Campaign Spending and Hidden Policy Intentions*

Brenton Kenkel†

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Abstract

Popular concerns about money in politics revolve around the idea that campaign spending is electorally effective but distorts policy outcomes, as well-funded candidates are disproportionately likely to favor special interests once elected. This account appears to assume an irrational electorate: why would voters reward high-spending candidates who go on to enact extreme policies? I develop a model of spending competition between candidates whose ability to fundraise is tied to the policies they will enact if elected, which in turn are private information. The electorate in the model updates its beliefs rationally, and yet I find that a fundraising advantage never results in an electoral disadvantage. Candidates that can raise money easily exploit this ability when it benefits them—but when it is disadvantageous, they refrain from spending so as to conceal their policy intentions, preventing the electorate from making an informed decision. Analyzing the effects of campaign finance reform, I find that reductions in the fundraising advantage never decrease the chance of electing the candidate whose policy intentions are closest to the median voter, even when such candidates are the ones who can raise money most easily.

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†Assistant Professor, Department of Political Science, Vanderbilt University. Email: `brenton.kenkel@vanderbilt.edu`.

1 Introduction

Campaign finance reform is a central concern of progressive activists in American politics. In April 2016, hundreds of activists were arrested in Washington, D.C., while protesting in connection with the Democracy Spring movement (Carpenter and Berman 2016). Their demands included "to end the corruption of big money in politics," which they described as an obstacle to "real democracy in the United States" (Democracy Spring 2016). Three months later, at the 2016 Democratic National Convention, the Democratic Party approved a platform that said "Big money is drowning out the voices of everyday Americans" and called for widespread reform of the campaign finance system (Democratic Platform Committee 2016, 25).

Two assumptions underlie these concerns about the role of money in politics. The first is that political spending is *effective*. If campaign spending had no effect on electoral outcomes, some voters might still be annoyed by political advertisements, but there would be no cause for concern about the health of democracy or the ability of citizens to influence policy. The second is that political spending *distorts policy*—that the candidates with the ability to spend a lot, whether through traditional campaign fundraising or external groups, enact policies that are not in the public interest. If we got money out of politics, the logic goes, then we would elect better politicians.

The problem with this line of reasoning is that the two assumptions seem to contradict each other. If well-funded candidates support policies that harm the public interest once elected, why is campaign spending effective? As Morton and Myerson (2012, 572) put it, if "the candidate who spends the most has also sold the most promises to special-interest contributors," then "[i]t is hard to imagine that voters would find candidates that provide such favors attractive." In a world where the only way to raise a lot of money is to sell out to special interests, high campaign spending should repel voters rather than attract them.

Perhaps the effectiveness and distortionary nature of campaign spending do not contradict each other, namely because voters are irrational. A rational Bayesian, knowing that high-spending candidates act contrary to the public interest once in office, would revise downward her assessment of a candidate who spent a lot—but voters are not rational Bayesians. Indeed, a growing empirical literature in political science reaches exactly this conclusion (e.g., Healy, Malhotra and Mo 2010; Achen and Bartels 2016), though there are reasons both theoretical (Ashworth, Bueno de Mesquita and Friedenberg 2016) and empirical (Fowler and Montagnes 2015; Fowler and Hall 2017) to doubt these findings. Regardless of whether voters really are rational, it should be clear that a world of rational Bayesian voters is the worst-case scenario for the hypothesis that unrestricted campaign spending distorts policy outcomes. If high campaign spending signals that the candidate will enact policy counter to the public interest, it ought to be least successful when voters properly interpret the signal.

In this paper, I use a formal model to investigate candidate and voter behavior when a candidate's ability to spend signals what kind of policy she will implement if elected and voters are rational Bayesians. In the model, candidates have private information about their ability to spend (i.e., how much effort they must exert per dollar raised) and the policy they will implement once elected. Spending has a direct positive effect on the electorate's assessment of a candidate, but it also sends a signal about the candidate's policy intentions. Voters understand this signal and make their vote choices accordingly. Perhaps surprisingly, I find that important components of the folk theory of money in politics hold up in this environment. Candidates with an advantage in fundraising (i.e., those who can raise money relatively easily) also have at least a weak electoral advantage, even if this type of candidate will enact policies far from the median voter's ideal point once elected. Consequently, campaign finance reforms that "equalize" different candidates' fundraising abilities increase the chance of electing candidates whose policy intentions are close to the median voter.

The argument relies on a simple signaling logic. A candidate who *could* raise a lot of money does not *have* to do so. If high spending would send such a poor signal to the median voter that it would sink the candidate's electoral chances, then the candidate will be better off not raising much money. When lavish spending is advantageous—either because it spends a positive signal about the candidate's policy intentions, or because voters place relatively little weight on policy—candidates that can raise money easily will do so, and they will tend to win the election. But in the opposite case, when the median voter would ideally like to elect the type of candidate who cannot raise money easily, there is no way for such candidates to distinguish themselves: refraining from spending is a signal that is easy to mimic. Then candidates with an (untapped) financial advantage will still have a 50-50 shot at the election, since their strategy leaves the electorate unable to distinguish between candidates. Therefore, a candidate with a fundraising advantage is never at an electoral disadvantage. This line of reasoning resembles that of Ashworth and Bueno de Mesquita (2014), who show that the electoral consequences of voter (ir)rationality are not straightforward when politicians strategically anticipate voter responses.

I investigate the effects of a hypothetical policy intervention to equalize candidates, reducing the magnitude of the fundraising advantage. When high spending signals policy intentions relatively far from the median voter, such a reform increases the chance of electing a candidate closer to the median voter. The more effort a candidate must exert to spend the same amount of money, the more likely she is to prefer to conceal her type and tie the election rather than spend enough to win the election. One might worry, though, that the same reform would decrease the chance of electing a candidate close to the median voter when this type of candidate has a financial advantage. Surprisingly, I find that there is no such offsetting effect. Equalizing reforms increase the amount that such a candidate must spend to separate herself, but not by so much that she is unwilling to do so. This asymmetry in effects is important, given that which type of candidate is advantaged may vary race by race, while campaign finance rules are set at broader levels.

The analysis also suggests how disclosure reform might work to promote candidates whose policy intentions are close to the ideological center. If revealing a candidate's funding sources gives the electorate information about what policies the candidate would enact once elected (e.g., because funds come from interest groups that are better informed than the public about a candidate's policy intentions), then disclosure has two effects. One is to inform the electorate when a high-spending candidate will enact policies counter to the public interest. But this already takes place endogenously in the model via the rational expectations of voters. When voters see high spending, they infer that the candidate has a fundraising advantage and whatever policy intentions go along with it; candidates only exploit this advantage if they can overcome (or benefit from) the electoral consequences of revealing their type. The second effect of disclosure reform—allowing candidates with policy intentions close to the median to signal that their money is "clean"—is more important. In the equilibrium of the model, when a fundraising advantage is associated with policy intentions very far from the center, candidates of all types refuse to spend for fear that the electorate will infer that they share these intentions. The electoral result would perhaps be different, even giving financially disadvantaged candidates an electoral advantage, if these candidates could spend without signaling incorrectly an association with special interests.

This paper contributes to the political economy literature on campaign spending, candidate valence, and voter beliefs. Much of the existing theoretical research suggests that advertisements funded by interest-group donations serve as a signal of underlying candidate quality. Prat (2002*a*,*b*) and Wittman (2007) each assume interest groups directly observe candidate quality prior to the election, but voters do not. Both find that it is worthwhile for high-quality candidates to make policy promises to special interests in exchange for advertising money (or endorsements). The high-valence candidate gains because the pressure group's willingness to donate to her signals her quality to the electorate; the pressure group gains because it can extract policy promises from a candidate in exchange for sending a signal that makes her more likely to win the election. Coate (2004) and Ashworth (2006) each use a similar framework, but in their models, pressure groups do not directly observe candidate valence. Instead, they assume that advertisements cannot transmit false information about underlying candidate quality, so only high-quality candidates would want to raise funds and run ads. In each of these models, as in Prat's and Wittman's, high-quality candidates have an incentive to give policy favors to interest groups—even though doing so reduces their standing among other voters—in order to be able to signal their quality via advertising. On balance, these analyses find that campaign finance reforms are welfare-improving for the electorate (Wittman 2007 is an exception), even though campaign advertising is informative and voters are rational.

Though I reach the same broad conclusions about the effects of reform, I consider a significantly different electoral environment. Drawing from the endogenous valence literature (Wiseman 2006; Meirowitz 2008; Ashworth and Bueno de Mesquita 2009; Serra 2010), I model candidate quality (or the electorate's perception of it) as the result of costly effort exertion rather than as a fixed attribute. The model here is particularly close to that of Meirowitz (2008), in which each candidate's only choice is how much to spend, making the election akin to an all-pay contest (Baye, Kovenock and de Vries 1996). I innovate on the existing literature on candidate effort selection by introducing a signaling component. Candidates have private information about the policies they intend to enact once elected, as in Banks (1990) and Callander and Wilkie (2007). I assume that a candidate's marginal cost of effort in the valence contest is related to these policy intentions. Whereas the literature on hidden policy intentions focuses on how the costliness of lying affects candidates' public announcements, I focus instead on signaling through the spending mechanism. I focus particularly on how the spending dynamics can change when high spending sends an undesirable signal to the median voter.

The remainder of the paper proceeds as follows. In the next section, I describe the model. I then solve for an equilibrium, which I show is unique under reasonable conditions on the electorate's beliefs. The subsequent section analyzes the effect of campaign finance reform on the equilibrium spending strategies and their electoral consequences. In the penultimate section, I consider two extensions to the model: one in which candidates may opt out of the spending contest and take public financing instead, and one in which candidate types are correlated. Finally, I offer concluding thoughts.

2 The Model

There is an election in which two candidates, labeled 1 and 2, are competing for support from a set N consisting of n voters (n odd).[1] The game consists of two stages. In the first stage, each candidate chooses how much to spend, $s_i \geq 0$. In the second stage, each voter selects a candidate, $v_j \in \{1, 2\}$.

Candidates build valence by spending, but it is costly to do so. A candidate's cost of spending depends on her type $t_i \in T = \{A, D\}$, where the type names stand for *A*dvantaged and *D*isadvantaged. It is easier for Advantaged candidates to spend than it is for Disadvantaged candidates: the marginal cost of spending is c_A for Advantaged candidates and $c_D > c_A$ for Disadvantaged candidates. A candidate's type also determines the policy she will implement if elected, denoted x_{t_i}, which lies in the policy space $X \subseteq \mathbb{R}$. Candidate types are drawn independently by Nature at the start of the game. It is common knowledge that the prior probability each candidate is Advantaged is $p_A \in (0, 1)$, with $p_D = 1 - p_A$ denoting the prior probability that a candidate is Disadvantaged. However, the realization of each candidate's type is private information, unknown to the other candidate and to the electorate. Therefore, a candidate cannot base her spending decisions on the other candidate's type, and voters must infer candidates' policy intentions from their spending decisions.

Each candidate's objective is to win office. The expected utility function for a candidate, informally expressed, is

$$Eu_i(s_i \mid t_i) = \Pr(i \text{ wins} \mid s_i) - c_{t_i} s_i. \tag{1}$$

Although candidates may have distinct policy intentions, their payoff is not a function of the winning candidate's policy choice—they are "office-motivated," not "policy-motivated" (Calvert 1985). In this sense, the model is closely related to existing theories of candidates who have privately known policy intentions but are office-motivated (Banks 1990; Callander and Wilkie 2007).

[1] Generic candidates are labeled i and take female pronouns; generic voters are labeled j and take male pronouns.

The game is symmetric from the perspective of the two candidates, who have identical type distributions, action spaces, and utility functions. To simplify the analysis, I restrict attention to symmetric strategy profiles, in which the spending strategy of type t of candidate 1 is the same as that of type t of candidate 2. A mixed strategy profile is hence a pair $\sigma = (\sigma_A, \sigma_D)$, where each σ_t is a probability measure with support on $\mathbb{R}_+$. For any mixed strategy σ_t, let F_t denote the corresponding cumulative distribution function, and let $\operatorname{supp} \sigma_t$ denote its support.

