be true if voters were exogenously informed about candidates' types, instead of inferring them from the candidates' own spending decisions.

4 The Effects of Reform

The results of the model can be used to analyze the effects of campaign finance reform on voter welfare—a common concern in models of spending and electoral competition (e.g., Prat 2002*a*; Ashworth 2006; Meirowitz 2008). In this section, I find that reform is generally beneficial from the standpoint of reducing the likelihood that the victorious candidate has extreme policy intentions. In particular, if non-centrists can raise money more easily, reforms that reduce the size of this advantage either reduce the chance that a non-centrist is elected or have no effect. On the other hand, if the cost of fundraising is lower for centrists, marginal reductions in the size of their advantage do *not* lower the chance that a centrist is elected.

The first step in analyzing the effects of reform is to determine an appropriate measure of voter welfare. The usual measure in welfare analysis, the average of the equilibrium expected utility across the population, is not appropriate for the present model. The voter utility function (2) is strictly increasing in a candidate's spending, representing the fact that campaign spending raises perceptions of his quality. More broadly, this utility function is a way of embedding a behavioral assumption that, all else equal, voters are more likely to prefer a candidate who has spent more.

Treating the raw expression of voter utility as the maximand can lead to peculiar conclusions in welfare analysis, such as that voters would be better off if there were always a tie in which both candidates spent 1 than if there were always a tie with both spending 0. To avoid conflating high aggregate spending with a positive outcome for voters, I focus on an alternative criterion: the *ex ante* probability that the winning candidate has extreme policy intentions. This is equivalent to leaving out the spending component of voters' utility and focusing on the average of the policy component, which increases with the probability of electing a centrist. Because candidate types are drawn independently, the probability of electing a non-centrist is bounded below by p_E^2, the probability that both candidates are non-centrists, and bounded above by $1 - p_C^2$, the probability that at least one is a non-centrist.

The next question is how to define campaign finance reform in terms of the parameters of the model. I examine reforms that seek to reduce disparities in the ease of fundraising across types of candidates. This category includes policies that raise the marginal cost of fundraising for advantaged candidates, such as imposing contribution limits to prevent candidates from raising a large sum from a small set of supporters. It also includes policies that make it easier for disadvantaged candidates to raise money, such as public provision of matching funds for small campaign donations. In technical terms, such policies can be thought of as moving the ratio of the two types' costs of fundraising, $\frac{1}{b}$, closer to 1. This is equivalent to increasing b when it is less than 1 (when non-centrists are advantaged) and decreasing b when it is greater than 1 (when centrists are advantaged). I assess the welfare consequences of this kind of policy by examining how changes in b affect the equilibrium probability that a non-centrist wins the election.

I begin by looking at the effects of reform when non-centrists can raise money more easily, i.e., $b < 1$. As discussed in Section 3.3, non-centrist candidates follow one of two strategies in equilibrium here: either they spend enough to be able to defeat centrists despite revealing their extreme policy intentions, or they settle for a tie by mimicking centrists and spending nothing. Intuitively, one would suspect that raising the cost of fundraising for non-centrists would push them toward the latter option, which in turn would reduce their chance of victory. This logic is borne out by examining each of the three forms that the equilibrium takes:

- Proposition 4 ($b \geq \frac{1}{2xp_C}$): All candidates spend 0 in equilibrium and tie, so the *ex ante* probability of victory by a non-centrist is $p_E^2 + \frac{1}{2}p_E p_C$. Marginal changes in b therefore do not affect voter welfare.

- Proposition 5 ($\frac{p_C}{2x} < b < \frac{1}{2xp_C}$): Centrists always spend 0, while non-centrists employ a mixed strategy that involves spending 0 with probability $\pi = \frac{\sqrt{2bxp_C} - p_C}{p_E}$. There are three circumstances in which a non-centrist may win the election: (1) both candidates are non-centrists, (2) one candidate is a non-centrist who spends more than 0, (3) one candidate is a centrist who spends 0, and the tie is broken in his favor. The probability of any of these occurring is:

$$\underbrace{p_E^2}_{(1)} + \underbrace{p_E p_C(1-\pi)}_{(2)} + \underbrace{\frac{1}{2}p_E p_C \pi}_{(3)} = p_E^2 + p_E p_C\left(1 - \frac{\pi}{2}\right).$$

  Since π increases with b, a marginal increase in b would cause the probability of a non-centrist winning the election to go down.

- Proposition 6 ($b \leq \frac{p_C}{2x}$): All candidates spend a positive amount, and non-centrists spend enough to defeat any centrist opponent. Since a non-centrist is guaranteed to win the election unless both candidates are centrists, the *ex ante* probability of electing a non-centrist is $p_E^2 + p_E p_C$. As in the first case, marginal changes in b do not affect voter welfare.

As expected, if non-centrist candidates can raise money more easily, reforms that reduce this advantage are either beneficial to voters or have no effect. Such reforms make it more expensive for non-centrists to use additional spending to compensate for their policy distance from the median voter. Non-centrists are thus more likely to mimic centrists and tie with them instead of defeating them, increasing the chance that a centrist will win the election.

It would be natural to suspect that the opposite logic holds when centrists can raise money more easily—that reducing their fundraising advantage would increase the chance of electing a non-centrist. On the contrary, such changes have no effect on the chance that a non-centrist wins the election. In the equilibrium when $b > 1$, which is given by Proposition 3, a centrist candidate always defeats a non-centrist opponent. The probability of victory by a non-centrist is simply the chance that both candidates are non-centrists, p_E^2, which is insensitive to changes in b. To see why this is unlike the previous case, recall that the equilibrium with $b > 1$ is not a mirror image of the equilibrium with $b < 1$. When non-centrists can raise money more easily, they must balance the positive effect of high spending against the negative effect of revealing their type. There is no such tradeoff when centrists are advantaged, as they receive an electoral benefit from revealing their type. A marginal increase in centrists' cost of fundraising would still leave them with no incentive to mimic non-centrists, and thus would not change the chance that a centrist will win the election. The only effect of such a change would be to reduce centrist candidates' equilibrium utility, which is unrelated to the welfare of the electorate.

On the whole, reforms that reduce disparities in the cost of fundraising have a neutral or positive effect on voter welfare. Throughout most of the parameter space, marginal changes in b have no effect on the probability that a candidate with extreme policy intentions is elected. The sole exception is when non-centrists are advantaged and the conditions of Proposition 5 hold, in which case increasing non-centrists' marginal cost of spending makes them more likely to mimic centrists and thus reduces their chance of victory. Most importantly, reform has asymmetric effects: it can reduce non-centrists' electoral fortunes when they have a fundraising advantage, but it does not affect centrists' chances in the reverse case. From the standpoint of bringing policy in line with the median voter's preferences, marginal increases in the cost of fundraising for the advantaged type (or decreases for the disadvantaged one) appear to have little downside.

5 Conclusion

I have analyzed a model of electoral competition in which candidates' ability to raise money is related to the policy they will enact in office. I have characterized an equilibrium of the model and shown that is unique among those satisfying the D1 refinement. The form of the equilibrium depends on which type of candidate can raise funds more easily. If centrists are advantaged, there is a fully separating equilibrium in mixed strategies, in which centrist candidates always defeat any non-centrist opponent. On the other hand, if non-centrists have a lower marginal cost of fundraising than centrists, the extent to which non-centrists reveal their type via high spending depends on the magnitude of their advantage. I have analyzed the effects of a potential campaign finance reform in the context of this model and found that it would only reduce the advantaged type's chance of winning the election when non-centrists are advantaged.

The analysis here suggests promising directions for both theoretical and empirical research.

One natural extension would be to directly model the interaction between candidates and interest groups, as in previous models of campaign finance (e.g., Prat 2002*a*; Wittman 2007), rather than the reduced-form approach here. Such an analysis could also incorporate candidates' choice to accept public financing, which lowers the marginal cost of fundraising but imposes a spending cap, as well as serving as a public signal that a candidate's finance does not come from special interests. Another avenue of further analysis would be to incorporate Downsian policy announcements into the model, as in other theories of endogenous valence (e.g., Ashworth and Bueno de Mesquita 2009). On the empirical side, the results here highlight the need for more direct evidence about what voters infer from candidate spending. Other than some experimental work (Houser and Stratmann 2008), there is little individual-level data on whether high spending causes voters to believe that candidates are in thrall to special interests. Empirical research along these lines would be useful for identifying the real-world conditions to which this paper's results would best apply.

A Appendix

Lemma 1 (Strict dominance)**.** *In any equilibrium,* $\operatorname{supp}\sigma_t \subseteq [0, \frac{1}{b_t}]$ *for each type* $t \in T$*.*

Proof. Suppose $v \in \operatorname{supp}\sigma_t$ with $v > 1/b_t$. Take any $\epsilon \in (0, v - 1/b_t)$ and let $S = (v - \epsilon, v + \epsilon)$. Using (7), we have

$$Eu_t(v') = \lambda(v') - b_t v' \leq 1 - b_t v' < 0$$

for all $v' \in S$. But notice that $Eu_t(0) = \lambda(0) - b_t \cdot 0 \geq 0$, so spending 0 strictly dominates any $v' \in S$. Since $v \in \operatorname{supp}\sigma_t$ implies $\sigma_t(S) > 0$, this cannot be an equilibrium strategy. □

Proposition 1. *There is no equilibrium in which* $\operatorname{supp}\sigma_C = \{v_C\}$ *and* $\operatorname{supp}\sigma_E = \{v_E\}$ *with* $v_C \neq v_E$*.*

Proof. Immediate from Corollary A.2 below. □

Lemma A.1. *Consider an equilibrium assessment* (σ, μ) *and a type* $t \in T$*. For almost all* $v \in \operatorname{supp}\sigma_t$*,* $U_t = Eu_t(v) = \lambda(v) - b_t v$*. This is true for all* v *such that* $\lambda(\cdot)$ *is continuous at* v *or* $\sigma_t(\{v\}) > 0$*.*

Proof. Suppose the claim is not true for almost all $v \in \operatorname{supp}\sigma_t$, so there exists a set of positive measure $S \subseteq \operatorname{supp}\sigma_t$ such that $Eu_t(v) \neq U_t$ for all $v \in S$. There cannot be any v such that $Eu_t(v) > U_t$, as this would contradict the optimality condition of equilibrium, so we must have $Eu_t(v) < U_t$ for all $v \in S$. But $S \subseteq \operatorname{supp}\sigma_t$ implies $\sigma_t(S) > 0$, so this contradicts the indifference condition of mixed-strategy equilibrium.