Voters' preferences depend on the candidates' valence and policy intentions. All voters value valence identically, but voters differ in their policy preferences: each voter has a distinct ideal point $x_j \in X$. A voter's payoff from candidate i winning, given that she spent s_i, is

$$u_j(i \mid s_i) = s_i - \beta |x_j - x_{t_i}|, \tag{2}$$

where $\beta > 0$ is the relative weight voters place on policy. Because candidates' types are private information, voters do not know their exact payoff from a particular candidate winning. Instead, voters must base their expectations on observable information—namely, the amount a candidate spends. Since the candidates do not know each others' types when making their spending decisions, the inference a voter makes about each candidate is solely a function of that candidate's spending. For every $s \geq 0$, let $\mu(s)$ denote the electorate's belief that a candidate who spends s is Advantaged.[2] A voter's expected utility from victory by a candidate who spends s is

$$Eu_j(s) = s - \beta \mu(s) |x_j - x_A| - \beta (1 - \mu(s)) |x_j - x_D|. \tag{3}$$

As the number of voters is odd, there is a median voter $m \in N$; without loss of generality, I assume $x_m = 0$. The policy distance from the median voter to an Advantaged candidate is thus $|x_A|$, while that to a Disadvantaged candidate is $|x_D|$. Let $\alpha = \beta(|x_A| - |x_D|)$ denote the difference in distance from the median across types, weighted by the relative importance of

[2]This notation reflects the implicit assumption that (1) all voters use the same updating rule and (2) the rule is the same for both candidates, even for spending decisions off the equilibrium path.

policy.[3] We can thus write the median voter's expected utility from a candidate who spends s as[4]

$$Eu_m(s) = s - \mu(s)\alpha. \tag{4}$$

I say Advantaged candidates are "in the majority" if $\alpha \leq 0$ and "in the minority" otherwise. Comparative statics on α reflect the effects of the policy difference between the two types of candidate and the relative importance of policy to the electorate.

As usual in models of electoral competition, there are numerous trivial equilibria in the voting stage in which no voter is pivotal. To eliminate these, I employ the usual assumption that no voter employs a weakly dominated strategy (e.g., Besley and Coate 1997). As voters have single-peaked preferences and identical belief systems, this assumption implies that the median voter's preference determines electoral outcomes. Let the median voter's strategy be $\xi : \mathbb{R}_+^2 \to [0,1]$, where $\xi(s,s')$ denotes his probability of electing a candidate who spends s over one who spends s'. In equilibrium, $Eu_m(s) > Eu_m(s')$ implies $\xi(s,s') = 1$. In line with the restriction to symmetric strategy profiles, I assume that the $\xi(s,s) = 1/2$ for all $s \in \mathbb{R}_+$. For cases in which $s \neq s'$ but $Eu_m(s_i) = Eu_m(s_j)$ (because of the difference in the electorate's beliefs), I let the median voter's choice rule be determined endogenously (Simon and Zame 1990). Typically this results in the median voter choosing the candidate who spends more when indifferent. In the main model, ties between candidates spending different amounts occur with probability zero, so the choice of sharing rule is not critical.

A candidate's expected utility for spending a particular amount depends on her type and the median voter's strategy. Under the strategy profile σ, the *ex ante* distribution of an arbitrary candidate's spending s_i is given by the probability measure $\tilde{\sigma} = p_A\sigma_A + p_D\sigma_D$. The *ex ante* probability of victory for a candidate who spends s is therefore

$$\lambda(s) = \int \xi(s,s_i)\, d\tilde{\sigma}(s_i). \tag{5}$$

[3]The baseline model of Meirowitz (2008) is the limiting case of this one where $\alpha = 0$ and $c_D = c_A$.

[4]For ease of exposition, equation (4) drops the constant $-\beta|x_D|$ that results when $x_j = 0$ is substituted into equation (3).

The expected utility of spending s for a candidate of type t, holding fixed her opponent's mixed strategy and the strategies of the voters, is therefore

$$Eu_t(s) = \lambda(s) - c_t s. \tag{6}$$

The *ex ante* expected utility of a candidate of type t under the mixed strategy profile σ is

$$U_t = \int Eu_t(s)\, d\sigma_t(s). \tag{7}$$

This is a two-stage game of incomplete information with a finite type space and an infinite action space, so I employ a solution concept akin to perfect Bayesian equilibrium (Fudenberg and Tirole 1991, pp. 331–333). An assessment (σ, μ) is an *equilibrium* if it meets the following requirements. First, the median voter's strategy is a best response given his beliefs. Second, each candidate's strategy is a best response given her type, the strategy of the other candidate, and the median voter's strategy. Under the restriction to symmetric strategy profiles, the optimality requirement is satisfied if and only if

$$U_t \geq Eu_t(s) \qquad \text{for all } t \in T \text{ and } s \in \mathbb{R}_+. \tag{8}$$

An important consequence of this requirement is the indifference condition of mixed-strategy equilibrium, which is that a candidate of type t must be indifferent across almost all spending choices in the support of her mixed strategy, σ_t. Third, the electorate's beliefs must be updated in accordance with Bayes' rule whenever possible. Specifically, I use the following criteria for consistency. If s is a mass point of either type's mixed strategy, the electorate applies Bayes' rule:

$$\mu(s) = \frac{p_A \sigma_A(\{s\})}{p_A \sigma_A(\{s\}) + p_D \sigma_D(\{s\})}. \tag{9}$$

If $s \in \operatorname{supp} \sigma_A \setminus \operatorname{supp} \sigma_D$, then $\mu(s) = 1$. Similarly, if $s \in \operatorname{supp} \sigma_D \setminus \operatorname{supp} \sigma_A$, then $\mu(s) = 0$. If

$s \in \operatorname{supp}\sigma_A \cap \operatorname{supp}\sigma_D$ but is not a mass point of either, the electorate's belief is the weighted ratio of densities if they exist:

$$\mu(s) = \frac{p_A F_A'(s)}{p_A F_A'(s) + p_D F_D'(s)}, \tag{10}$$

where F_t is the cumulative distribution function corresponding to σ_t. I place no restriction in case $s \in \operatorname{supp}\sigma_A \cap \operatorname{supp}\sigma_D$ but $F_A'(s)$ or $F_D'(s)$ fails to exist.[5] Finally, beliefs for $s \notin \operatorname{supp}\sigma_A \cup \operatorname{supp}\sigma_D$ are unrestricted.

To rule out equilibria that are supported by implausible off-the-path beliefs, I employ the D1 refinement (Cho and Kreps 1987). D1 requires, in essence, that voters ascribe any off-the-path spending choice to the type of candidate that could potentially benefit most from making it. Other models of electoral competition with privately known policy intentions use similar refinements (e.g., Banks 1990; Callander and Wilkie 2007). For this model, the restriction that D1 places on beliefs is reasonable. I show in the Appendix that in any equilibrium, there is a cutpoint $\hat{s}$ such that no Disadvantaged candidate spends more than $\hat{s}$ and no Advantaged candidate spends less (Lemma 5). D1 imposes the natural requirement that off-the-path deviations below this cutpoint be ascribed to Disadvantaged candidates, and those above it to Advantaged candidates (Lemma 6). I say that an equilibrium (σ, μ) is *essentially unique* under D1 if any other equilibrium that survives D1 entails the same spending strategies.

3 Results

In this section, I characterize an equilibrium of the model for each segment of the parameter space. In each case, the equilibrium described is essentially unique under D1; others may exist, but they involve implausible assumptions about off-the-path beliefs (i.e., ascribing high

[5]The lack of restriction is immaterial in the main model. The only instance where a point s enters the support of both types' strategies but is not a mass point of either is in Proposition 1. In that case, the median voter's expected utility would still be strictly increasing in s regardless of his beliefs at $\bar{s}_D^*$, the common point of the two types' support, implying the same best responses for the candidates.

spending to Disadvantaged types). When Advantaged candidates are in the majority, the form of the equilibrium is simple: Advantaged candidates always spend enough to separate themselves and ensure victory over a Disadvantaged rival. On the other hand, when Advantaged candidates are in the minority, the strategic calculus becomes more complex. If a candidate spends enough to reveal that she is Advantaged, the median voter makes a negative inference about the policies she will enact if elected. Therefore, such candidates will only reveal themselves if they are willing to spend enough to make up for the negative policy inference. Whether this is the case depends on the relative magnitude of the spending advantage and the policy gap between types of candidates.

3.1 General Remarks

I begin with some general observations about the shape of the equilibrium. These are stated formally and proved in a sequence of lemmas in the Appendix. The first observation concerns the relationship between a candidate's spending and her electoral fortunes.

Remark 1. *On the equilibrium path, spending more implies a greater probability of victory.*

The reason this result holds is that campaign spending is costly to the candidate. Since candidates are office-motivated, a candidate would never incur an additional cost of fundraising if there were not an electoral payoff. However, we should be careful in how we interpret this result. It refers only to spending decisions that are made on the equilibrium path—not to all possible counterfactual spending decisions. In particular, when Advantaged candidates are in the minority, it is conceivable that spending more might lead the median voter to update his beliefs negatively about the candidate's policy intentions, making him less likely to vote for that candidate. But no candidate would ever choose to do so, and hence any negative effect of greater spending will be purely counterfactual. Observationally, higher spending should be associated with a higher chance of winning.

The second observation is that it is indeed better to be Advantaged at fundraising, even if

Advantaged candidates are in the minority on policy matters.

Remark 2. *In equilibrium, Advantaged types are weakly better off:* $U_A \geq U_D$.

It is obvious why this is true when Advantaged candidates are in the majority, as then all good things go together. What is more curious is that there is no utility disadvantage even when Advantaged candidates are very far on policy from the median voter (i.e., α is very high). What drives the result is that Advantaged candidates always have the option of mimicking their Disadvantaged counterparts. A candidate may, in essence, pretend to be Disadvantaged by following the spending strategy of Disadvantaged candidates. Doing so yields the same probability of victory at the same (or lower) cost of fundraising, and thus $U_A \geq U_D$. In other words, an Advantaged candidate can choose to "reveal" herself only when doing so is beneficial; otherwise, she may conceal her type by not exploiting her fundraising advantage.

Next, we have that Advantaged candidates spend more than their Disadvantaged counterparts.

Remark 3. *In equilibrium, Disadvantaged (Advantaged) types never spend more (less) than* $\hat{s} = (U_A - U_D)/(c_D - c_A)$.

This result follows from the positive relationship between spending and the probability of victory summarized in Remark 1. If it is worth it to a Disadvantaged candidate to spend s, in that the increase in the chance of victory justifies the personal cost, then it must also be worth it for an Advantaged candidate, for whom the personal cost is lower. In equilibrium, therefore, no Advantaged candidate will spend less than the most that a Disadvantaged candidate is willing to spend. The two types' mixed strategies may overlap at one point at most, namely $\hat{s}$. On the equilibrium path, if a candidate spends any amount besides $\hat{s}$, voters infer her type with certainty.

Two features of $\hat{s}$, the cutpoint derived in Remark 3, merit additional attention. First, as I show in the Appendix, this is also the cutpoint for off-the-path beliefs under the D1 refinement.

In order for an equilibrium to survive D1, an off-the-path deviation must be attributed to a Disadvantaged type if it is below $\hat{s}$ and to an Advantaged type if it is above $\hat{s}$. This restriction is natural in light of Remark 3. Second, the value of the cutpoint depends on the equilibrium utilities. Consequently, it cannot be used to solve for equilibrium strategies, though the result is useful in demonstrating uniqueness.

The final baseline result is that a fundraising advantage confers an electoral advantage. At this point, it is important to distinguish between the (interim) probability of victory from the candidate's perspective and the (*ex ante*) probability of victory by a particular type of candidate from the electorate's perspective. The interim question for the candidate is: Given the equilibrium strategies of both types, once a candidate learns her own type, what would she say the chance of her winning the election is? Under the restriction to symmetric equilibria, her chances against a candidate of the same type must be exactly 50-50.