The second claim is immediate for mass points (those where $\sigma_t(\{v\}) > 0$), again by the indifference condition. To establish the other part, take any $v \in \operatorname{supp}\sigma_t$ such that $\lambda(\cdot)$ is continuous at v, and suppose $Eu_t(v) \neq U_t$. For the same reasons as above, this gives $Eu_t(v) < U_t$. Because $\lambda(\cdot)$ is continuous at v, so is $Eu_t(\cdot)$. Therefore, there exists $\epsilon > 0$ such that $Eu_t(v') < U_t$ for all $v' \in (v - \epsilon, v + \epsilon) = S$.[9] But $v \in \operatorname{supp}\sigma_t$ implies $\sigma_t(S) > 0$, so this again contradicts the indifference condition of equilibrium. □

Lemma 2. *Consider an equilibrium assessment* (σ, μ)*, a type* $t \in T$*, and* $v \in \operatorname{supp}\sigma_t$ *such that* $U_t = Eu_t(v) = \lambda(v) - b_t v$*. For all* $v' < v$*,* $\lambda(v) > \lambda(v')$ *and* $Eu_m(v) > Eu_m(v')$*. This holds for almost all* $v \in \operatorname{supp}\sigma_t$*, and for all* $v \in \operatorname{supp}\sigma_t$ *such that* $\lambda(\cdot)$ *is continuous at* v *or* $\sigma_t(\{v\}) > 0$*.*

Proof. Take a $v \in \operatorname{supp}\sigma_t$ such that $U_t = \lambda(v) - b_t v$ and suppose that $\lambda(v') \geq \lambda(v)$ for some $v' < v$. This gives $Eu_t(v') = \lambda(v') - b_t v' > U_t$, which contradicts the assumption of equilibrium. Moreover, by (4) and (6), $Eu_m(v') \geq Eu_m(v)$ implies $\lambda(v') \geq \lambda(v)$, and thus yields the same contradiction. The final claim of the lemma is immediate from Lemma A.1. □

Lemma 3. *Let* (σ, μ) *be an equilibrium. If* $b \geq 1$*, then* $U_C \geq U_E$*. Conversely, if* $b \leq 1$*, then* $U_C \leq U_E$*.*

Proof. Let $b \geq 1$ and suppose the result does not hold, so there exists an equilibrium in which $U_C < U_E$. By Lemma A.1, there exists $v \in \operatorname{supp}\sigma_E$ such that $U_E = \lambda(v) - bv$. This gives $Eu_C(v) = \lambda(v) - v \geq \lambda(v) - bv > U_C$, which contradicts the assumption of equilibrium. The proof for the case of $b \leq 1$ is analogous. □

[9] The same logic applies if $\lambda(\cdot)$ is right-continuous at v, as long as $\sigma_t((v, v + \epsilon)) > 0$ for all $\epsilon > 0$.

Lemma 4. *Let (σ, μ) be an equilibrium and define*

$$\hat{a} \equiv \frac{U_E - U_C}{1 - b}. \tag{9}$$

If $b < 1$, then $\max \operatorname{supp} \sigma_C \leq \hat{a} \leq \min \operatorname{supp} \sigma_E$. If $b > 1$, then $\max \operatorname{supp} \sigma_E \leq \hat{a} \leq \min \operatorname{supp} \sigma_C$.

Proof. Suppose $b \neq 1$ and take $s, t \in T$ such that $b_t < b_s$. (If $b < 1$, then $t = E$ and $s = C$; if $b > 1$, $t = C$ and $s = E$.) Lemma 3 gives $U_t \geq U_s$ and thus $\hat{a} = (U_t - U_s)/(b_s - b_t)$. Suppose $\max \operatorname{supp} \sigma_s > \hat{a}$, so there exists $v > \hat{a}$ such that $U_s = \lambda(v) - b_s v$. This gives

$$v > \frac{U_t - (\lambda(v) - b_s v)}{b_s - b_t}.$$

Since $b_s > b_t$, rearranging terms yields

$$b_s v - b_t v > U_t - \lambda(v) + b_s v.$$

This gives $U_t < \lambda(v) - b_t v = Eu_t(v)$, which contradicts the assumption of equilibrium. An analogous contradiction is obtained if $\min \operatorname{supp} \sigma_t < \hat{a}$. □

Lemma 5. *Suppose $b \neq 1$. Let (σ, μ) be an equilibrium and let $\hat{a}$ be defined as in (9). The equilibrium survives the D1 refinement if and only if the following holds for all off-the-path $v \leq \max\{1 - U_C, (1 - U_E)/b\}$:*

- *If $b < 1$, $v < \hat{a}$ implies $\mu(v) = 0$ and $v > \hat{a}$ implies $\mu(v) = 1$.*
- *If $b > 1$, $v < \hat{a}$ implies $\mu(v) = 1$ and $v > \hat{a}$ implies $\mu(v) = 0$.*

Proof. As in the similar models of Banks (1990) and Callander and Wilkie (2007), it is sufficient to consider the probability of winning that would give a candidate an incentive to deviate to an off-the-path spending choice. Take any $v \in \Re_+ \setminus (\operatorname{supp} \sigma_C \cup \operatorname{supp} \sigma_E)$. Let π_t denote the minimal probability of victory that would give type t a weak incentive to deviate to v, so $\pi_t = U_t + b_t v$. If $\pi_C > 1$ and $\pi_E > 1$, neither type can have a weak incentive to deviate to v, so D1 places no restriction on beliefs. Otherwise, if $\pi_C < \pi_E$, then it is easier to induce a candidate of type C to deviate to v, so D1 requires that $\mu(v) = 0$. Conversely, if $\pi_C > \pi_E$, D1 requires that $\mu(v) = 1$.

If $v > \max\{1 - U_C, (1 - U_E)/b\}$, then $\pi_C > 1$ and $\pi_E > 1$, so D1 places no restrictions on beliefs. Otherwise, note that

$$\hat{a} - v = \frac{U_E - U_C - (1 - b)v}{1 - b} = \frac{\pi_E - \pi_C}{1 - b}.$$

If $b < 1$, then $\pi_E \lesseqgtr \pi_C$ if and only if $\hat{a} \lesseqgtr v$. If $b > 1$, then $\pi_E \lesseqgtr \pi_C$ if and only if $\hat{a} \gtreqless v$. □

Lemma A.2. *If (σ, μ) is an equilibrium, $\max \operatorname{supp} \sigma_t \leq (1 - U_t)/b_t$ for each $t \in T$.*

Proof. Suppose not, so there exists an equilibrium with $\max \operatorname{supp} \sigma_t > (1 - U_t)/b_t$ for some type t. By Lemma A.1, there exists $v > (1 - U_t)/b_t$ such that $U_t = \lambda(v) - b_t v$. This gives

$$\begin{aligned} U_t &= \lambda(v) - b_t v \\ &\leq 1 - b_t v \\ &< 1 - b_t \frac{1 - U_t}{b_t} = U_t, \end{aligned}$$

a contradiction. $\square$

Corollary A.1. *If (σ, μ) is an equilibrium and $\sigma_t(\{v\}) > 0$ for some type $t \in T$ and value $v \in \Re_+$, then $v < (1 - U_t)/b_t$.*

Proof. Suppose not, so by Lemma A.2, we must have $v = (1-U_t)/b_t$. Since $\sigma_t(\{v\}) > 0$, Lemma A.1 gives $U_t = \lambda(v) - b_t v$. Combining these, we yield

$$U_t = \lambda(v) - b_t \frac{1 - U_t}{b_t} = (\lambda(v) - 1) + U_t.$$

This implies $\lambda(v) = 1$, which contradicts $\sigma_t(\{v\}) > 0$. $\square$

Lemma A.3. *Let (σ, μ) be an equilibrium with a type $t \in V$ and value $v \in \Re_+$ such that $\sigma_t(\{v\}) > 0$. There exists $\delta > 0$ such that $\mu(v') > \mu(v)$ for all $v' \in (v, v + \delta)$.*

Proof. Consider an assessment with $\sigma_t(\{v\}) = \pi > 0$ for a type $t \in T$ and a value $v \in \Re_+$. There are two immediate consequences of v being a mass point. First, since the *ex ante* probability of facing a candidate spending v is at least πp_t, it follows from (6) that $\lambda(v) \leq 1 - \frac{\pi p_t}{2} < 1$. Second, if $Eu_m(v') > Eu_m(v)$ for some $v' \in \Re_+$, it follows from (6) that $\lambda(v') \geq \lambda(v) + \frac{p_t \pi}{2}$.

Now suppose the claim in the lemma is not true, so for all $\delta > 0$, there exists $v' \in (v, v+\delta)$ with $\mu(v') \leq \mu(v)$. Let $\delta = \frac{\pi p_t}{2b_t}$ and take such a v'. It is immediate from (3) that $Eu_m(v') > Eu_m(v)$, as $v' > v$ and $\mu(v') \leq \mu(v)$. This gives

$$\begin{aligned} Eu_t(v') &= \lambda(v') - b_t v' \\ &\geq \lambda(v) + \frac{\pi p_t}{2} - b_t v' \\ &> \lambda(v) + \frac{\pi p_t}{2} - b_t(v + \delta) \\ &= \lambda(v) - b_t(v) = Eu_t(v). \end{aligned}$$

This contradicts the assumption of equilibrium, as Lemma A.1 implies that $U_t = Eu_t(v)$. $\square$

Corollary A.2. *Let (σ, μ) be an equilibrium. If $\sigma_E(\{v\}) > 0$ for some $v \in \Re_+$, then $\sigma_C(\{v\}) > 0$. That is, any mass point of E's mixed strategy must also be a mass point of C's mixed strategy.*

Proof. Suppose not, so there is an equilibrium where $\sigma_E(\{v\}) > 0$ and $\sigma_C(\{v\}) = 0$ for some $v \in \Re_+$. Bayes' rule gives $\mu(v) = 1$, so for any $\delta > 0$, we have $\mu(v') \leq \mu(v)$ for all $v' \in (v, v + \delta)$. This contradicts Lemma A.3. $\square$

Lemma A.4. *Suppose $b \neq 1$, let (σ, μ) be an equilibrium that satisfies D1, let $\hat{a}$ be defined as in (9), and consider a type $t \in T$. For all $v \neq \hat{a}$, $\sigma_t(\{v\}) = 0$. That is, no type's mixed strategy may have a mass point other than $\hat{a}$.*

Proof. Suppose the claim in the lemma is not true, so there is an equilibrium that satisfies D1 with $\sigma_t(\{v\}) > 0$ for a type $t \in T$ and value $v \neq \hat{a}$. By Corollary A.1, $v < \max\{1 - U_C, (1 - U_E)/b\}$. Therefore, by Lemma 5, off-the-path beliefs are pinned down by D1 in a neighborhood of v. Moreover, Lemma 4 implies that $v \notin \operatorname{supp} \sigma_s$, where $s \neq t$ is the other type of candidate. It follows from these results that for some $\delta > 0$, we have $\mu(v') = \mu(v)$ for all $v' \in (v, v + \delta)$. This contradicts Lemma A.3. $\square$

Corollary A.3. *Suppose $b > 1$ and let (σ, μ) be an equilibrium that satisfies D1. Then $\sigma_t(\{v\}) = 0$ for all $t \in T$ and $v \in \Re_+$. That is, neither type's mixed strategy may contain a mass point.*

Proof. Suppose not, so there is an equilibrium that satisfies D1 where $\sigma_t(\{v\}) > 0$ for some $t \in T$ and $v \in \Re_+$. By Lemma A.4, this implies $v = \hat{a}$, where $\hat{a}$ is defined as in (9). Let $\delta = \frac{1-U_t}{b_t} - \hat{a}$, noting that $\delta > 0$ by Corollary A.1, and take any $v \in (\hat{a}, \hat{a} + \delta)$. By Lemma 4, $v \notin \operatorname{supp} \sigma_E$. Therefore, if $v \in \operatorname{supp} \sigma_C$, Bayes' rule gives $\mu(v) = 0$. Otherwise, $v \notin (\operatorname{supp} \sigma_C \cup \operatorname{supp} \sigma_E)$, so by Lemma 5, $\mu(v) = 0$. Hence $\mu(v) \leq \mu(\hat{a})$ for all $v \in (\hat{a}, \hat{a} + \delta)$, which contradicts Lemma A.3. $\square$

Lemma A.5. *Let (σ, μ) be an equilibrium that satisfies D1, let $\hat{a}$ be defined as in (9), and consider a type $t \in T$. If there exist $\underline{v}, \bar{v} \in \operatorname{supp} \sigma_t$ such that $\underline{v} < \bar{v} < \hat{a}$ or $\hat{a} < \underline{v} < \bar{v}$, then $[\underline{v}, \bar{v}] \subseteq \operatorname{supp} \sigma_T$.*

Proof. Suppose not, so there exist v', v'' such that $\underline{v} \leq v' < v'' \leq \bar{v}$ and $(v', v'') \cap \operatorname{supp} \sigma_t = \emptyset$. Let $\hat{v}'$ and $\hat{v}''$ denote the boundary points of this gap, so that $\hat{v}' = \max\{v \leq v' \,|\, v \in \operatorname{supp} \sigma_t\}$ and $\hat{v}'' = \min\{v \geq v'' \,|\, v \in \operatorname{supp} \sigma_t\}$. Notice that $\underline{v} \leq \hat{v}' < \hat{v}'' \leq \bar{v}$, so either $\hat{a} < \hat{v}'$ or $\hat{a} > \hat{v}''$. Therefore, because $\hat{v}', \hat{v}'' \in \operatorname{supp} \sigma_t$, Lemma A.1 implies that there exist $\tilde{v}' \leq \hat{v}'$ and $\tilde{v}'' \geq \hat{v}''$ such that $\tilde{v}', \tilde{v}'' \in \operatorname{supp} \sigma_t \setminus \{\hat{a}\}$ and $U_t = \lambda(\tilde{v}') - b_t \tilde{v}' = \lambda(\tilde{v}'') - b_t \tilde{v}''$. It is immediate from Lemmas 4, 5, and A.2 that under D1, $\mu(v)$ is constant on $[\tilde{v}', \tilde{v}'']$. Therefore, by (3), $Eu_m(\cdot)$ is strictly increasing on $[\tilde{v}', \tilde{v}'']$. This in turn implies that $\lambda(\cdot)$ is weakly increasing on $[\tilde{v}', \tilde{v}'']$.

Now suppose $\lambda(\hat{v}'') > \lambda(\hat{v}')$, so by (6) there exists a type $s \in T$ and a set $S \subseteq \operatorname{supp} \sigma_s$ such that $\sigma_s(S) > 0$ and $Eu_m(\hat{v}') < Eu_m(v) \leq Eu_m(\hat{v}'')$ for all $v \in S$. Because $Eu_m(\cdot)$ is strictly increasing on $[\tilde{v}', \tilde{v}'']$, we have $S \cap ([\tilde{v}', \hat{v}') \cup (\hat{v}'', \tilde{v}'']) = \emptyset$. In addition, recall that $\sigma_s([\hat{v}', \hat{v}'']) = 0$. These results imply $\sigma_s(S \setminus [\tilde{v}', \tilde{v}'']) > 0$, so by Lemma A.1 there exists $v \in S \setminus [\tilde{v}', \tilde{v}'']$ such that $U_s = \lambda(v) - b_s v$. If $v > \tilde{v}''$, then we have $Eu_m(v) \leq Eu_m(\hat{v}'') < Eu_m(\tilde{v}'')$ and $v > \tilde{v}''$, contradicting Lemma 2. Otherwise, if $v < \tilde{v}'$, then we have $Eu_m(\tilde{v}') < Eu_m(\hat{v}') < Eu_m(v)$ and $\tilde{v}' > v$, again contradicting Lemma 2. Therefore, $\lambda(\hat{v}'') = \lambda(\hat{v}')$. This also shows that $\lambda(\cdot)$ is continuous at $\hat{v}''$, as otherwise, by (6), we would have $\Pr(Eu_m(v_{-i}) = Eu_m(\hat{v}'')) > 0$ and thus $\lambda(\hat{v}'') > \lambda(\hat{v}')$. However, we now have that $\hat{v}'' \in \operatorname{supp} \sigma_t$, $\lambda(\cdot)$ is continuous at $\hat{v}''$, and $\lambda(\hat{v}'') = \lambda(\hat{v}')$ with $\hat{v}'' > \hat{v}'$, contradicting Lemma 2. $\square$

Lemma A.6. *Suppose $b > 1$ and let (σ, μ) be an equilibrium that satisfies D1. Then the CDF of E's mixed strategy is*

$$F_E(v) = \begin{cases} 0 & v < 0, \\ \frac{bv}{p_E} & 0 \leq v \leq \frac{p_E}{b}, \\ 1 & v > \frac{p_E}{b}. \end{cases} \tag{10}$$

Proof. It follows from Corollary A.3 and Lemma A.5 that $\operatorname{supp} \sigma_E = [\underline{v}_E, \bar{v}_E]$ with $\bar{v}_E > \underline{v}_E$. I begin by establishing that $\lambda(\cdot)$ is continuous on $[\underline{v}_E, \bar{v}_E)$. By Lemmas 5 and 4, $\mu(v) = 1$ for all $v \in [0, \bar{v}_E)$, so $Eu_m(\cdot)$ is strictly increasing on this interval. Suppose there is a point $v_0 \in [\underline{v}_E, \bar{v}_E)$ at which $\lambda(\cdot)$ is not continuous. By (6), this implies there exists a set S such that $\sigma_C(S) + \sigma_E(S) > 0$ and $Eu_m(v) = Eu_m(v_0)$ for all $v \in S$. This gives

$$Eu_m(v) = v - \mu(v)x = v_0 - x = Eu_m(v_0)$$

for all such v. This implies $(1 - \mu(v))x = v_0 - v$ and thus $v \leq v_0$ for all such v. However, $v \leq v_0$ implies $\mu(v) = 1$, which in turn gives $v = v_0$ and thus $S = \{v_0\}$ Since σ_E contains no mass points and $\min \operatorname{supp} \sigma_C > v_0$, we have $\sigma_C(S) + \sigma_E(S) = 0$, a contradiction. Therefore, $\lambda(\cdot)$ must be continuous on $[\underline{v}_E, \bar{v}_E)$. In turn, by Lemma A.1, $U_E = \lambda(v) - bv$ for all $v \in [\underline{v}_E, \bar{v}_E)$.

Now suppose $\underline{v}_E > 0$. By Lemma 4, $\min \operatorname{supp} \sigma_C \geq \hat{a} \geq \bar{v}_E > \underline{v}_E$. It then follows from Lemma A.4 that $\sigma_C(\{\underline{v}_E\}) + \sigma_E(\{\underline{v}_E\}) = 0$. Therefore, by Lemma 2, there does not exist a set $S \subseteq \Re_+$ such that $\sigma_C(S)+\sigma_E(S) > 0$ and $Eu_m(v) \leq Eu_m(\underline{v}_E)$ for all $v \in S$. This implies $\lambda(\underline{v}_E) = 0$ and thus

$$U_E = -b\underline{v}_E < 0 \leq \lambda(0) = Eu_E(0),$$

which contradicts the assumption of equilibrium. Therefore, $\operatorname{supp} \sigma_E = [0, \bar{v}_E]$.