Remark 4. *In equilibrium, Advantaged candidates have a greater interim chance of victory than do Disadvantaged candidates.*

What Remark 4 shows is that an Advantaged candidate's chances against a Disadvantaged opponent must be 50-50 or greater. However, it may still be the case that there is a low chance of the election producing an Advantaged victor, namely if the prior probability of Advantaged types is low. The implication of the remark for this *ex ante* question is that the probability that the eventual victor is Advantaged must be at least

$$\Pr(\text{both Advantaged}) + \frac{1}{2}\Pr(\text{exactly one Advantaged}) = p_A^2 + \frac{2p_A(1-p_A)}{2} = p_A.$$

The logic of Remark 4 follows from the previous remarks. We have seen that greater spending is associated with a greater chance of victory along the equilibrium path (Remark 1) and that Advantaged candidates spend more in equilibrium (Remark 3). It then follows that Advantaged candidates have a greater chance of winning. Again, it is somewhat surprising that this result holds even if Advantaged candidates' policy intentions put them in the minority, meaning high spending sends a negative policy signal to the median voter. The key, as before, is that Advantaged candidates have the option to conceal their type by mimicking the spending strategy of Disadvantaged types. If candidates' types were exogenously revealed before spending decisions were made, then there would be parameters of the model under which Advantaged candidates would be electorally disadvantaged. However, as long as candidates may strategically conceal their types, those with a fundraising advantage have the electoral upper hand.

This final remark also illuminates the connection between electoral strategies and the policy component of the median voter's welfare. When Advantaged candidates are in the majority, of course, the median voter's first-best would be for Advantaged candidates always to defeat Disadvantaged opponents. As I show below, this is exactly what the equilibrium in this case entails. Conversely, when Advantaged candidates are in the minority, the median voter, at least in terms of policy, would prefer that Disadvantaged candidates prevail over Advantaged opponents. Remark 4 shows that this first-best is not feasible. Given what we have seen so far, the best the median voter can hope for on policy is for Advantaged and Disadvantaged candidates to tie on average—as is the case in a pooling equilibrium, where both types employ the same mixed strategy. Therefore, when Advantaged candidates are in the minority, there is a dilemma between disclosure and the median voter's policy preference. The best chance for the median voter to elect a candidate closer to her own ideal point comes when the candidates employ strategies that do not reveal their types. If information is being revealed, then candidates farther from the median voter's ideal point are disproportionately likely to win. This dilemma arises from Advantaged candidates' ability to strategically conceal their types in the spending phase.

3.2 Equilibrium When Centrists Are Advantaged

First, I consider the case in which Advantaged candidates will implement policies relatively close to the median voter's ideal point; i.e., Advantaged candidates are in the majority. Formally, the condition for this case is $\alpha \leq 0$. Because candidates' spending decisions signal their policy intentions, the fundraising advantage is doubly potent in this case. Spending a high amount has both the direct benefit of raising one's perceived valence and the indirect benefit of signaling one's centrist policy intentions. Naturally, then, in equilibrium Advantaged candidates always spend enough to separate themselves from Disadvantaged types.

The following proposition characterizes the equilibrium. As is typical in endogenous valence models (e.g., Meirowitz 2008), the equilibrium is in mixed strategies. In a pure strategy profile, ties would occur with positive probability; it would then be profitable to spend slightly more than the tying amount so as to win instead of tie. Therefore, there cannot be a pure strategy equilibrium.[6] Instead, in equilibrium, Disadvantaged candidates mix uniformly over $[0, p_D/c_D]$, and Advantaged candidates mix uniformly over $[p_D/c_D, p_D/c_D + p_A/c_A]$. Figure 1 plots the CDFs that correspond to these strategies.

Proposition 1. *If Advantaged candidates are in the majority, then there is an equilibrium (σ^*, μ^*) that is essentially unique under D1 in which Disadvantaged candidates employ a mixed strategy whose CDF is*

$$F_D^*(s) = \begin{cases} 0 & s < 0, \\ c_D s/p_D & 0 \leq s \leq \bar{s}_D^*, \\ 1 & s > \bar{s}_D^*, \end{cases} \tag{11}$$

and Advantaged candidates employ a mixed strategy whose CDF is

$$F_A^*(s) = \begin{cases} 0 & s < \bar{s}_D^*, \\ c_A(s - \bar{s}_D^*)/p_A & \bar{s}_D^* \leq s \leq \bar{s}_A^*, \\ 1 & s > \bar{s}_A^*, \end{cases} \tag{12}$$

[6] Nor, by the same token, a mixed strategy equilibrium with mass points. As we will see below, this same logic does not always hold when Advantaged candidates are in the minority, as then spending slightly more than a tying amount might have a negative offsetting effect on the median voter's policy utility.

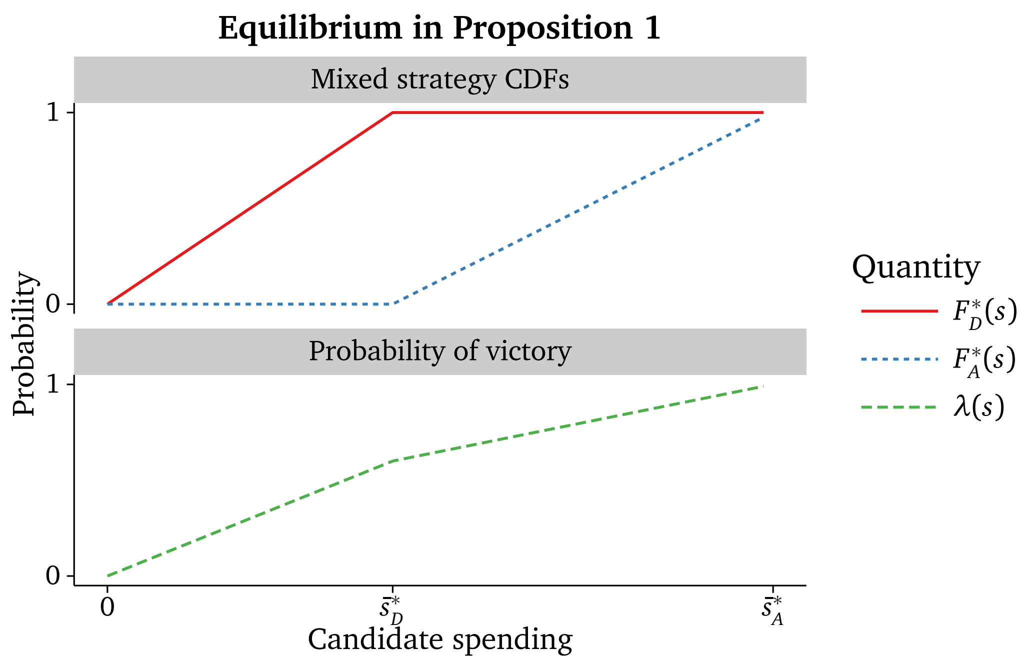

Figure 1. CDFs and probability of victory by spending in equilibrium when Advantaged candidates are in the majority, as given in Proposition 1.

where $\bar{s}_D^* = p_D/c_D$ *and* $\bar{s}_A^* = \bar{s}_D^* + p_A/c_A$*. The electorate's beliefs are*

$$\mu^*(s) = \begin{cases} 0 & s < \bar{s}_D^*, \\ p_A & s = \bar{s}_D^*, \\ 1 & s > \bar{s}_D^*. \end{cases} \tag{13}$$

The most striking feature of the equilibrium is that in an election between one candidate of each type, the Advantaged candidate is sure to prevail.[7] Therefore, in terms of the median voter's policy preferences, the electoral outcome of this equilibrium is a first-best. Since the signaling effect of high spending works in favor of Advantaged candidates when they are in the majority, in this case they exploit that advantage to its fullest.

In this equilibrium, there is no gap between the mixed strategies of Advantaged and Disadvantaged candidates. A Disadvantaged candidate could defeat an Advantaged opponent with

[7]A tie between an Advantaged candidate and a Disadvantaged one is possible if both candidates spend p_D/c_D, but this is a zero-probability event.

positive probability if she spent infinitesimally more than p_D/c_D. So why do Disadvantaged candidates allow their Advantaged counterparts to separate themselves and reap an electoral advantage? The key is the cost-benefit tradeoff to spending. As illustrated in Figure 1, the marginal effect of spending on the probability of victory is lower in the range of spending employed by Advantaged candidates than in the range employed by Disadvantaged candidates. As required in a mixed strategy equilibrium, Disadvantaged candidates are indifferent across all spending choices in $[0, p_D/c_D]$; this means the marginal electoral benefit exactly equals the marginal cost. At the top of this range, the marginal cost remains the same but the marginal benefit for the probability of victory shifts downward. Therefore, Disadvantaged candidates are unwilling to pay the additional cost it would take to mimic an Advantaged type.

Another interesting property of the equilibrium when Advantaged candidates are in the majority is that the strategies do not depend on α, the magnitude of the policy difference between types of candidates. The exact shape of the strategies depends only on the relative frequency of each type of candidate (p_A and p_D) and their respective marginal costs of fundraising (c_A and c_D). In fact, the equilibrium takes the form described in Proposition 1 even if there is no policy difference between types ($\alpha = 0$). From the candidates' perspective as they make their policy decisions, the magnitude of the policy difference is only important to whichever type is in the minority. If a candidate reveals herself as being in the minority, she must spend α more than the other type to equalize herself in the eyes of the median voter. But in this case, where Advantaged candidates are in the majority, this is a price that their Disadvantaged counterparts are unwilling to pay, as we have just seen. The magnitude of the policy difference is considerably more important for the form of the equilibrium in the converse case—when Advantaged candidates are in the minority in terms of their policy intentions—to which I now turn.

3.3 Equilibrium When Centrists Are Disadvantaged

I now consider the case in which candidates with a fundraising advantage are relatively far from the median voter in their policy intentions. The formal condition for Advantaged candidates to be in the minority is that $\alpha > 0$. The equilibrium in this case is not simply the mirror image of the equilibrium when Advantaged candidates are in the majority. The form of the equilibrium depends on the magnitude of α, the difference between Advantaged and Disadvantaged candidates in their policy distance from the median voter. The farther Advantaged candidates are from the median voter, relative to their Disadvantaged counterparts, the more likely they are to conceal their type by spending little.

To see why the magnitude of the policy distance determines the likelihood of concealment, imagine a pooling equilibrium in which all types spend the same amount, $\tilde{s}$. This represents the limiting case of total concealment by Advantaged types.[8] The median voter, updating his beliefs according to Bayes' rule, infers that a candidate who spends $\tilde{s}$ is Advantaged with probability p_A, the prior probability. The median voter's utility from electing a candidate who spends $\tilde{s}$ is therefore $Eu_m(\tilde{s}) = \tilde{s} - p_A\alpha$, per Equation 4. The candidates' strategies form an equilibrium only if it is not profitable for either type to deviate to spending a different amount. Under the D1 restriction on off-the-path beliefs, any deviation to $s > \tilde{s}$ will be ascribed to an Advantaged type, giving the median voter a payoff of $Eu_m(s) = s - \alpha$. A candidate who makes such a deviation will win the election if and only if $Eu_m(s) > Eu_m(\tilde{s})$, which is equivalent to

$$s > \tilde{s} + (1 - p_A)\alpha = \tilde{s} + p_D\alpha.$$

In other words, a candidate must spend an extra $p_D\alpha$ in order to increase her chance of victory by 1/2 (since, in equilibrium, the election always ends in a tie). The deviation is unprofitable for an Advantaged candidate only if the marginal cost of the additional fundraising exceeds the electoral benefit: $c_A p_D \alpha \geq 1/2$. Therefore, in order for Advantaged candidates to fully conceal

[8] Remark 3 implies that any fully pooling equilibrium must be in pure strategies.

their types in equilibrium, the policy difference between Advantaged types and the median voter must be sufficiently large: $\alpha \geq 1/2c_A p_D$.

If this condition holds and there is a pooling equilibrium, it must entail zero spending. Imagine that both types pool on $\tilde{s} > 0$, and consider a deviation to $s = \tilde{s} - \epsilon$. Under the D1 restriction on off-the-path beliefs, a "low" deviation like this one is ascribed to a Disadvantaged type. So the deviation would reduce the valence component of the median voter's utility from electing the candidate by ϵ, but increase the policy component by $p_A\alpha$. For any $\epsilon < p_A\alpha$, then, a candidate who deviated would win the election rather than tying. Since the deviation yields a better electoral outcome at lower cost, it would be profitable for either type of candidate. Consequently, the only possible pooling equilibrium entails $\tilde{s} = 0$, in which case there is no downward deviation of this type available.

The foregoing arguments have shown that total concealment by Advantaged types is possible only if $\alpha \geq 1/2c_A p_D$ and that this kind of pooling equilibrium, if it exists, involves zero spending. In fact, if the condition on α holds, this is the unique equilibrium outcome under D1, as the following proposition states.

Proposition 2. *If $\alpha \geq 1/2c_A p_D$, then there is an equilibrium (σ^*, μ^*) that is essentially unique under D1 in which both types of candidates spend 0 for certain. The electorate's beliefs are $\mu^*(0) = p_A$ and $\mu^*(s) = 1$ for all $s > 0$.*

When the policy distance between Advantaged types and the median voter is not great enough for there to be total concealment in equilibrium, there may still be partial concealment. In an equilibrium with partial concealment, all Disadvantaged candidates spend nothing, while Advantaged candidates employ a mixed strategy. With a certain probability, call it π, an Advantaged type spends nothing, in essence mimicking the Disadvantaged type. On the other hand, with probability $1 - \pi$, an Advantaged type "separates" herself by spending enough to defeat candidates who spend nothing. Under such an equilibrium, the median voter infers that a candidate who spends 0 is Advantaged with probability $\mu(0) = \pi p_A/(\pi p_A + p_D)$, making his expected utility from electing such a candidate

$$Eu_m(0) = -\frac{\pi p_A}{\pi p_A + p_D}\alpha.$$

As before, any spending $s > 0$ is attributed to an Advantaged candidate under D1, resulting in $Eu_m(s) = s - \alpha$. Therefore, an Advantaged candidate who spends a positive amount must spend

$$s > \frac{p_D}{\pi p_A + p_D}\alpha,$$

or else she would be better off electorally spending nothing.

The Advantaged type's strategy in an equilibrium with partial concealment cannot entail a mixture between spending zero and a single positive value. As in the previous case of Advantaged candidates in the majority (and as in Meirowitz 2008), the all-pay logic precludes either type's mixed strategy from having a mass point besides zero. If there were a value $\tilde{s} > 0$ on which the Advantaged type's mixed strategy placed positive probability, it would be profitable for an Advantaged type to deviate to spending infinitesimally more, so as to defeat rather than tie an Advantaged opponent who spent $\tilde{s}$. (The same logic does not preclude a mass point at 0 since a slight upward deviation would be attributed to an Advantaged candidate, causing the deviant to lose the election when $\alpha > 0$.) The equilibrium strategy for Advantaged types therefore entails spending zero with probability π and drawing from a continuous mixture with probability $1 - \pi$. The exact form of the mixture, which as in Proposition 1 is uniform over an interval, is given in the following proposition. The corresponding cumulative distribution function is plotted in Figure 2.

Proposition 3. *If $p_D/2c_A < \alpha < 1/2c_A p_D$, then there is an equilibrium (σ^*, μ^*) that is essentially unique under D1 in which Disadvantaged candidates spend 0 for certain and Advantaged candidates*

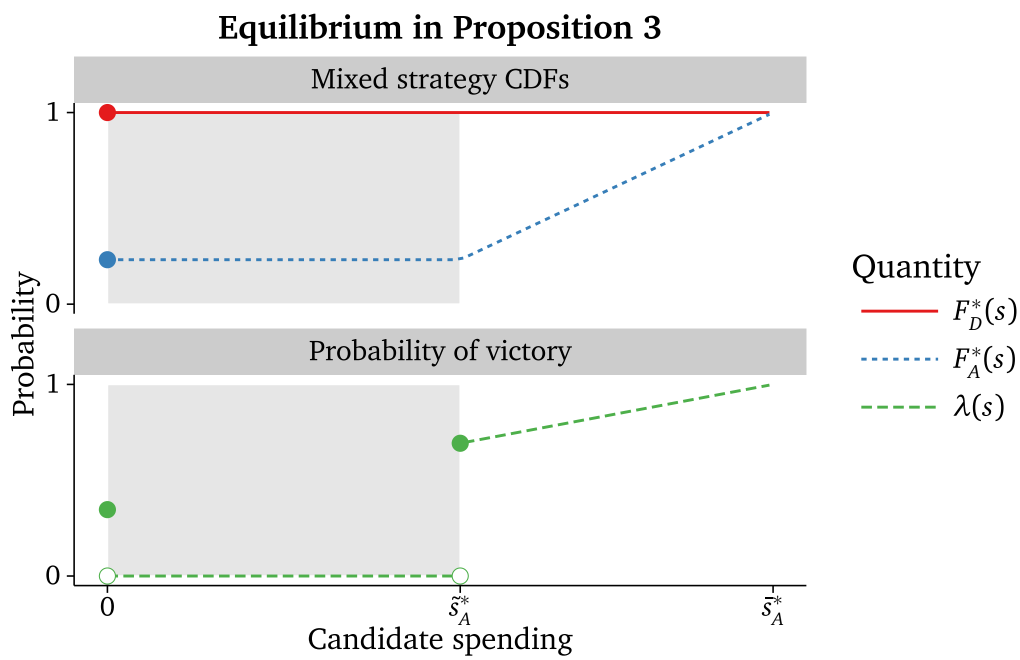

Figure 2. CDFs and probability of victory by spending in an equilibrium with partial concealment when Advantaged candidates are in the minority, as given in Proposition 3. Shaded regions represent spending levels off the equilibrium path.

employ a mixed strategy whose CDF is

$$F_A^*(s) = \begin{cases} 0 & s < 0, \\ \pi^* & 0 \leq s \leq \tilde{s}_A^*, \\ \pi^* + c_A(s - \tilde{s}_A^*)/p_A & \tilde{s}_A^* < s < \bar{s}_A^*, \\ 1 & s \geq \bar{s}_A^*, \end{cases} \tag{14}$$

where $\pi^* = (\sqrt{2\alpha c_A p_D} - p_D)/p_A$*,* $\tilde{s}_A^* = (\pi^* p_A + p_D)/2c_A$*, and* $\bar{s}_A^* = \tilde{s}_A^* + p_A(1 - \pi^*)/c_A$*. The electorate's beliefs are* $\mu^*(0) = \pi^* p_A/(\pi^* p_A + p_D)$ *and* $\mu^*(s) = 1$ *for all* $s > 0$.

Unlike in the previous case of full concealment, here the election does not always end in a tie. Consider an election between one Advantaged candidate and one Disadvantaged candidate. With probability

$$\pi^* = \frac{\sqrt{2\alpha c_A p_D} - p_D}{p_A},$$

the Advantaged candidate spends zero, concealing her type, and the election ends in a tie. Otherwise, with probability $1-\pi^*$, the Advantaged candidate spends more, revealing her type. Since she spends enough to overcome the median voter's downward shift in policy utility, in this contingency the election ends in victory for the Advantaged candidate.

Two important comparative statics emerge from the above expression for the probability of concealment. First, the chance that an Advantaged candidate conceals her type increases with α, the difference between Advantaged and Disadvantaged types' policy distance from the median voter. In fact, it is easy to check that this probability approaches one at the top of the range on α for which partial concealment is an equilibrium ($\alpha \to 1/2c_A p_D$) and zero at the bottom ($\alpha \to p_D/2c_A$). Second, the probability of concealment also increases with c_A, the marginal cost of fundraising for an Advantaged type. The more effort that an Advantaged type must expend to raise the same amount of funds, the less she will be willing to pay the extra cost it takes to defeat rather than tie an opponent who spends nothing.

An interesting feature of the partial concealment equilibrium is that the probability of victory as a function of spending is non-monotonic. Figure 2 shows that $\lambda(s)$ dips between 0 and $\tilde{s}_A^*$, the lower bound of the positive portion of Advantaged types' mixed strategy. Because all positive spending is attributed to Advantaged candidates, the median voter would rather elect a candidate who spends nothing—who has a chance of being Disadvantaged and thus would implement policies strictly closer to his ideal point, under the parameter conditions here—than one who spends a bit more. This non-monotonicity does not contradict Remark 1, however, since the values where the probability dips are off the equilibrium path. Although it would be possible for a candidate to win less by spending more, such outcomes are strictly counterfactual.

When the difference between the two types' policy distance from the median voter is small enough, the equilibrium is fully separating. If both types are known, then α is the extra amount an Advantaged candidate must spend to defeat a Disadvantaged opponent. Therefore, when α is sufficiently small, even partial concealment is no longer sustainable in equilibrium, as Advantaged candidates would rather pay the extra cost to win rather than tie. However, the

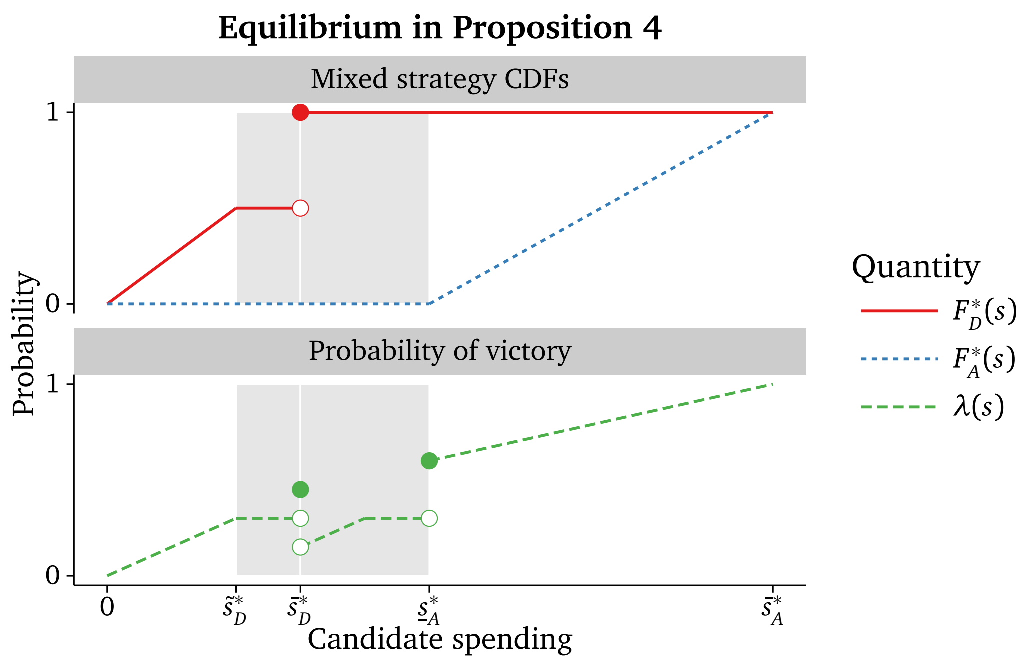

Figure 3. CDFs and probability of victory by spending in a fully separating equilibrium when Advantaged candidates are in the minority, as given in Proposition 4. Shaded regions represent spending levels off the equilibrium path.

fully separating equilibrium here is not a mirror image of the one when Advantaged candidates are in the majority, as described in Proposition 1. First, there is a gap between the supports of the Advantaged and Disadvantaged types' mixed strategies. The width of the gap is α, the extra amount an Advantaged candidate must pay to make the median voter indifferent between herself and a Disadvantaged opponent. Second, the Disadvantaged type's strategy is not continuous; it places positive mass on its upper bound. The following proposition formally describes the equilibrium in this case, and Figure 3 plots the equilibrium spending strategies and probability of victory.