It is now possible to establish the claim in the lemma. Because $\lambda(\cdot)$ is continuous at 0, we have $U_E = \lambda(0) - 0 = 0$. This in turn gives $\lambda(v) - bv = 0$ and thus $\lambda(v) = bv$ for each $v \in [0, \bar{v}_E)$. In addition, since $\min \operatorname{supp} \sigma_C \geq \bar{v}_E$, it follows from Lemma 2 that $\lambda(v) = p_E F_E(v)$ for all $v \in [0, \bar{v}_E)$. This gives $F_E(v) = \frac{bv}{p_E}$ for all such v. Notice that $F_E(\bar{v}_E) = 1$ by construction, and absence of mass points in σ_E implies

$$F_E(\bar{v}_E) = \lim_{v \to \bar{v}_E^-} F_E(v) = \frac{b\bar{v}_E}{p_E},$$

and thus $\bar{v}_E = \frac{p_E}{b}$. This gives the result. □

Lemma A.7. *Suppose $b \neq 1$ and let (σ, μ) be an equilibrium that satisfies D1. Take types $s, t \in T$ such that $b_t < b_s$, and let $\bar{v}_s = \max \operatorname{supp} \sigma_s$ and $\underline{v}_t = \min \operatorname{supp} \sigma_t$. Then $Eu_m(\bar{v}_s) = Eu_m(\underline{v}_t)$.*

Proof. Suppose the claim does not hold, so either $Eu_m(\bar{v}_s) > Eu_m(\underline{v}_t)$ or $Eu_m(\bar{v}_s) < Eu_m(\underline{v}_t)$. It follows from Lemma 4 that $\bar{v}_s \leq \underline{v}_t$. Equality would yield an immediate contradiction, so $\bar{v}_s < \underline{v}_t$.

I first establish that $\underline{v}_t$ is not a mass point in t's mixed strategy. Suppose not, so $\sigma_t(\{\underline{v}_t\}) > 0$. Since $\underline{v}_t \notin \operatorname{supp} \sigma_s$, Corollary A.2 implies $t = C$ and thus $s = E$. Since $\min \operatorname{supp} \sigma_C > \max \operatorname{supp} \sigma_E$, Lemma 4 gives $b > 1$. But then the presence of a mass point in σ_C contradicts Corollary A.3. Therefore, $\sigma_t(\{\underline{v}_t\}) = 0$; moreover, by Lemma A.4, σ_t may not place positive mass on any other point either.

First suppose that $Eu_m(\bar{v}_s) > Eu_m(\underline{v}_t)$. Since $\sigma_t(\{\underline{v}_t\}) = 0$, by definition of support there exists $\delta > 0$ such that $[\underline{v}_t, \underline{v}_t + \delta) \subseteq \operatorname{supp} \sigma_t$. Bayes' rule gives $\mu(v) = \mu(\underline{v}_t)$ for all $v \in [\underline{v}_t, \underline{v}_t + \delta)$. Let $S = (\underline{v}_t, \underline{v}_t + \delta) \cap (\underline{v}_t, \underline{v}_t + (Eu_m(\bar{v}_s) - Eu_m(\underline{v}_t)))$. For all $v \in S$,

$$\begin{aligned} Eu_m(v) &= v + \mu(v)x \\ &= v + \mu(\underline{v}_t)x \\ &< \underline{v}_t + Eu_m(\bar{v}_s) - Eu_m(\underline{v}_t) + \mu(\underline{v}_t)x \\ &= Eu_m(\bar{v}_s). \end{aligned}$$

Since $\sigma_t(S) > 0$, this contradicts Lemma 2.

We thus must have $Eu_m(\bar{v}_s) < Eu_m(\underline{v}_t)$. Since $\operatorname{supp} \sigma_s \cap \operatorname{supp} \sigma_t = \emptyset$, we have $\mu(v) = \mu(\underline{v}_t)$ for all $v \in \operatorname{supp} \sigma_t$. Then, by (3), we have $Eu_m(v') > Eu_m(v)$ for all $v, v' \in \operatorname{supp} \sigma_t$ such that $v' > v$. An analogous result also holds for s, since $\mu(\cdot)$ is also constant on $\operatorname{supp} \sigma_s$. Because $Eu_m(\underline{v}_t) > Eu_m(\bar{v}_s)$, this in turn gives $Eu_m(v) > Eu_m(v')$ for all $v \in \operatorname{supp} \sigma_t$ and $v' \in \operatorname{supp} \sigma_s$. Since σ_t contains no mass points, it follows from (6) that $\lambda(\cdot)$ is continuous on any interval within $\operatorname{supp} \sigma_t$. As noted above, since $\sigma_t(\{\underline{v}_t\}) = 0$, there exists $\delta > 0$ such that $[\underline{v}_t, \underline{v}_t + \delta) \subseteq \operatorname{supp} \sigma_t$. Therefore, $\lambda(\cdot)$ is right-continuous at $\underline{v}_t$ and thus, by Lemma A.1 (see footnote 9), $U_t = Eu_t(\underline{v}_t)$.

There are now two cases to consider: either $\sigma_s(\{\bar{v}_s\}) = 0$ or $\sigma_s(\{\bar{v}_s\}) > 0$. In the former case, since neither $\bar{v}_s$ nor $\underline{v}_t$ is a mass point and $Eu_m(v) \leq Eu_m(\bar{v}_s) < Eu_m(\underline{v}_t) \leq Eu_m(v')$ for all $v \in \operatorname{supp} \sigma_s$ and $v' \in \operatorname{supp} \sigma_t$, we have $\lambda(\underline{v}_t) = \lambda(\bar{v}_s)$. Since $U_t = Eu_t(\underline{v}_t)$ and $\underline{v}_t > \bar{v}_s$, this contradicts Lemma 2. In the latter case, since $\sigma_t(\{\bar{v}_s\}) > 0$, it follows from Lemma A.4 that $\bar{v}_s = \hat{a}$; Lemma 5 then gives $\mu(v) = \mu(\underline{v}_t)$ for all $v \in (\bar{v}_s, \underline{v}_t)$. This implies that $Eu_m(\cdot)$ is continuous on $(\bar{v}_s, \underline{v}_t]$. Since

$Eu_m(\underline{v}_t) > Eu_m(\bar{v}_s)$, by continuity there is $v \in (\bar{v}_s, \underline{v}_t)$ such that $Eu_m(v) > Eu_m(\bar{v}_s)$. In fact, we have $\lambda(v) = \lambda(\underline{v}_t)$, because $\underline{v}_t$ is not a mass point and $Eu_m(v') < Eu_m(v) < Eu_m(\underline{v}_t) \leq Eu_m(v'')$ for all $v' \in \operatorname{supp} \sigma_s$ and $v'' \in \operatorname{supp} \sigma_t$. Since $U_t = Eu_t(\underline{v}_t)$ and $\underline{v}_t > v$, this contradicts Lemma 2. □

Lemma A.8. *Suppose $b > 1$ and let (σ, μ) be an equilibrium that satisfies D1. Then the CDF of C's mixed strategy is*

$$F_C(v) = \begin{cases} 0 & v < \frac{p_E}{b}, \\ \frac{1}{p_C}(v - \frac{p_E}{b}) & \frac{p_E}{b} \leq v \leq p_C + \frac{p_E}{b}, \\ 1 & v > p_C + \frac{p_E}{b}. \end{cases} \tag{11}$$

Proof. Let $\underline{v}_t = \min \operatorname{supp} \sigma_t$ and $\bar{v}_t = \max \operatorname{supp} \sigma_t$ for each $t \in T$. By Corollary A.3 and Lemma A.5, $\operatorname{supp} \sigma_C = [\underline{v}_C, \bar{v}_C]$ with $\bar{v}_C > \underline{v}_C$. Notice that $\underline{v}_C = \bar{v}_E$; otherwise, if $\underline{v}_C > \bar{v}_E$, Bayes' rule gives $\mu(\underline{v}_C) = 0$ and $\mu(\bar{v}_E) = 1$, and thus

$$Eu_m(\underline{v}_C) = \underline{v}_C > \bar{v}_E - x = Eu_m(\bar{v}_E),$$

contradicting Lemma A.7. We therefore have $\underline{v}_C = \bar{v}_E = \frac{p_E}{b}$ by Lemma A.8.

Note that $\mu(v) = 0$ for all $v \in (\frac{p_E}{b}, \bar{v}_C]$ and $\mu(v) = 1$ for all $v \in [0, \frac{p_E}{b})$, so $Eu_m(\cdot)$ is strictly increasing on each of these intervals. Moreover, we have

$$\frac{p_E}{b} - x \leq \frac{p_E}{b} - \mu\left(\frac{p_E}{b}\right) x \leq \frac{p_E}{b}$$

and thus

$$\lim_{v \to \frac{p_E}{b}^-} Eu_m(v) \leq Eu_m\left(\frac{p_E}{b}\right) \leq \lim_{v \to \frac{p_E}{b}^+} Eu_m(v),$$

so $Eu_m(\cdot)$ is strictly increasing on the whole interval $[0, \bar{v}_C]$. Since neither type's strategy may contain a mass point (by Corollary A.3), this gives

$$\lambda(v) = p_E F_E(v) + p_C F_C(v)$$

for all $v \in [0, \bar{v}_C]$. Clearly $\lambda(\cdot)$ is continuous on $[0, \bar{v}_C]$, so we have

$$U_C = \lambda(\underline{v}_C) - \underline{v}_C = p_E - \frac{p_E}{b}$$

by Lemma A.1. From the indifference condition of equilibrium, we then have

$$Eu_C(v) = p_E + p_C F_C(v) - v = p_E - \frac{p_E}{b} = U_C$$

for each $v \in [\underline{v}_C, \bar{v}_C]$. This gives $F_C(v) = \frac{1}{p_C}(v - \frac{p_E}{b})$, as in (11), for all $v \in [\underline{v}_C, \bar{v}_C]$. Moreover, since $F_C(\bar{v}_C) = 1$ by definition, the above expression also gives $\bar{v}_C = p_C + \frac{p_E}{b}$, as in (11). □

Lemma A.9. *Suppose $b < 1$ and let (σ, μ) be an equilibrium that satisfies D1. If $\sigma_E(\{v\}) > 0$, then $v = 0$ and $\operatorname{supp} \sigma_C = \{0\}$.*