Proposition 4. *If $0 < \alpha \leq p_D/2c_A$, then there is an equilibrium (σ^*, μ^*) that survives D1 in which*

Disadvantaged candidates employ a mixed strategy whose CDF is

$$F_D^*(s) = \begin{cases} 0 & s < 0, \\ c_D s/p_D & 0 \le s \le \tilde{s}_D^*, \\ c_D \tilde{s}_D^*/p_D & \tilde{s}_D^* < s < \bar{s}_D^*, \\ 1 & s \ge \bar{s}_D^*, \end{cases} \tag{15}$$

and Advantaged candidates employ a mixed strategy whose CDF is

$$F_A^*(s) = \begin{cases} 0 & s < \underline{s}_A^*, \\ c_A(s - \underline{s}_A^*)/p_A & \underline{s}_A^* \le s \le \bar{s}_A^*, \\ 1 & s > \bar{s}_A^*, \end{cases} \tag{16}$$

where $\tilde{s}_D^* = (p_D - 2c_A\alpha)/c_D$*,* $\bar{s}_D^* = (p_D - c_A\alpha)/c_D$*,* $\underline{s}_A^* = \bar{s}_D^* + \alpha$*, and* $\bar{s}_A^* = \underline{s}_A^* + p_A/c_A$*. The electorate's beliefs are* $\mu^*(s) = 0$ *for all* $s \le \bar{s}_D^*$ *and* $\mu^*(s) = 1$ *for all* $s > \bar{s}_D^*$*. If* $0 < \alpha < p_D/2c_A$*, this equilibrium is essentially unique under D1.*

It is intuitive that there will be full separation when the difference in policy distances between the two types of candidate is sufficiently small. The form that the Disadvantaged type's strategy takes under full separation is less intuitive. In the equilibrium described here, Disadvantaged candidates randomize between drawing from a uniform mixture over $[0, \tilde{s}_D^*]$ and spending the mass point $\bar{s}_D^*$. To see why the mass point is necessary, imagine a strategy profile in which each type mixed over a continuum, as in Proposition 1. At the low point of the Advantaged type's mixed strategy, at best she would defeat almost every Disadvantaged opponent and lose to almost every Advantaged opponent. She could get the same electoral outcome at strictly less cost by deviating to the high point of the Disadvantaged type's mixed strategy. Therefore, the strategy profile cannot be an equilibrium. When the high point of the Disadvantaged type's mixed strategy is a mass point, such a deviation becomes less attractive, since it results in a positive probability of a tie with a Disadvantaged opponent. Hence the mass point in the equilibrium here.

To sum up, when Advantaged candidates are in the minority, they retain their electoral advantage by strategically concealing their type when beneficial. Whether it is optimal to mimic a Disadvantaged opponent by spending nothing depends on how much farther an Advantaged candidate is from the median voter on policy and on her marginal cost of fundraising. The probability of concealment by Advantaged candidates ranges from 0 to 1, varying continuously with the parameters of the model.

4 Equalizing Reforms

Imagine an "equalizing reform" that reduces the gap in fundraising ability between Advantaged and Disadvantaged candidates. For example, Meirowitz (2008, 690) points out that caps on individual donations to a candidate raise the marginal cost of fundraising, by making candidates contact more individuals to raise the same amount of money. If some candidates have a differential advantage in fundraising due to their connections to interest groups, then it is reasonable to conclude in the context of this model that such policies would raise the marginal cost of spending proportionally more for Advantaged candidates than for Disadvantaged ones. In this section, I will consider the electoral consequences of such equalizing reforms for candidate behavior and electoral outcomes in the equilibrium of the model. I will focus on the question of how the probability of electing an Advantaged candidate changes with c_A, the marginal cost of fundraising for Advantaged types.

One potential concern for equalizing reforms is that they might sometimes be beneficial (e.g., when Advantaged candidates are in the minority) and other times harmful (e.g., when Advantaged candidates are in the majority). If campaign finance regulations were tailored to each individual race, this would not be a concern; policymakers could enact reforms when beneficial and prevent them otherwise. However, if the same campaign finance rules are set at the federal or even state level, they are bound to cover races with wide variation in the relevant parameters of the model: namely, each type's prior probability, policy distance from the median voter, and marginal cost of fundraising. To what extent do the benefits, if any, of equalizing reforms depend on the distribution of these parameters across races?

Perhaps surprisingly, the results of the model so far show that an across-the-board equalizing reform *never* decreases the chance of electing the type of candidate closest to the median voter. To see why, consider the *ex ante* probability of electing the closer candidate in equilibrium across each portion of parameter space, using the equilibria characterized in the previous section. When Advantaged candidates are in the majority, an Advantaged candidate always defeats a Disadvantaged opponent, per Proposition 1. Therefore, a marginal increase in c_A does not affect the *ex ante* probability that the election produces an Advantaged winner. When Advantaged candidates are in the minority, the probability that a Disadvantaged candidate defeats an Advantaged opponent is

$$\iint \xi^*(s_D, s_A)\, d\sigma_D^*(s_D)\, d\sigma_A^*(s_A) = \begin{cases} 0 & \alpha \leq p_D/2c_A \text{ (Proposition 4)}, \\ \pi^*/2 & p_D/2c_A < \alpha < 1/2c_A p_D \text{ (Proposition 3)}, \\ 1/2 & \alpha \geq 1/2c_A p_D \text{ (Proposition 2)}. \end{cases} \tag{17}$$

From Proposition 3 we have that π^* increases with c_A, so a marginal increase in c_A either increases or does not affect the chance of electing a Disadvantaged candidate.

Consequently, the asymmetry in the form of the equilibrium between the two cases (Advantaged candidates in the majority or the minority) turns out to be substantively important for the electoral consequences of reform. When Advantaged candidates are in the majority, increasing c_A causes them to spend less, but still enough to defeat a Disadvantaged opponent. Conversely, when Advantaged candidates are in the minority, increasing c_A makes it more attractive for such candidates to conceal their type—and thus to tie with, rather than defeat, Disadvantaged opponents who are closer to the median voter. In no context does an increase in c_A make it less likely that the eventual winner will enact policy relatively close to the median voter. Therefore, if the objective is to elect such candidates, the effect of equalizing reforms appears to be benign.

Two points of caution are in order, though. The first is that this comparative statics analysis does not consider candidate entry, which is treated exogenously in the model. When Advantaged candidates are in the majority, an increase in c_A does decrease the equilibrium expected utility of an Advantaged type, $U_A = p_D(1 - c_A/c_D)$. Intuitively, this might deter entry by Advantaged candidates. However, given that Disadvantaged candidates have an even lower equilibrium expected utility, $U_D = 0$, it is not entirely clear what the effect would be in a general equilibrium analysis.

The second point of caution is that the results so far assume the two candidates' types are independent. High spending by Advantaged candidates in equilibrium is driven in part by their expectation that their opponent is Advantaged with probability p_A. In a correlated-types extension in the next section, I investigate the extent to which independence is responsible for the innocuousness of equalizing reforms when Advantaged candidates are in the majority.

5 Extensions (Preliminary)

I consider two extensions to the model. In the first, candidates may take lump-sum public financing as an alternative to participating in the spending competition. In the second, candidate types are negatively correlated—i.e., the election is disproportionately likely to be between one Advantaged and one Disadvantaged candidate.

5.1 Public Financing

An alternative kind of equalizing reform is public financing, which allows a candidate to spend money without participating in costly fundraising. The *game with public finance* is as follows.

1. Candidates learn their types, $t_i \in T$.

2. Each candidate simultaneously chooses whether to take public financing, $P_i \in \{0, 1\}$.

3. These choices are observed by the candidates and the electorate, who update their beliefs accordingly.

4. Each candidate who chose $P_i = 0$ simultaneously spends $s_i \in \mathbb{R}_+$, at marginal cost c_{t_i}. Each candidate who chose $P_i = 0$ spends $\ell \geq 0$ at no cost.

5. The electorate observes the spending choices and updates its beliefs.

6. The median voter selects a winner.

I assume that the effect of public finance on the spending component of the electorate's utility is the same as the equivalent amount of privately raised money. However, depending on the strategy profile, the electorate may make a different inference about the type of a candidate that privately raises ℓ than one who takes public financing. As before, I focus on symmetric equilibria, so that the electorate's belief system is symmetric across the two candidates. Because the candidates learn about each other from the choice of whether or not to take public finance, the equilibria of the spending subgames may be different than that of the main model.

Whether this extension is directly comparable to the main model depends on one's interpretation of the relationship between policy intention and fundraising advantage. If candidates are Advantaged only because they have been "bought," and would otherwise implement policies closer to the median voter, then the notion of an Advantaged candidate taking public financing is incoherent. To facilitate comparisons, I will instead operate under the assumption that a candidate's policy intentions are a fixed attribute, as in previous models of hidden policy intentions (Banks 1990; Callander and Wilkie 2007). On this interpretation, interest groups "buy" candidates because they are like-minded, not the other way around.

The first result is that if the amount provided by public financing is sufficiently generous, then there is a pooling equilibrium in which all candidates take it.

Proposition 5. *If $\ell \geq 1/2c_A - p_D\alpha$, there is an equilibrium of the game with public finance in which all candidates select public finance.*

In such an equilibrium, an Advantaged candidate defeats a Disadvantaged opponent with probability $1/2$. If Advantaged candidates are in the minority, this is the best feasible scenario for the median voter in terms of policy. Notice that the condition for the pooling equilibrium to hold is equivalent to

$$\alpha \geq \frac{1}{2c_A p_D} - \frac{\ell}{p_D},$$

which, if $\alpha > 0$, is strictly weaker than the condition for Proposition 2 to hold. In other words, even a small amount of public finance can push a race just below the threshold for full concealment into this range.

However, that does not necessarily imply that public finance is universally beneficial from the standpoint of electing candidates closer to the median voter. In particular, unless we imagine that the availability of public finance can be set differently across races, we must check whether the addition of public finance gives Disadvantaged candidates a better chance when Advantaged candidates are in the majority. The previous proposition implies that if ℓ is great enough, there will indeed be a pooling equilibrium even when Advantaged candidates are in the majority. But what if ℓ is small? I find that the original equilibrium with Advantaged candidates in the majority holds up, as stated in the following proposition.

Proposition 6. *If $\alpha \leq 0$ and $\ell \leq 1/c_D - p_A\alpha$, there is an equilibrium of the game with public finance that is outcome-equivalent to the equilibrium in Proposition 1, with no candidate selecting public finance.*

Taken together, these two results imply that a small amount of public financing has asymmetric effects, just like marginal increases to c_A. When Advantaged candidates are in the minority, public financing can push a marginal case (one where Advantaged candidates partially separate, but with a high probability of concealment) toward total concealment, reducing the chance of electing such a candidate. However, when Advantaged candidates are in the majority, a bit of public financing need not threaten their electoral advantage.

5.2 Correlated Types

I now return to the original model, except changing the joint distribution of candidate types. If a candidate has a fundraising advantage because of connections to a special interest group, it could be relatively unlikely that the same group is also funding her opponent. As the simplest model of this possibility, called the *game with correlated types*, I assume the prior probability of each type is 1/2, and that the interim probability of facing a candidate the same type as oneself is $q \in (0, 1/2)$. 1 gives the joint distribution of types under this assumption.

	$t_2 = A$	$t_2 = D$
$t_1 = A$	$q/2$	$(1-q)/2$
$t_1 = D$	$(1-q)/2$	$q/2$

Table 1. Joint probability distribution of types in the game with correlated types.

In the limiting case $q = 1/2$, we have the original model with independent types and $p_A = p_D = 1/2$. In the other limiting case, $q = 0$, every election involves exactly one Advantaged and one Disadvantaged candidate.

As before, I focus on symmetric equilibria. Even so, the median voter's belief system is more complex in the original model. When a candidate learns her own type, she non-trivially updates her beliefs about her opponent's type. Therefore, a candidate's spending choice signals not only her own type, but also that of her opponent. So for any pair of spending choices (s_1, s_2), denote the median voter's beliefs

$$\mu_{AA}(s_1, s_2) = \Pr(t_1 = A, t_2 = A | s_1, s_2),$$

and so on for the other possible type pairings.

I focus on the case of Advantaged candidates in the majority, to examine under what conditions the (lack of) comparative statics on c_A for the probability of victory by an Advantaged candidate hold up. From inspection of Proposition 1, it is obvious that the equilibrium cannot take its original form if an Advantaged candidate is nearly certain to face a Disadvantaged opponent. Imagine an equilibrium where an Advantaged candidate defeats a Disadvantaged opponent for sure, as in the independent case. Then a Disadvantaged candidate, nearly certain her opponent is Advantaged, must spend nothing. This in turn means the Advantaged candidate's best response must be to spend barely more than nothing. Yet then it would be profitable for a Disadvantaged candidate to spend enough to beat an Advantaged opponent—meaning this cannot be an equilibrium. So if interdependence of types breaks the electoral dominance of Advantaged candidates, does it also break the insensitivity of that dominance to c_A?

The following propositions characterize an equilibrium across the range of $q \in (0, 1/2)$ when Advantaged candidates are in the majority. For sufficiently small violations of the independence condition, the basic form of the equilibrium remains as before. Disadvantaged and Advantaged types mix over separate intervals, and an Advantaged candidate always prevails over a Disadvantaged opponent. However, once the negative correlation between types is large, this no longer holds. Instead, there is an equilibrium in which Disadvantaged candidates mix over $[0, 1/c_D]$ and Advantaged candidates mix over some upper portion of this range. Advantaged candidates are still relatively likely to win, but no longer certain—and the probability of their victory decreases with c_A.

Proposition 7. *If $\alpha \leq 0$ and $c_A \leq c_D q/(1-q)$, there exists an equilibrium of the game with correlated types in which an Advantaged candidate defeats a Disadvantaged opponent with probability one.*

Proposition 8. *If $\alpha \leq 0$ and $c_A > c_D q/(1-q)$, there exists an equilibrium of the game with correlated types in which an Advantaged candidate defeats a Disadvantaged opponent with probability*

$$\frac{1}{2}\left(1 + \frac{c_D - c_A}{(1-q)c_D - qc_A}\right), \tag{18}$$

which is decreasing in c_A.

These results suggest some caution about assuming that an equalizing reform would not affect races where Advantaged candidates are in the majority, at least to the extent that we think candidate types might be interdependent. As types become more negatively correlated, Disadvantaged candidates focus more on remaining competitive against an Advantaged opponent. The concomitant increase in spending by Disadvantaged types makes it more expensive for Advantaged types to assure victory against a Disadvantaged opponent. If their marginal cost of fundraising is high enough, assured victory is not worth the cost. Through this mechanism of increased competition from Disadvantaged candidates, equalizing reforms now reduce the equilibrium probability of electing an Advantaged candidate.

6 Conclusion

I have shown how some common intuitions about money in politics—that candidates with an edge in fundraising get elected disproportionately, even if their ability to raise money signals a willingness to enact policies outside the public interest—can hold up in an environment with rational Bayesian voters. The results of the analysis support the idea that campaign finance reform can increase the chance of electing candidates whose policies will benefit the median voter, particularly reforms that equalize the marginal cost of fundraising across candidates.

There are numerous directions for future research. A natural next step would be to incorporate policy announcements into the model, like Ashworth and Bueno de Mesquita (2009) in the endogenous valence literature or Banks (1990) in the hidden policy intentions literature. If candidates make policy announcements before engaging in spending competition, these will affect not only the electoral incentives in the subsequent campaign (as in Ashworth and Bueno de Mesquita 2009) but also perhaps their beliefs about each other's ability to raise funds. In addition, if candidates are punished for lying, policy announcements might provide a way for financially disadvantaged candidates to separate themselves when advantaged opponents with extreme policy intentions would want to conceal their type.

Another avenue for future research would be to endogenize the interaction between candidates and special interests that determines a candidate's policy intentions and marginal cost of fundraising. An analysis along these lines could examine how the policy preferences and financial resources of groups determine the distribution of candidate types in the contest stage. For example, one could compare the distribution of candidate policy intentions and their ability to raise funds when there is a single extreme interest group to when there are two interest groups roughly equidistant from the median voter.

A Appendix

A.1 Properties of Equilibrium

Lemma 1. *Let (σ, μ) be an equilibrium. For each type $t \in T$, $Eu_t = U_t$ σ_t-almost everywhere.*

Proof. Take either type t, and suppose there exists a set $S \subseteq \mathbb{R}_+$ such that $\sigma_t(S) > 0$ and $U_t \neq Eu_t(s)$ for all $s \in S$. If $U_t < Eu_t(s')$ for some $s' \in S$, then it would be profitable for a candidate of type t to deviate to a strategy that places probability one on s', contradicting the assumption of equilibrium. But if $U_t > Eu_t(s)$ for all $s \in S$, then a candidate of type t could strictly benefit by deviating to a strategy that places probability zero on S, which again violates the assumption of equilibrium. □

Lemma 2. *Let (σ, μ) be an equilibrium. For each type $t \in T$ and each $s \in \operatorname{supp} \sigma_t$, if σ_t places positive probability on s or λ is continuous at s, then $Eu_t(s) = U_t$.*

Proof. Take either type t. The first part of the claim, concerning mass points of σ_t, is immediate from Lemma 1. To prove the second part, take any $s \in \operatorname{supp} \sigma_t$ such that λ is continuous at s, and suppose $Eu_t(s) < U_t$. It is apparent from (6), the definition of Eu_t, that continuity of λ at s implies continuity of Eu_t at s. Therefore, there exists $\epsilon > 0$ such that $Eu_t(s') < U_t$ for all s' in an ϵ-neighborhood of s, contradicting the indifference condition of equilibrium.[9] □

Lemma 3. *Let (σ, μ) be an equilibrium. For each type $t \in T$ and each $s \in \mathbb{R}_+$ such that $Eu_t(s) = U_t$,*

$$\begin{aligned} \lambda(s) &> \lambda(s') && \textit{for all } s' < s, \\ Eu_m(s) &> Eu_m(s') && \textit{for all } s' < s. \end{aligned} \tag{19}$$

[9] The same logic applies if λ is right-continuous at s, as long as $\sigma_t((s, s+\epsilon)) > 0$ for all $\epsilon > 0$.

Proof. Take either type t, and take any s such that $Eu_t(s) = U_t$. In equilibrium, then, we have $Eu_t(s) \geq Eu_t(s')$ for all s'. For $s' < s$, this implies $\lambda(s) > \lambda(s')$, proving the first claim. The second claim is then immediate from the fact that $\lambda(s) > \lambda(s')$ only if $Eu_m(s) > Eu_m(s')$. $\square$

Lemma 4. *In any equilibrium* (σ, μ)*,* $U_A \geq U_D$.

Proof. The assumption $c_A < c_D$ implies $Eu_A \geq Eu_D$. Therefore, the optimality condition of equilibrium gives

$$U_A = \max_{s \in \mathbb{R}_+} Eu_A(s) \geq \max_{s \in \mathbb{R}_+} Eu_D(s) = U_D. \qquad \square$$

Lemma 5. *In any equilibrium* (σ, μ)*,* $\max \operatorname{supp} \sigma_D \leq \hat{s} \leq \min \operatorname{supp} \sigma_A$*, where*

$$\hat{s} = \frac{U_A - U_D}{c_D - c_A}. \tag{20}$$

Proof. Let (σ, μ) be an equilibrium. For all $s > \hat{s}$,

$$(c_D - c_A)s > U_A - U_D \geq \lambda(s) - c_A s - U_D.$$

A rearrangement of terms yields

$$U_D > \lambda(s) - c_D s = Eu_D(s),$$

so a Disadvantaged candidate's mixed strategy may not place positive probability on $(\hat{s}, \infty)$. An analogous argument establishes that $\min \operatorname{supp} \sigma_A \geq \hat{s}$. $\square$

A.2 Equilibria Surviving D1

Lemma 6. *An equilibrium* (σ, μ) *survives the D1 refinement if and only if*

$$\begin{aligned} s < \hat{s} \quad &\Rightarrow \quad \mu(s) = 0, \\ s > \hat{s} \quad &\Rightarrow \quad \mu(s) = 1, \end{aligned}$$

for all $s \leq \max_{t \in T}\{(1 - U_t)/c_t\}$.

Proof. The claim is true for all $s \neq \hat{s}$ on the equilibrium path by Bayes' rule and Lemma 5, so now consider off-the-path values of s. As in Banks (1990) and Callander and Wilkie (2007), beliefs under D1 depend on the probability of victory that would be necessary to give a candidate an incentive to deviate to an off-the-path spending choice. For each $s \geq 0$ and each type t, let $q_t(s) = U_t + c_t s$ denote the minimal probability of victory that would give a candidate of type t a weak incentive to deviate to spending s. Notice that

$$\hat{s} - s = \frac{U_A - U_D - s(c_D - c_A)}{c_D - c_A} = \frac{q_A(s) - q_D(s)}{c_D - c_A}.$$

Therefore, if $\hat{s} < s \leq (1-U_A)/c_A$, then $q_A(s) < q_D(s)$ and $q_A(s) \leq 1$, so D1 requires that a deviation to s be ascribed to an Advantaged candidate. Similarly, if $s < \hat{s}$ and $s \leq (1-U_D)/c_D$, then $q_D(s) < q_A(s)$ and $q_D(s) \leq 1$, so D1 requires that a deviation to s be ascribed to a Disadvantaged candidate. Finally, if $s > \max_{t \in T}\{(1-U_t)/c_t\}$, then $q_A(s) > 1$ and $q_D(s) > 1$, so D1 places no restriction on beliefs. $\square$

Lemma 7. *Let (σ, μ) be an equilibrium that survives D1.*

(a) Neither type of candidate's strategy contains any mass points besides $\hat{s}$.

(b) If Advantaged candidates are in the majority, then neither type's strategy contains any mass points.

(c) If Advantaged candidates are in the minority and the Advantaged type's strategy places positive probability on $\hat{s}$, then $\hat{s} = 0$ and the Disadvantaged type spends 0 for certain.

Proof. I begin by proving a necessary intermediate claim, namely that no $s \geq (1-U_t)/c_t$ can be a mass point for a candidate of type t. Suppose not, so $\sigma_t(\{s'\}) > 0$ with $s' \geq (1-U_t)/c_t$. We then have, by Lemma 2,

$$U_t = \lambda(s') - c_t s' \leq \lambda(s') - (1-U_t).$$

Rearranging terms gives $\lambda(s') \geq 1$. This is a contradiction, since a candidate who spends s' has a positive probability of tying and thus cannot win the election for certain.

To prove claim (a), suppose some type t places probability $\pi > 0$ on $s' \neq \hat{s}$. Because $s' < (1-U_t)/c_t$ and $s' \neq \hat{s}$, we have from Lemma 6 that the electorate's beliefs are constant in an ϵ-neighborhood of s'. A candidate who spent $s \in (s', s'+\epsilon)$ would thus defeat any candidate who spent s'. Therefore, by spending infinitesimally more than s' and thereby defeating rather than tying those who spend s', a candidate would raise her chance of victory by at least $\pi p_t/2 > 0$. This contradicts the assumption of equilibrium.

To prove claim (b), suppose that Advantaged candidates are in the majority and that some type t places positive probability on $\hat{s}$. By Lemma 6, there exists $s' > \hat{s}$ such that $\mu(s) = 1$ for all $s \in (\hat{s}, s')$. This implies that Eu_m is strictly increasing on $[\hat{s}, s')$, as Advantaged candidates are in the majority, so any candidate who spent $s \in (\hat{s}, s')$ would defeat a candidate who spent $\hat{s}$. As in the proof of the last claim, a sufficiently small deviation would therefore be profitable, violating the assumption of equilibrium.