Proof. Suppose the first claim does not hold, so there is an equilibrium that satisfies D1 where $\sigma_E(\{v\}) > 0$ for some $v > 0$. It follows from Lemma A.4 that $v = \hat{a}$, and Bayes' rule gives $\mu(\hat{a}) > 0$. For any $v \in [0, \hat{a})$, we have $v \notin \operatorname{supp} \sigma_E$ by Lemma 4. Then for all such v we have $\mu(v) = 0$ by

Bayes' law if $v \in \operatorname{supp} \sigma_C$, and $\mu(v) = 0$ by Lemma 5 otherwise. Then for all $\epsilon \in (0, \hat{a}] \cap (0, \mu(\hat{a})x)$,

$$\begin{aligned} Eu_m(\hat{a} - \epsilon) &= \hat{a} - \epsilon + \mu(\hat{a} - \epsilon)x \\ &= \hat{a} - \epsilon \\ &> \hat{a} - \mu(\hat{a})x = Eu_m(\hat{a}), \end{aligned}$$

and thus $\lambda(\hat{a} - \epsilon) \geq \lambda(\hat{a})$. Since $U_E = Eu_E(\hat{a})$ by Lemma A.1, this contradicts Lemma 2. Therefore, if $\sigma_E(\{v\}) > 0$, then $v = 0$. It is then immediate from Lemma 4 that $\operatorname{supp} \sigma_C = \{0\}$. □

Lemma A.10. *Suppose $b < 1$ and let (σ, μ) be an equilibrium that satisfies D1. If $\sigma_E(\{0\}) \in (0, 1)$, then*

$$F_E(v) = \begin{cases} 0 & v < 0, \\ \pi & 0 \leq v \leq \frac{1}{2b}(p_C + \pi p_E), \\ \frac{\pi}{2} + \frac{1}{p_E}(bv - \frac{p_C}{2}) & \frac{1}{2b}(p_C + \pi p_E) < v < \frac{1}{b} - \frac{1}{2b}(p_C + \pi p_E), \\ 1 & v \geq \frac{1}{b} - \frac{1}{2b}(p_C + \pi p_E), \end{cases} \tag{12}$$

where

$$\pi = \frac{\sqrt{2bxp_C} - p_C}{p_E}. \tag{13}$$

Proof. Suppose $\sigma_E(\{0\}) \in (0, 1)$ and let π denote its value. Note that $\operatorname{supp} \sigma_C = \{0\}$ by Lemma A.9. Bayes' law gives $\mu(0) = \pi p_E/(p_C + \pi p_E) < 1$, so $Eu_m(0) = -\mu(0)x > -x$. In addition, by Lemma A.1,

$$U_E = \frac{1}{2}(p_C + \pi p_E). \tag{14}$$

Since $\pi < 1$, it follows from Lemmas A.4 and A.5 that $\operatorname{supp} \sigma_E = \{0\} \cup [\tilde{v}_E, \bar{v}_E]$ with $\bar{v}_E > \tilde{v}_E$. Notice that $\tilde{v}_E > 0$, as otherwise we have

$$Eu_m(v) = v - x < -\mu(0)x = Eu_m(0)$$

for all $v \in (0, (1 - \mu(0))x) \cap (\tilde{v}_E, \underline{v}_E]$, contradicting Lemma 2. Therefore, $\tilde{v}_E > 0$ and Bayes' law gives $\mu(v) = 1$ for all $v \in [\tilde{v}_E, \bar{v}_E]$.

The next task is to derive the value of $\tilde{v}_E$. Because $\mu(v) = 1$ for all $v \in [\tilde{v}_E, \bar{v}_E]$, $Eu_m(\cdot)$ is continuous and strictly increasing on this interval. Therefore, we must have $Eu_m(\tilde{v}_E) \geq Eu_m(0)$; otherwise, continuity implies that there exists $\epsilon > 0$ such that $Eu_m(v) < Eu_m(0)$ for all $v \in [\tilde{v}_E, \tilde{v}_E + \epsilon]$, contradicting Lemma 2. This gives $Eu_m(v) > Eu_m(0)$ for all $v \in (\tilde{v}_E, \bar{v}_E]$. Since σ_E has no mass points other than 0, this in turn implies that $\lambda(\cdot)$ is continuous on $(\tilde{v}_E, \bar{v}_E]$. By Lemma A.1,

$$U_E = \lim_{v \to \tilde{v}_E^+} \lambda(v) - bv = p_C + \pi p_E - b\tilde{v}_E.$$

Combined with (14), this gives $\tilde{v}_E = \frac{1}{2b}(p_C + \pi p_E)$.

We can now derive the form of F_E. Notice that $\lambda(v) = p_C + p_E F_E(v)$ for all $v \in (\tilde{v}_E, \bar{v}_E]$. By Lemma A.1 and (14), we thus have

$$U_E = p_C + p_E F_E(v) - bv = \frac{1}{2}(p_C + \pi p_E)$$

for all $v \in (\tilde{v}_E, \bar{v}_E]$, which gives

$$F_E(v) = \frac{\pi}{2} + \frac{1}{p_E}\left(bv - \frac{p_C}{2}\right).$$

Finally, note that $F_E(\bar{v}_E) = 1$ by construction, so this expression also implies $\bar{v}_E = \frac{1}{b} - \frac{1}{2b}(p_C + \pi p_E)$. These findings yield the expression of F_E given in (12).

The final task is to derive the expression for π. Suppose $Eu_m(\tilde{v}_E) > Eu_m(0)$, which, by (6), implies $\lambda(\cdot)$ is continuous on $[\tilde{v}_E, \bar{v}_E]$ and thus $U_E = Eu_E(\tilde{v}_E)$. Notice that $\mu(v) = 1$ for all $v \in (0, \tilde{v}_E)$ by Lemma 5, so $Eu_m(\cdot)$ is continuous on $(0, \bar{v}_E]$. By continuity, there exists $v' < \tilde{v}_E$ such that $Eu_m(v') > Eu_m(0)$ and thus $\lambda(v') = \lambda(\tilde{v}_E) = p_C + \pi p_E$, contradicting Lemma 2. Therefore, $Eu_m(\tilde{v}_E) = Eu_m(0)$, which implies

$$\tilde{v}_E = \frac{xp_C}{p_C + \pi p_E}.$$

Combining this with the other expression for $\tilde{v}_E$ derived above, we yield

$$\frac{xp_C}{p_C + \pi p_E} = \frac{p_C + \pi p_E}{2b}.$$

Rearranging terms gives (13). □

Lemma A.11. *Suppose $b < 1$ and $x > \frac{p_C}{2b}$. If (σ, μ) is an equilibrium that satisfies D1, then* $\operatorname{supp}\sigma_C = \{0\}$ *and $\sigma_E(\{0\}) > 0$.*

Proof. Let $\underline{v}_t = \min\operatorname{supp}\sigma_t$ and $\bar{v}_t = \max\operatorname{supp}\sigma_t$ for each $t \in T$. I claim that $\sigma_E(\{\underline{v}_E\}) > 0$. Suppose this does not hold, so $\sigma_E(\{\underline{v}_E\}) = 0$.

The first task is to show that $\underline{v}_E > \bar{v}_C$. Suppose not, so $\underline{v}_E = \bar{v}_C$. Note that $\operatorname{supp}\sigma_E = [\underline{v}_E, \bar{v}_E]$ with $\bar{v}_E > \underline{v}_E$ by Lemma A.5, and $\mu(v) = 1$ for all $v \in (\underline{v}_E, \bar{v}_E]$ by Bayes' rule and Lemma 4. If $\bar{v}_C = 0$, then we must have $\sigma_C(\{0\}) > 0$. Bayes' rule then gives $\mu(0) = 0$. For all $v \in (\underline{v}_E, \bar{v}_E] \cap (0, x)$, we thus have

$$Eu_m(v) = v - x < 0 = Eu_m(0),$$

contradicting Lemma 2. Otherwise, in the case where $\bar{v}_C > 0$, we have $\mu(v) = 0$ for all $v \in [0, \bar{v}_C)$ by Lemmas 5 and 4. For all $\epsilon \in (0, \bar{v}_E - \underline{v}_E) \cap (0, \frac{x}{2}) \cap (0, \underline{v}_E)$, we thus have

$$Eu_m(\underline{v}_E + \epsilon) = \underline{v}_E + \epsilon - x < \underline{v}_E - \epsilon = Eu_m(\underline{v}_E - \epsilon),$$

again contradicting Lemma 2. Therefore, $\underline{v}_E > \bar{v}_C$, so Bayes' rule gives $\mu(\underline{v}_E) = 1$ and $\mu(\bar{v}_C) = 0$. We then have

$$Eu_m(\bar{v}_C) = \bar{v}_C = \underline{v}_E - x = Eu_m(\underline{v}_E)$$

by Lemma A.7, and thus $\underline{v}_E = \bar{v}_C + x$.