To prove claim (c), suppose that Advantaged candidates are in the minority and that their mixed strategy places positive probability on $\hat{s} > 0$. By Bayes' rule, then, the electorate's beliefs are $\mu(\hat{s}) > 0$. However, under D1, we have $\mu(s) = 0$ for all $s \in [0, \hat{s})$, per Lemma 6. Since Advantaged candidates are in the minority, this means there exists $s' < \hat{s}$ such that $Eu_m(s) > Eu_m(\hat{s})$ for all $s \in (s', \hat{s})$. We have $U_A = Eu_A(\hat{s})$, as Advantaged candidates spend $\hat{s}$ with positive probability, so this contradicts Lemma 3. $\square$

Lemma 8. *Let (σ, μ) be an equilibrium that survives D1. For each type $t \in T$, $\operatorname{supp} \sigma_t \setminus \{\hat{s}\}$ is convex.*

Proof. Take either type $t \in T$, and suppose $\operatorname{supp}\sigma_t \setminus \{\hat{s}\}$ is not convex, so there exist $s', s'' \in \operatorname{supp}\sigma_t \setminus \{\hat{s}\}$ such that $s' < s''$ and $\sigma_t((s', s'')) = 0$. Since $s' \neq \hat{s}$ and $s'' \neq \hat{s}$, neither of these is a mass point of σ_t, per Lemma 7(a). Therefore, there exists $\delta > 0$ such that $[s' - \delta, s'] \cup [s'', s'' + \delta] \subseteq \operatorname{supp}\sigma_t \setminus \{\hat{s}\}$. Let $S = [s' - \delta, s'' + \delta]$. By Lemma 6, the electorate's beliefs μ are constant on S, which in turn implies the median voter's expected payoff Eu_m is continuous and strictly increasing on S. Consequently, the set of $s \notin S$ such that $Eu_m(s) \in Eu_m(S)$ has $\tilde{\sigma}$-measure zero, per Lemma 3. Two implications follow from this claim. First, because there is not positive mass on any s such that $Eu_m(s) \in Eu_m(S)$, the probability of victory λ is continuous on S. Second, because the set of s such that $Eu_m(s) \in Eu_m((s', s''))$ has $\tilde{\sigma}$-measure zero, $\lambda(s') = \lambda(s'')$. We therefore have $Eu_t(s') > Eu_t(s'')$. But, by Lemma 2, continuity of λ implies $Eu_t(s') = Eu_t(s'') = U_t$, a contradiction. □

Lemma 9. *In any equilibrium (σ, μ) that survives D1, $Eu_m(\max\operatorname{supp}\sigma_D) = Eu_m(\min\operatorname{supp}\sigma_A)$.*

Proof. Let $\bar{s}_D = \max\operatorname{supp}\sigma_D$, and let $\underline{s}_A = \min\operatorname{supp}\sigma_A$. The claim is trivial if $\bar{s}_D = \underline{s}_A$, so suppose $\bar{s}_D < \underline{s}_A$ and $Eu_m(\bar{s}_D) \neq Eu_m(\underline{s}_A)$. The first step of the proof is to establish that $U_A = Eu_A(\underline{s}_A)$. We know that $\underline{s}_A$ is not a mass point of σ_A, as the only amount on which an Advantaged candidate's strategy may place positive mass is 0, by Lemma 7. Therefore, there exists $s' > \underline{s}_A$ such that $[\underline{s}_A, s'] \subseteq \operatorname{supp}\sigma_A$. By Bayes' rule, the electorate's beliefs are $\mu(s) = 1$ for all $s \in [\underline{s}_A, s']$, so Eu_m is continuous and strictly increasing on this interval. Since $\bar{s}_D$ is the only possible mass point of either type's strategy and $Eu_m(\bar{s}_D) \neq Eu_m(\underline{s}_A)$, this in turn implies λ is right-continuous at $\underline{s}_A$. Then, by Lemma 2 (see note 9), $U_A = Eu_A(\underline{s}_A)$.

We are now prepared to show that $Eu_m(\bar{s}_D) = Eu_m(\underline{s}_A)$. Suppose $Eu_m(\bar{s}_D) > Eu_m(\underline{s}_A)$. By definition, then, $\lambda(\bar{s}_D) \geq \lambda(\underline{s}_A)$. Since $\bar{s}_D < \underline{s}_A$ and $U_A = Eu_A(\underline{s}_A)$, this contradicts the optimality requirement of equilibrium, per Lemma 3. On the other hand, suppose $Eu_m(\bar{s}_D) < Eu_m(\underline{s}_A)$. If $\bar{s}_D$ is not a mass point of a Disadvantaged candidate's mixed strategy, then we have $\lambda(\bar{s}_D) = \lambda(\underline{s}_A)$, again contradicting the optimality requirement of equilibrium. Otherwise, we have $\hat{s} = \bar{s}_D$, so the electorate's beliefs μ are constant on $(\bar{s}_D, \underline{s}_A]$ under D1, per Lemma 6. The median voter's expected utility function Eu_m is thereby continuous and strictly increasing on $(\bar{s}_D, \underline{s}_A]$, so there exists $s'' \in (\bar{s}_D, \underline{s}_A)$ such that $Eu_m(s'') > Eu_m(\bar{s}_D)$. Since the set of s such that $Eu_m(s'') \leq Eu_m(s) \leq Eu_m(\underline{s}_A)$ has $\tilde{\sigma}$-measure zero, we have $\lambda(\underline{s}_A) = \lambda(s'')$, once again contradicting the optimality requirement of equilibrium. Consequently, we must have $Eu_m(\bar{s}_D) = Eu_m(\underline{s}_A)$. □

A.3 Advantaged Candidates in the Majority

Lemma 10. *If Advantaged candidates are in the majority, then in any equilibrium (σ, μ) that survives D1, the CDF of the Disadvantaged type's mixed strategy is given by* (11)*, and the CDF of the Advantaged type's mixed strategy is given by* (12).

Proof. Assume $\alpha \leq 0$, and let (σ, μ) be an equilibrium that survives D1. Since Advantaged candidates are in the majority, we have from Lemma 7(b) that neither type's strategy contains any mass points. Consequently, the support of each type t's mixed strategy is a closed interval, $\operatorname{supp}\sigma_t = [\underline{s}_t, \bar{s}_t]$, per Lemma 8. Moreover, we have $0 \leq \underline{s}_D < \bar{s}_D \leq \hat{s} \leq \underline{s}_A < \bar{s}_A$, per Lemma 5. Let $S = [0, \bar{s}_A]$. Because Advantaged candidates are in the majority, the median voter's expected utility Eu_m is strictly increasing on S. Then, since neither type's mixed strategy contains any mass points, the probability of victory function λ is continuous and non-decreasing on S. Therefore, each type's expected utility function Eu_t is continuous on $[0, \bar{s}_A]$, and we have $U_t = Eu_t(s)$ for all $s \in \operatorname{supp} \sigma_t$, per Lemma 2.

I begin by characterizing the Disadvantaged type's mixed strategy. First, since $\lambda(\underline{s}_D) = 0$ and $U_D = Eu_D(\underline{s}_D)$, we must have $\underline{s}_D = 0$, or else it would be profitable for Disadvantaged candidates to deviate to spending 0. It follows immediately that $U_D = 0$. Next, to derive the expression for Disadvantaged candidates' mixed strategy, take any $s \in \operatorname{supp} \sigma_D$. Because Eu_m is strictly increasing on S and neither type's strategy contains any mass points, we have $\lambda(s) = p_D F_D(s)$. Moreover, by continuity of Eu_D at s and the indifference condition of equilibrium, we have

$$Eu_D(s) = p_D F_D(s) - c_D s = 0 = Eu_D(0).$$

Rearranging terms gives $F_D(s) = c_D s / p_D$. Then, from $F_D(\bar{s}_D) = 1$, we yield $\bar{s}_D = p_D / c_D$. Therefore, the Disadvantaged type's mixed strategy must satisfy (11).

The characterization of the Advantaged type's mixed strategy is similar. Recall from Lemma 9 that $Eu_m(\bar{s}_D) = Eu_m(\underline{s}_A)$. As Eu_m is strictly increasing on S, this implies $\underline{s}_A = \bar{s}_D = p_D / c_D$. Now, to derive the expression for Advantaged candidates' mixed strategy, take any $s \in \operatorname{supp} \sigma_A$. We have $\lambda(s) = p_D + p_A F_A(s)$, again because Eu_m is strictly increasing on S and neither type's mixed strategy contains any mass points. By continuity of Eu_A at s and the indifference condition of equilibrium, we have

$$Eu_A(s) = p_D + p_A F_A(s) - c_A s = p_D - c_A \frac{p_D}{c_D} = Eu_A(\underline{s}_A).$$

Rearranging terms gives

$$F_A(s) = \frac{c_A}{p_A} \left[s - \frac{p_D}{c_D} \right].$$

Then, from $F_A(\bar{s}_A) = 1$, we yield $\bar{s}_A = p_D / c_D + p_A / c_A$. Therefore, the Advantaged type's mixed strategy must satisfy (12). $\square$

Proposition 1. *If Advantaged candidates are in the majority, then there is an equilibrium (σ^*, μ^*) that is essentially unique under D1 in which Disadvantaged candidates employ a mixed strategy whose CDF is*

$$F_D^*(s) = \begin{cases} 0 & s < 0, \\ c_D s / p_D & 0 \le s \le \bar{s}_D^*, \\ 1 & s > \bar{s}_D^*, \end{cases} \tag{11}$$

and Advantaged candidates employ a mixed strategy whose CDF is

$$F_A^*(s) = \begin{cases} 0 & s < \bar{s}_D^*, \\ c_A (s - \bar{s}_D^*) / p_A & \bar{s}_D^* \le s \le \bar{s}_A^*, \\ 1 & s > \bar{s}_A^*, \end{cases} \tag{12}$$

where $\bar{s}_D^* = p_D/c_D$ *and* $\bar{s}_A^* = \bar{s}_D^* + p_A/c_A$. *The electorate's beliefs are*

$$\mu^*(s) = \begin{cases} 0 & s < \bar{s}_D^*, \\ p_A & s = \bar{s}_D^*, \\ 1 & s > \bar{s}_D^*. \end{cases} \tag{13}$$

Proof. The first task is to confirm that (σ^*, μ^*) is an equilibrium, which requires confirming that there are no profitable deviations available and that the given beliefs are consistent with the candidates' strategies. Since Advantaged candidates are in the majority and μ^* is weakly increasing, the median voter's expected utility Eu_m is strictly increasing. Consequently, as neither type's strategy contains any mass points, the probability of victory by a candidate who spends s is $\lambda(s) = p_A F_A^*(s) + p_D F_D^*(s)$. Each type's expected utility is continuous in s regardless of the median voter's behavior when indifferent, so the choice of sharing rule is immaterial.

I begin by checking for profitable deviations. For every point in the support of a Disadvantaged type's mixed strategy, $s \in [0, \bar{s}_D^*]$, we have

$$Eu_D(s) = p_D F_D^*(s) - c_D s = 0,$$

confirming the indifference condition for Disadvantaged types. It is not profitable for a Disadvantaged candidate to mimic an Advantaged one, since for any $s \in (\bar{s}_D^*, \bar{s}_A^*]$ we have

$$Eu_D(s) = p_D + p_A F_A^*(s) - c_D s = (c_A - c_D)(s - \bar{s}_D^*) < 0.$$

Similarly, for an Advantaged candidate, for all $s \in [\bar{s}_D^*, \bar{s}_A^*]$, we have

$$Eu_A(s) = p_D + p_A F_A^*(s) - c_A s = p_D - c_A \bar{s}_D^*,$$

confirming the indifference condition for Advantaged types. It is not profitable for an Advantaged candidate to deviate to spending less, since for any $s < \bar{s}_D^*$ we have

$$Eu_A(s) = p_D F_D^*(s) - s = (c_D - c_A)s < p_D - c_A \bar{s}_D^*.$$

Finally, it is not profitable for either type to deviate to spending $s > \bar{s}_A^*$, since doing so yields the same chance of victory as spending $\bar{s}_A^*$ but at greater cost.

Next, I confirm that the electorate's beliefs are consistent with the application of Bayes' rule on the path of play and that the off-the-path beliefs survive D1. It is obvious that the on-the-path beliefs are consistent with Bayes' rule. Then, notice that the cutpoint for beliefs under D1 is $\hat{s} = (U_A - U_D)/(c_D - c_A) = p_D/c_D = \bar{s}_D^*$, so the equilibrium survives D1, per Lemma 6. The final claim of the proposition, that the equilibrium is essentially unique under D1, follows from Lemma 10. $\square$

A.4 Advantaged Candidates in the Minority

Lemma 11. *Let* (σ, μ) *be an equilibrium that survives D1. If Advantaged candidates are in the minority, then the Disadvantaged type's strategy places positive probability on* $\hat{s}$*, and there exists* $\delta > 0$ *such that neither type's strategy places positive probability on* $(\hat{s}, \hat{s} + \delta)$.

Proof. Suppose $\alpha > 0$, so Advantaged candidates are in the minority. I begin by showing that the Disadvantaged type's strategy places positive probability on $\hat{s}$. For the sake of contradiction, suppose not, so $\sigma_D(\{\hat{s}\}) = 0$. Since $\hat{s}$ is the only possible mass point of either type's strategy, per Lemma 7(a), this means σ_D contains no mass points. Moreover, since Advantaged candidates are in the minority, the lack of a mass point in σ_D implies there is none in σ_A either, per Lemma 7(c). Since neither type t's mixed strategy contains a mass point, the support of each is an interval $[\underline{s}_t, \bar{s}_t]$, per Lemma 8. From Lemma 5, we have $\bar{s}_D \leq \hat{s} \leq \underline{s}_A$.