We can now derive E's utility and show that a contradiction results. Since $\sigma_E(\{\underline{v}_E\}) = 0$, it follows from Lemmas 5 and 4 that σ_E contains no mass points. In addition, by Bayes' rule and Lemma 5, $\mu(v) = 0$ for all $v \in [0, \bar{v}_C]$ and $\mu(v) = 1$ for all $v \in [\underline{v}_E, \bar{v}_E]$, so $Eu_m(\cdot)$ is strictly increasing on each of these intervals. Since $Eu_m(\underline{v}_E) = Eu_m(\bar{v}_C)$ and σ_E contains no mass points, it follows that

$$\lambda(v) = p_C + p_E F_E(v)$$

for each $v \in (\underline{v}_E, \bar{v}_E]$. Clearly, $\lambda(\cdot)$ is continuous on this interval, so $U_E = Eu_E(v)$ for each $v \in (\underline{v}_E, \bar{v}_E]$. Moreover, by (6), we have $\lambda(\bar{v}_C) \geq \frac{p_C}{2}$. Combining these gives

$$\begin{aligned}
U_E &= \lim_{v \to \underline{v}_E^+} \lambda(v) - bv \\
&= p_C - b\underline{v}_E \\
&\leq \lambda(\bar{v}_C) + \frac{p_C}{2} - b\underline{v}_E \\
&= \lambda(\bar{v}_C) + \frac{p_C}{2} - b\bar{v}_C - bx \\
&< \lambda(\bar{v}_C) + \frac{p_C}{2} - b\bar{v}_C - \frac{p_C}{2} \\
&= Eu_E(\bar{v}_C),
\end{aligned}$$

which contradicts the assumption of equilibrium. Therefore, $\sigma_E(\{\underline{v}_E\}) > 0$. It is then immediate from Lemma A.9 that $\underline{v}_E = 0$ and $\operatorname{supp} \sigma_C = \{0\}$. $\square$

Lemma A.12. *Suppose $b < 1$ and $x > \frac{p_C}{2b}$, and let (σ, μ) be an equilibrium that satisfies D1. Then $\operatorname{supp} \sigma_E = \{0\}$ if and only if $x \geq \frac{1}{2bp_C}$.*

Proof. Note that $\operatorname{supp} \sigma_C = \{0\}$ and $\sigma_E(\{0\}) = \pi > 0$ by Lemma A.11. Bayes' law gives

$$\mu(0) = \frac{\pi p_E}{p_C + \pi p_E},$$

and E's utility is

$$U_E = \lambda(0) = \frac{p_C + \pi p_E}{2}. \tag{15}$$

Suppose $x \geq \frac{1}{2bp_C}$ and $\pi < 1$. By Lemma A.10, $\operatorname{supp} \sigma_E = \{0\} \cup [\tilde{v}_E, \bar{v}_E]$, where $\tilde{v}_E = \frac{1}{2b}(p_C + \pi p_E)$ and $\bar{v}_E > \tilde{v}_E$. Bayes' rule implies $\mu(v) = 1$ for all $v \in [\tilde{v}_E, \bar{v}_E]$, so $Eu_m(\cdot)$ is continuous on this interval. Notice that

$$\begin{aligned}
Eu_m(\tilde{v}_E) - Eu_m(0) &= \left[\frac{p_C + \pi p_E}{2b} - x\right] - \left[-\frac{\pi p_E}{p_C + \pi p_E} x\right] \\
&= \frac{p_C + \pi p_E}{2b} - \frac{p_C}{p_C + \pi p_E} x \\
&\leq \frac{p_C + \pi p_E}{2b} - \frac{p_C}{p_C + \pi p_E} \frac{1}{2bp_C} \\
&= \frac{1}{2b}\left[(p_C + \pi p_E) - \frac{1}{p_C + \pi p_E}\right].
\end{aligned}$$

Since $\pi < 1$, we have $p_C + \pi p_E < \frac{1}{p_C + \pi p_E}$ and thus $Eu_m(\tilde{v}_E) < Eu_m(0)$. Continuity of $Eu_m(\cdot)$ then implies that there exists $\epsilon > 0$ such that $Eu_m(v) < Eu_m(0)$ for all $v \in [\tilde{v}_E, \tilde{v}_E + \epsilon]$, which contradicts Lemma 2.

Now suppose $x < \frac{1}{2bp_C}$ and $\pi = 1$, so $U_E = \frac{1}{2}$ and $Eu_m(0) = -p_E x$. Notice that $x < \frac{1}{2bp_C}$ implies $p_C x < \frac{1}{2b}$. Take any $v \in (p_C x, \frac{1}{2b})$. We have

$$\begin{aligned}
Eu_m(v) &= v - \mu(v)x \\
&\geq v - x \\
&> -(1 - p_C)x = Eu_m(0),
\end{aligned}$$

so $\lambda(v) = 1$. This gives

$$Eu_E(v) = 1 - bv > 1 - b\frac{1}{2b} = \frac{1}{2} = U_E,$$

which contradicts the assumption of equilibrium. $\square$

Proposition 3. *Suppose $b > 1$. There exists an equilibrium (σ^*, μ^*) that satisfies D1 in which*

$$F_E^*(v) = \begin{cases} 0 & v < 0, \\ \frac{bv}{p_E} & 0 \leq v \leq \bar{v}_E^*, \\ 1 & v > \bar{v}_E^*, \end{cases}$$

and

$$F_C^*(v) = \begin{cases} 0 & v < \bar{v}_E^*, \\ \frac{1}{p_C}(v - \frac{p_E}{b}) & \bar{v}_E^* \leq v \leq \bar{v}_C^* \\ 1 & v > \bar{v}_C^*, \end{cases}$$

where $\bar{v}_E^* = \frac{p_E}{b}$ and $\bar{v}_C^* = \frac{p_E}{b} + p_C$. Beliefs are $\mu^*(v) = 1$ for all $v < \bar{v}_E^*$, $\mu^*(\bar{v}_E^*) = p_E$, and $\mu^*(v) = 1$ for all $v > \bar{v}_E^*$. In any other equilibrium (σ, μ) that satisfies D1, $\sigma = \sigma^*$.

Proof. The first task is to confirm that (σ^*, μ^*) is an equilibrium, which requires confirming that there are no profitable deviations available and that the given beliefs are consistent with the candidates' strategies. Since $\mu^*(\cdot)$ is constant on $[0, \frac{p_E}{b})$ and on $(\frac{p_E}{b}, \infty)$, with $0 < \mu^*(\frac{p_E}{b}) < 1$, we have that $Eu_m(\cdot)$ is everywhere strictly increasing in v. Since neither type's strategy contains any mass points, it then follows from (6) that $\lambda(v) = p_E F_E^*(v) + p_C F_C^*(v)$ for all v.

I begin by confirming there is no profitable deviation for candidates of type E. For all $v \in [0, \frac{p_E}{b}]$,

$$Eu_E(v) = p_E F_E^*(v) - bv = 0,$$

confirming the indifference condition for E. Next, for all $v \in (\frac{p_E}{b}, p_C + \frac{p_E}{b}]$,

$$\begin{aligned} Eu_E(v) &= p_E + p_C F_C^*(v) - bv \\ &= p_E + v - \frac{p_E}{b} - bv \\ &= (1 - b)\left(v - \frac{p_E}{b}\right) < 0 = U_E, \end{aligned}$$

so a deviation to such v would not be profitable. Then, since $\lambda(v) = \lambda(p_C + \frac{p_E}{b}) = 1$ for all $v > p_C + \frac{p_E}{b}$, we have $Eu_E(v) < Eu_E(p_C + \frac{p_E}{b}) \leq U_E$ for all such v as well.

I next confirm that there is no profitable deviation available for C. For all $v \in [\frac{p_E}{b}, p_C + \frac{p_E}{b}]$,

$$Eu_C(v) = p_E + p_C F_C^*(v) - v = p_E - \frac{p_E}{b},$$

confirming the indifference condition for C. Since $\lambda(v) = \lambda(p_C + \frac{p_E}{b}) = 1$ for all $v > p_C + \frac{p_E}{b}$, we have $Eu_C(v) < Eu_C(p_C + \frac{p_E}{b}) = U_C$ and hence it would not be profitable to deviate to such v. Next, for all $v \in [0, \frac{p_E}{b})$,

$$\begin{aligned} Eu_C(v) &= p_E F_E^*(v) - v \\ &= (b - 1)v \\ &< p_E - \frac{p_E}{b} = U_C, \end{aligned}$$

so a deviation to such v would not be profitable.

The last step to verify that (σ^*, μ^*) is an equilibrium is to confirm consistency of beliefs. Bayes' rule gives $\mu^*(v) = 1$ for all $v \in [0, \frac{p_E}{b})$, $\mu^*(\frac{p_E}{b}) = p_E$, and $\mu^*(v) = 0$ for all $v \in (\frac{p_E}{b}, p_C + \frac{p_E}{b}]$. Therefore, the on-the-path beliefs are consistent. In addition, we have

$$\hat{a} = \frac{U_E - U_C}{1-b} = \frac{-(p_E - \frac{p_E}{b})}{1-b} = \frac{p_E}{b},$$

so, by Lemma 5, the off-the-path beliefs satisfy D1.

The claim of uniqueness of the equilibrium strategies is immediate from Lemmas A.6 and A.8. □

Proposition 4. *Suppose $b < 1$ and $x \geq \frac{1}{2bp_C}$. There exists an equilibrium (σ^*, μ^*) that satisfies D1 in which $\operatorname{supp} \sigma_C^* = \operatorname{supp} \sigma_E^* = \{0\}$, $\mu^*(0) = p_E$, and $\mu^*(v) = 1$ for all $v > 0$. In any other equilibrium (σ, μ) that satisfies D1, $\sigma = \sigma^*$.*

Proof. As in the previous proposition, the first task is to confirm that (σ^*, μ^*) is an equilibrium. To rule out profitable deviations, begin by noting that $U_C = U_E = \lambda(0) = \frac{1}{2}$. We have

$$Eu_m(0) = -\mu^*(0)x = -p_E x.$$

Therefore, for any $v \in (0, p_C x]$,

$$Eu_m(v) = v - x \leq -p_E x = Eu_m(0).$$

This gives $Eu_t(v) \leq \lambda(0) - b_t v < \lambda(0) = U_t$ for each type $t \in T$, so a deviation to $v \in (0, p_C x]$ would not be profitable. Conversely, for any $v > p_C x$, $Eu_m(v) > Eu_m(0)$ and thus $\lambda(v) = 1$. This gives

$$\begin{aligned} Eu_t(v) &= 1 - b_t v \\ &< 1 - bp_C x \\ &\leq 1 - bp_C \frac{1}{2bp_C} \\ &= \frac{1}{2} = U_t, \end{aligned}$$

so such a deviation is not profitable. To confirm that the given beliefs are consistent, observe that Bayes' rule gives $\mu^*(0) = p_E$. In addition, we have $\hat{a} = (U_E - U_C)/(1-b) = 0$, so the off-the-path beliefs satisfy D1 by Lemma 5.

Finally, notice that $x \geq \frac{1}{2bp_C}$ implies $x > \frac{p_C}{2b}$. Uniqueness of the equilibrium strategies is then immediate from Lemmas A.11 and A.12. □

Proposition 5. *Suppose $b < 1$ and $\frac{p_C}{2b} < x < \frac{1}{2bp_C}$. There exists an equilibrium (σ^*, μ^*) that satisfies D1 in which $\operatorname{supp} \sigma_C^* = \{0\}$ and*

$$F_E^*(v) = \begin{cases} 0 & v < 0, \\ \pi & 0 \leq v \leq \tilde{v}_E^*, \\ \frac{\pi}{2} + \frac{1}{p_E}(bv - \frac{p_C}{2}) & \tilde{v}_E^* < v < \bar{v}_E^*, \\ 1 & v \geq \bar{v}_E^*, \end{cases}$$

where $\pi = \frac{\sqrt{2bxp_C} - p_C}{p_E}$, $\tilde{v}_E^* = \frac{1}{2b}(p_C + \pi p_E)$, *and* $\bar{v}_E^* = \frac{1}{b} - \tilde{v}_E^*$. *Beliefs are* $\mu(0) = \frac{\pi p_E}{p_C + \pi p_E}$ *and* $\mu(v) = 1$ *for all* $v > 0$. *In any other equilibrium* (σ, μ) *that satisfies D1,* $\sigma = \sigma^*$.

Proof. As in the previous propositions, the first task is to confirm that (σ^*, μ^*) is an equilibrium. To rule out profitable deviations, I begin by deriving the form of $\lambda(\cdot)$. Note that $\lambda(0) = \frac{1}{2}(p_C + \pi p_E)$ and thus $U_C = U_E = \frac{1}{2}(p_C + \pi p_E)$. We have

$$Eu_m(0) = -\mu^*(0)x = -\frac{\pi p_E}{p_C + \pi p_E}x.$$

Since $\mu^*(v) = 1$ for all $v > 0$, $Eu_m(\cdot)$ is continuous and strictly increasing on $(0, \infty)$. Notice that

$$\begin{aligned}
Eu_m(\tilde{v}_E^*) - Eu_m(0) &= \left[\frac{p_C + \pi p_E}{2b} - x\right] - \left[-\frac{\pi p_E}{p_C + \pi p_E}x\right] \\
&= \frac{p_C + \pi p_E}{2b} - \frac{xp_C}{p_C + \pi p_E} \\
&= \frac{\sqrt{2bxp_C}}{2b} - \frac{xp_C}{\sqrt{2bxp_C}} \\
&= 0.
\end{aligned}$$

This implies $Eu_m(v) \le Eu_m(0)$ and thus $Eu_t(v) \le \lambda(0) - b_t v < U_t$ for all $v \in (0, \tilde{v}_E^*]$ and $t \in T$.[10] Moreover, we then have $\lambda(v) = p_C + p_E F_E^*(v)$ for all $v > \tilde{v}_E^*$. For all $v \in (\tilde{v}_E^*, \bar{v}_E^*]$, this gives

$$Eu_E(v) = p_C + p_E\left[\frac{\pi}{2} + \frac{bv - \frac{p_C}{2}}{p_E}\right] = \frac{p_C + \pi p_E}{2} = U_E,$$

confirming E's indifference condition, and

$$Eu_C(v) = \lambda(v) - v < \lambda(v) - bv = U_E = U_C,$$

confirming that C has no incentive to deviate. For all $v > \bar{v}_E^*$, we have $Eu_t(v) = 1 - b_t v < 1 - b_t\bar{v}_E^* = Eu_t(\bar{v}_E) \le U_t$ for each $t \in T$, so neither type has an incentive to deviate. To confirm that the given beliefs are consistent, observe that Bayes' rule gives $\mu^*(0) = \frac{\pi p_E}{p_C + \pi p_E}$ and $\mu^*(v) = 1$ for all $v \in [\tilde{v}_E^*, \bar{v}_E^*]$. In addition, we have $\hat{a} = (U_E - U_C)/(1 - b) = 0$, so the off-the-path beliefs satisfy D1 by Lemma 5.

Finally, uniqueness of the equilibrium strategies is immediate from Lemmas A.10, A.11, and A.12. $\square$

Proposition 6. *Suppose* $b < 1$ *and* $x \le \frac{p_C}{2b}$. *There exists an equilibrium* (σ^*, μ^*) *that satisfies D1 in which*

$$F_C^*(v) = \begin{cases} 0 & v < 0, \\ \frac{v}{p_C} & 0 \le v \le \tilde{v}_C^*, \\ 1 - \frac{2bx}{p_C} & \tilde{v}_C^* < v < \bar{v}_C^*, \\ 1 & v \ge \bar{v}_C^*, \end{cases}$$

[10] It does not violate the indifference condition for E that $\tilde{v}_E^* \in \operatorname{supp}\sigma_E^*$ and $Eu_t(\tilde{v}_E^*) < U_t$, because $\sigma_E^*(\{\tilde{v}_E^*\}) = 0$.

and

$$F_E^*(v) = \begin{cases} 0 & v < \underline{v}_E^*, \\ \frac{b}{p_E}(v - p_C - (1-b)x) & \underline{v}_E^* \leq v \leq \bar{v}_E^*, \\ 1 & v > \bar{v}_E^*, \end{cases}$$

where $\tilde{v}_C^* = p_C - 2bx$, $\bar{v}_C^* = p_C - bx$, $\underline{v}_E^* = p_C + (1-b)x$, *and* $\bar{v}_E^* = \frac{1}{b}(1 - (1-b)(p_C - bx))$. *Beliefs are* $\mu(v) = 0$ *for all* $v \leq \bar{v}_C^*$ *and* $\mu(v) = 1$ *for all* $v > \bar{v}_C^*$. *If* $x < \frac{p_C}{2b}$, *then in any other equilibrium* (σ, μ) *that satisfies D1,* $\sigma = \sigma^*$.

Proof. As in the other propositions, the first task is to show that (σ^*, μ^*) is an equilibrium. To rule out profitable deviations, I begin by deriving the form of $\lambda(\cdot)$. Note that $\bar{v}_E^* > \underline{v}_E^* > \bar{v}_C^* > \tilde{v}_C^*$, and it follows from $x \leq \frac{p_C}{2b}$ that $\tilde{v}_C^* \geq 0$. Since $\mu^*(\cdot)$ is constant on $[0, \bar{v}_C^*]$ and on $(\bar{v}_C^*, \infty)$, $Eu_m(\cdot)$ is strictly increasing on each of these intervals. Notice that

$$\begin{aligned} Eu_m(\bar{v}_C^*) - Eu_m(\underline{v}_E^*) &= \bar{v}_C^* - (\underline{v}_E^* - x) \\ &= p_C - bx - (p_C + (1-b)x) + x \\ &= 0, \end{aligned}$$

so $Eu_m(\bar{v}_C^*) = Eu_m(\underline{v}_E^*)$. Using (6), we have

$$\lambda(v) = \begin{cases} p_C F_C^*(v) & v < \bar{v}_C^*, \\ p_C(1 - \frac{bx}{p_C}) & v = \bar{v}_C^*, \\ p_C F_C^*(v - x) & \bar{v}_C^* < v < \underline{v}_E^*, \\ p_C(1 - \frac{bx}{p_C}) & v = \underline{v}_E^*, \\ p_C + p_E F_E^*(v) & v > \underline{v}_E^*. \end{cases}$$

To check for profitable deviations for C, note that $\sigma_C^*(\{\bar{v}_C\}) > 0$ and thus

$$U_C = \lambda(\bar{v}_C^*) - \bar{v}_C^* = p_C - bx - (p_C - bx) = 0$$

by Lemma A.1. To confirm C's indifference condition, notice that $Eu_C(v) = p_C F_C^*(v) - v = 0 = U_C$ for all $v \in [0, \tilde{v}_C^*]$. For all $v \in (\tilde{v}_C^*, \bar{v}_C^*)$, we have $\lambda(v) = \lambda(\tilde{v}_C^*)$ and thus $Eu_C(v) < Eu_C(\tilde{v}_C^*) = U_C$, so C would have no incentive to deviate to any such v. Similarly, $\lambda(v) \leq \lambda(\bar{v}_C^*)$ for all $v \in (\bar{v}_C^*, \underline{v}_E^*]$, so we have $Eu_C(v) < Eu_C(\bar{v}_C^*) = U_C$ for any such v. Next, for all $v \in (\underline{v}_E^*, \bar{v}_E^*]$,

$$\begin{aligned} Eu_C(v) &= p_C + b(v - p_C - (1-b)x) - v \\ &= (1-b)(p_C - bx - v) \\ &< 0, \end{aligned}$$

so such a deviation would not be profitable for C. Finally, note that $\lambda(v) = \lambda(\bar{v}_E^*) = 1$ for any $v > \bar{v}_E^*$, and thus $Eu_C(v) < Eu_C(\bar{v}_E^*) < 0 = U_C$ for any such v.

To check for profitable deviations for E, note that $\lambda(\cdot)$ is continuous on $(\underline{v}_E^*, \bar{v}_E^*]$ and thus

$$U_E = \lambda(\bar{v}_E^*) - b\bar{v}_E^* = (1-b)(p_C - bx)$$

by Lemma A.1. To confirm E's indifference condition, notice that

$$\begin{aligned} Eu_E(v) &= p_C + p_E F_E^*(v) - bv \\ &= p_C + b(v - p_C - (1-b)x) - bv \\ &= (1-b)(p_C - bx) = U_E \end{aligned}$$

for all $v \in (\underline{v}_E^*, \bar{v}_E^*]$. For all $v \in [0, \tilde{v}_C^*]$, we have $Eu_E(v) = (1-b)v < (1-b)(p_C - bx) = U_E$, so a deviation to such v would not be profitable. Since $\lambda(v) = \lambda(\tilde{v}_C^*)$ for all $v \in (\tilde{v}_C^*, \bar{v}_C^*)$, we have $Eu_E(v) < Eu_E(\tilde{v}_C^*) < U_E$ for any such v. At the mass point in C's mixed strategy, we have $Eu_E(\bar{v}_C^*) = (1-b)(p_C - bx) = U_E$, so there is no strict incentive for E to deviate to $\bar{v}_C^*$. Next, since $\lambda(v) \leq \lambda(\bar{v}_C^*)$ for all $v \in (\bar{v}_C^*, \underline{v}_E^*]$, we have $Eu_E(v) < Eu_E(\bar{v}_C^*) = U_E$ and thus a deviation to such v is not profitable.[11] Finally, because $\lambda(v) = \lambda(\bar{v}_E^*) = 1$ for all $v > \bar{v}_E^*$, we have $Eu_E(v) < Eu_E(\bar{v}_E^*) = U_E$, so a deviation to such v is not profitable.

The last step to show that the given assessment is an equilibrium is to confirm the consistency of beliefs. Since $\underline{v}_E^* > \bar{v}_C^*$, Bayes' rule gives $\mu^*(v) = 0$ for all $v \in [0, \tilde{v}_C^*] \cup \{\bar{v}_C^*\}$ and $\mu^*(v) = 1$ for all $v \in [\underline{v}_E^*, \bar{v}_E^*]$. Therefore, the on-the-path beliefs are consistent. In addition, we have

$$\hat{a} = \frac{U_E - U_C}{1-b} = p_C - bx = \bar{v}_C^*,$$

so, by Lemma 5, the off-the-path beliefs satisfy D1.

To show uniqueness, suppose $x < \frac{p_C}{2b}$ and let (σ, μ) be an equilibrium. First, suppose there exists v such that $\operatorname{supp} \sigma_C = \{v\}$, $\sigma_C(\{v\}) = 1$ and $\lambda(v) \geq \frac{p_C}{2}$. By Lemma A.1, $U_C = Eu_C(v) = \lambda(v) - v$. In addition, Lemma 4 implies $v = \min(\operatorname{supp} \sigma_C \cup \operatorname{supp} \sigma_E)$. Therefore, using (6) and Lemma 2, we have $\lambda(v') \geq 2\lambda(v)$ for any v' such that $Eu_m(v') > Eu_m(v)$. We then have $U_E \geq 2\lambda(v) - b(v+x)$. Otherwise, if $U_E < 2\lambda(v) - b(v+x)$, then for any $v' \in (v + x, \frac{1}{b}(2\lambda(v) - U_E))$ we have

$$Eu_m(v') = v' - \mu(v')x \geq v' - x > v \geq Eu_m(v)$$

and thus $Eu_m(v') \geq 2\lambda(v) - bv' > U_E$, contradicting the assumption of equilibrium. However, Lemma A.4 gives $v = \hat{a}$ and thus

$$\begin{aligned} v &= \frac{U_C - U_E}{1-b} \\ &\geq \frac{(2\lambda(v) - b(v+x)) - (\lambda(v) - v)}{1-b} \\ &= v + \frac{\lambda(v) - bx}{1-b} \\ &\geq v + \frac{\frac{p_C}{2} - bx}{1-b} \\ &= v + \frac{\frac{p_C}{2b} - x}{\frac{1}{b} - 1} \\ &> v, \end{aligned}$$

a contradiction. Therefore, there does not exist v such that $\operatorname{supp} \sigma_C = \{v\}$. It then follows from Lemma A.9 that σ_E contains no mass points, so $\operatorname{supp} \sigma_E = [\underline{v}_E, \bar{v}_E]$ with $\bar{v}_E > \underline{v}_E$.

Next, suppose σ_C contains no mass points, so $\operatorname{supp} \sigma_C = [\underline{v}_C, \bar{v}_C]$. Notice that $\underline{v}_E > \bar{v}_C$ (see the

[11] It does not violate the indifference condition for E that $\underline{v}_E^* \in \operatorname{supp} \sigma_E^*$ and $Eu_E(\underline{v}_E^*) < U_E$, because $\sigma_E^*(\{\underline{v}_E^*\}) = 0$.

proof of Lemma A.11), so $\mu(v) = 1$ for all $v \in [\underline{v}_E, \bar{v}_E]$ and thus $Eu_m(\cdot)$ is strictly increasing on this interval. Since neither type's strategy contains any mass points, this implies $\lambda(\cdot)$ is continuous on $[\underline{v}_E, \bar{v}_E]$ and hence, by Lemma A.1, $U_E = Eu_E(\underline{v}_E)$. However, notice that $Eu_m(\underline{v}_E) = Eu_m(\bar{v}_C)$ by Lemma A.7, contradicting Lemma 2. Therefore, σ_C must contain a mass point. Using Lemmas 4, A.4, and A.5, this gives $\operatorname{supp} \sigma_C = [\underline{v}_C, \tilde{v}_C] \cup \{\bar{v}_C\}$ with $\tilde{v}_C > \underline{v}_C$ and $\sigma_C(\{\bar{v}_C\}) = \pi > 0$.

We can now characterize the CDF corresponding to σ_C. Notice that $\mu(v) = 0$ for all $v \in [\underline{v}_C, \tilde{v}_C] \cup \{\bar{v}_C\}$, so $Eu_m(\cdot)$ is strictly increasing on this set. Since $\bar{v}_C$ is the only mass point in either type's strategy, this implies $\lambda(\cdot)$ is continuous on $[\underline{v}_C, \tilde{v}_C)$ and thus $U_C = Eu_C(\underline{v}_C)$. In addition, Lemma 2 and (6) give $\lambda(\underline{v}_C) = 0$. Then we must have $\underline{v}_C = 0$, as otherwise

$$U_C = -\underline{v}_C < 0 \leq Eu_C(0),$$

contradicting the assumption of equilibrium. Since $\underline{v}_C = 0$, we have $U_C = \lambda(0) = 0$. Next, notice that $\mu(v) = 1$ for all $v \in [\underline{v}_E, \bar{v}_E]$, so $Eu_m(\cdot)$ is strictly increasing on this interval. We have $Eu_m(\underline{v}_E) = Eu_m(\bar{v}_C)$ by Lemma A.7; since $\bar{v}_C$ is the only mass point in either type's strategy, this implies $\lambda(\cdot)$ is continuous on $(\underline{v}_E, \bar{v}_E]$. Moreover, since $\mu(\underline{v}_E) = 1$ and $\mu(\bar{v}_C) = 0$, it follows from $Eu_m(\underline{v}_E) = Eu_m(\bar{v}_C)$ that $\underline{v}_E = \bar{v}_C + x$. Therefore, by Lemma A.1,

$$U_E = \lim_{v \to \underline{v}_E^+} \lambda(v) - bv = p_C - b(\bar{v}_C + x). \tag{16}$$

Lemma A.4 then gives

$$\bar{v}_C = \frac{U_E - U_C}{1-b} = \frac{p_C - b\bar{v}_C - bx}{1-b}$$

and thus $\bar{v}_C = p_C - bx$. Since $Eu_C(\bar{v}_C) = U_C = 0$ and $\lambda(\bar{v}_C) = p_C(1 - \frac{\pi}{2})$ by (6), we have

$$Eu_C(\bar{v}_C) = p_C\left(1 - \frac{\pi}{2}\right) - (p_C - bx) = 0,$$

and thus $\pi = \frac{2bx}{p_C}$. Since $x < \frac{p_C}{2b}$, we have $\pi \in (0, 1)$ as required.

To find $\tilde{v}_C$, notice that we must have $\tilde{v}_C < \bar{v}_C$; otherwise, if $\tilde{v}_C = \bar{v}_C$, then for all $v \in (\tilde{v}_C - \frac{\pi p_C}{2}, \tilde{v}_C) \cap [0, \tilde{v}_C)$,

$$\begin{aligned}
Eu_C(v) &= \lambda(v) - v \\
&\leq \lambda(\bar{v}_C) - \frac{\pi p_C}{2} - v \\
&< \lambda(\bar{v}_C) - \tilde{v}_C \\
&= \lambda(\bar{v}_C) - \bar{v}_C = U_C,
\end{aligned}$$

contradicting the assumption of equilibrium. Since $\tilde{v}_C < \bar{v}_C$ and $U_C = Eu_C(\bar{v}_C)$, Lemma 2 gives $Eu_m(\tilde{v}_C) < Eu_m(\bar{v}_C)$. Then, by (6), $\lambda(v) = p_C F_C(v)$ for all $v \in [0, \tilde{v}_C]$; because $\bar{v}_C$ is the only mass point in either type's strategy, $\lambda(\cdot)$ is continuous on this interval. Lemma A.1 then gives $Eu_C(v) = U_C = 0$ for all $v \in [0, \tilde{v}_C]$. This gives $p_C F_C(v) - v = 0$, and thus $F_C(v) = \frac{v}{p_C}$, for all such v. Since $F_C(\tilde{v}_C) = 1 - \pi$ by construction, this implies $\tilde{v}_C = p_C(1 - \pi) = p_C - 2bx$. We have thus shown that F_C must be the same as in the proposition.

The final task in the uniqueness proof is to characterize the CDF corresponding to σ_E. It has already been shown that $\underline{v}_E = \bar{v}_C + x = p_C + (1-b)x$. Then, by (16),

$$U_E = p_C - b\underline{v}_E = (1-b)(p_C - bx).$$

In addition, because $Eu_m(\underline{v}_E) = Eu_m(\bar{v}_C)$, we have $\lambda(v) = p_C + p_E F_E(v)$ for all $v \in (\underline{v}_E, \bar{v}_E]$. Since $\bar{v}_C$ is the only mass point in either type's strategy, $\lambda(\cdot)$ is continuous on this interval. Lemma A.1 then gives

$$Eu_E(v) = p_C + p_E F_E(v) - bv = (1-b)(p_C - bx) = U_E,$$

and thus $F_E(v) = \frac{b}{p_E}(v - p_C - (1-b)x)$ for all $v \in (\underline{v}_E, \bar{v}_E]$. Since $F_E(\bar{v}_E) = 1$ by construction, this in turn gives $\bar{v}_E = \frac{1}{b}(1 - (1-b)(p_C - bx))$. We have thus shown that F_E must be the same as in the proposition. $\square$

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