To show that the lack of a mass point in σ_D yields a contradiction, there are two cases to consider. First, suppose $\bar{s}_D = \underline{s}_A = \hat{s}$. By Bayes' rule, $\mu(s) = 0$ for all $s \in [\underline{s}_D, \hat{s})$ and $\mu(s) = 1$ for all $s \in (\hat{s}, \bar{s}_A]$. Taking the left- and right-hand limits of the median voter's expected utility at $\hat{s}$ gives

$$\lim_{s \to \hat{s}^+} Eu_m(s) = \hat{s} - \alpha < \hat{s} = \lim_{s \to \hat{s}^-} Eu_m(s).$$

Therefore, it would be profitable for an Advantaged candidate to deviate to spending slightly less than $\hat{s}$, contradicting the assumption of equilibrium. Second, suppose $\bar{s}_D < \underline{s}_A$. In this case, we have $\mu(s) = 1$ for all $s \in [\underline{s}_A, \bar{s}_A]$. In addition, because neither type's mixed strategy contains any mass points, the probability of victory λ is continuous on this interval. Therefore, by Lemma 2, $U_A = Eu_A(\underline{s}_A)$. However, recall that $Eu_m(\underline{s}_A) = Eu_m(\bar{s}_D)$ in any equilibrium that survives D1, per Lemma 9. Since $\bar{s}_D < \underline{s}_A$, it would thus be profitable for the Advantaged type to deviate to spending $\bar{s}_D$, again contradicting the assumption of equilibrium. As both cases yield a contradiction, we can conclude that σ_D places positive mass on $\hat{s}$.

The last step is to prove that there exists $\delta > 0$ such that $(\hat{s}, \hat{s} + \delta)$ lies outside the support of both types' strategies. Since the Disadvantaged type's mixed strategy places positive mass on $\hat{s}$, Bayes' rule implies $\mu(\hat{s}) < 1$. Because the equilibrium survives D1, we have $\mu(s) = 1$ for all $s \in (\hat{s}, \bar{s}_A]$, per Lemma 6. Similar to before, taking the right-hand limit of the median voter's utility at $\hat{s}$ gives

$$\lim_{s \to \hat{s}^+} Eu_m(s) = \hat{s} - \alpha < \hat{s} - \mu(\hat{s})\alpha = Eu_m(\hat{s}).$$

Any candidate would be strictly better off spending $\hat{s}$ than any amount just above it, so there must exist $\delta > 0$ such that $\sigma_t((\hat{s}, \hat{s} + \delta)) = 0$ for each type t. □

Lemma 12. *Suppose $\alpha > p_D/2c_A$. In any equilibrium that survives D1,*

(a) Disadvantaged types spend 0 for certain.

(b) Advantaged types spend 0 with probability $\pi^* > 0$, where

$$\pi^* = \min\left\{\frac{\sqrt{2\alpha c_A p_D} - p_D}{p_A}, 1\right\}. \tag{21}$$

Proof. Throughout the proof, let π denote the probability that an Advantaged candidate spends 0, so $\pi = \sigma_A(\{0\})$.

To prove claim (a), it will suffice to show that $\pi > 0$, as then Lemma 7(c) gives $\sigma_D(\{0\}) = 1$. For a proof by contradiction, suppose there is an equilibrium (σ, μ) that survives D1 in which $\pi = 0$. We know from Lemma 7(c) that the Advantaged type's strategy cannot contain any mass point besides 0. Consequently, the support of the Advantaged type's strategy is an interval, $\operatorname{supp}\sigma_A = [\underline{s}_A, \bar{s}_A]$, per Lemma 8. On the other hand, since Advantaged candidates are in the minority, we have from Lemma 11 that the Disadvantaged type's strategy places positive mass on $\hat{s} < \underline{s}_A$. We then have from Lemma 9 that $Eu_m(\underline{s}_A) = Eu_m(\hat{s})$. Bayes' rule gives $\mu(\underline{s}_A) = 1$ and $\mu(\bar{s}_D) = 0$, so we have $\underline{s}_A = \hat{s} + \alpha$. Since the Advantaged type's strategy contains no mass points, $\lambda(s) \to p_D$ as $s \to \underline{s}_A$ from the right. In addition, because a candidate who spends $\hat{s}$ either defeats or ties any Disadvantaged opponent, $\lambda(\hat{s}) \geq p_D/2$. Combining these with the fact that $\alpha > p_D/2c_A$ gives

$$\begin{aligned}
U_A &= \lim_{s \to \underline{s}_A^+} Eu_A(s) \\
&= p_D - c_A(\hat{s} + \alpha) \\
&< \frac{p_D}{2} - c_A\hat{s} \\
&\leq Eu_A(\hat{s}).
\end{aligned}$$

Therefore, it would be profitable for the Advantaged type to deviate to spending $\hat{s}$, contradicting the assumption of equilibrium. We conclude that $\pi > 0$ in any equilibrium that survives D1, which in turn implies that Disadvantaged candidates spend 0 for certain.

Before moving on to the proof of claim (b), it is worth noting two implications of the results just derived. First, the electorate's beliefs about a candidate who spends 0 are

$$\mu(0) = \frac{\pi p_A}{\pi p_A + p_D}. \tag{22}$$

Second, we know from Lemma 3 that on the equilibrium path, a candidate who spends nothing has zero probability of defeating a candidate who spends more. Consequently, the chance of victory for a candidate who spends 0 is

$$\lambda(0) = \frac{1}{2}\left(\pi p_A + p_D\right). \tag{23}$$

The proof of claim (b) consists of two steps. First, assume $\alpha \geq 1/2c_A p_D$, so $\pi^* = 1$. For a proof by contradiction, suppose there is an equilibrium (σ, μ) that survives D1 in which $\pi < 1$. We can then write the support of the Advantaged type's strategy as $\operatorname{supp}\sigma_A = \{0\} \cup [\tilde{s}_A, \bar{s}_A]$, where $0 < \tilde{s}_A < \bar{s}_A$. Because off-the-path beliefs must satisfy D1, Lemma 6 gives $\mu(s) = 1$ for all $s \in (0, \bar{s}_A]$. Then, applying the same line of argument as in the proof of Lemma 9, we must have $Eu_m(\tilde{s}_A) = Eu_m(0)$, which implies $\tilde{s}_A = (1 - \mu(0))\alpha$. A candidate who spends slightly more than $\tilde{s}_A$ thereby defeats those who spend 0 rather than tying. Taking the limit of the Advantaged type's utility as it approaches $\tilde{s}_A$ from the right gives

$$\begin{aligned}\lim_{s\to\tilde{s}_A^+} Eu_A(s) &= \pi p_A + p_D - c_A \frac{p_D}{\pi p_A + p_D}\alpha \\ &\leq \pi p_A + p_D - \frac{1}{2}\frac{1}{\pi p_A + p_D} \\ &< \frac{1}{2}(\pi p_A + p_D) \\ &= Eu_A(0),\end{aligned}$$

where the last inequality holds because $\pi < 1$ implies $\pi p_A + p_D < 1 < 1/(\pi p_A + p_D)$. Therefore, an Advantaged candidate is better off spending 0 than any amount just above $\tilde{s}_A$, which contradicts the assumption of equilibrium. We conclude that $\pi = 1$ in any equilibrium surviving D1 if $\alpha \geq 1/2c_A p_D$.

Second, assume $p_D/2c_A < \alpha < 1/2c_A p_D$, so $\pi^* < 1$. For a proof by contradiction, suppose there is an equilibrium (σ, μ) that survives D1 in which $\pi = 1$. Because both candidates spend 0 for certain, we have $U_A = U_D = \lambda(0) = 1/2$ and $\mu(0) = p_A$. A candidate could assure victory by spending $s > p_D\alpha$, since

$$Eu_m(s) > p_D\alpha - \alpha = -p_A\alpha = Eu_m(0).$$

Therefore, for any $s \in (p_D\alpha, 1/2c_A)$ (where $p_D\alpha < 1/2c_A$ because $\alpha < 1/2c_A p_D$), we have

$$Eu_A(s) = 1 - c_A s > \frac{1}{2} = U_A,$$

contradicting the assumption of equilibrium. We conclude that $\pi < 1$ in any equilibrium surviving D1 if $p_D/2c_A < \alpha < 1/2c_A p_D$. Moreover, as in the previous part of the proof, we have $\operatorname{supp}\sigma_A = \{0\} \cup [\tilde{s}_A, \bar{s}_A]$, where $\tilde{s}_A = (1 - \mu(0))\alpha > 0$. Taking limits as the Advantaged type's utility approaches $\tilde{s}_A$ from the right, the indifference condition of equilibrium gives

$$\lim_{s\to\tilde{s}_A^+} Eu_A(s) = \pi p_A + p_D - c_A \frac{p_D}{\pi p_A + p_D}\alpha = \frac{1}{2}(\pi p_A + p_D) = Eu_A(0).$$

Multiplying both sides by $\pi p_A + p_D$ and rearranging terms gives $\pi = (\sqrt{2\alpha c_A p_D} - p_D)/p_A$, as claimed. $\square$

Proposition 2. *If $\alpha \geq 1/2c_A p_D$, then there is an equilibrium (σ^*, μ^*) that is essentially unique under D1 in which both types of candidates spend 0 for certain. The electorate's beliefs are $\mu^*(0) = p_A$ and $\mu^*(s) = 1$ for all $s > 0$.*

Proof. As in the proof of Proposition 1, we must confirm that there are no profitable deviations available and that the given beliefs are consistent with the candidates' strategies. In the proposed equilibrium, the election always ends in a tie with both candidates spending nothing, so each type's utility is $U_A = U_D = \lambda(0) = 1/2$. A candidate who spent $s \in (0, p_D\alpha)$ would lose to a candidate who spent 0, so neither type has an incentive to deviate to spending such an amount.

Given his beliefs, the median voter is indifferent between a candidate who spends 0 and one who spends $p_D\alpha$. Regardless of the sharing rule employed in case of indifference, a candidate who spent $s \geq p_D\alpha$ would receive a payoff of

$$Eu_t(s) \leq 1 - c_A s \leq 1 - c_A p_D \alpha \leq \frac{1}{2},$$

so such a deviation would also be unprofitable. It is obvious that the on-the-path beliefs (namely, $\mu(0) = p_A$) are consistent with Bayes' rule, so the proposed assessment is an equilibrium. In addition, the cutpoint for beliefs under D1 is $\hat{s} = (U_A - U_D)/(c_D - c_A) = 0$, so the equilibrium survives D1, per Lemma 6. Its essential uniqueness under D1 follows from Lemma 12. $\square$

Lemma 13. *If $p_D/2c_A < \alpha < 1/2c_A p_D$, then in any equilibrium that survives D1, the Disadvantaged type spends 0 for certain, and the CDF of the Advantaged type's mixed strategy is given by* (14).

Proof. Assume $p_D/2c_A < \alpha < 1/2c_A p_D$, and let (σ, μ) be an equilibrium that survives D1. We already have from Lemma 12 that Disadvantaged candidates spend 0 for certain and that Advantaged candidates spend 0 with probability $\pi^* = (\sqrt{2\alpha c_A p_D} - p_D)/p_A > 0$. In addition, using the same logic as in the final part of the proof of Lemma 12, we can derive that the support of the Advantaged type's strategy is $\{0\} \cup [\tilde{s}_A, \bar{s}_A]$, where

$$\tilde{s}_A = \frac{p_D}{\pi^* p_A + p_D}\alpha = \frac{1}{2c_A}(\pi^* p_A + p_D).$$

All that remains is to derive $\bar{s}_A$ and the probability distribution of the Advantaged type's strategy on $[\tilde{s}_A, \bar{s}_A]$. Under D1, we have from Lemma 6 that $\mu(s) = 1$ for all $s \in (0, \bar{s}_A]$, so the median voter's expected utility is strictly increasing on this interval. Because $Eu_m(\tilde{s}_A) = Eu_m(0)$, this implies $\lambda(s) = p_D + p_A F_A(s)$ for all $s \in (\tilde{s}_A, \bar{s}_A]$. Moreover, since neither candidate's strategy contains any mass points besides 0, the probability of victory λ is continuous on $(\tilde{s}_A, \bar{s}_A]$. By the indifference condition of equilibrium,

$$Eu_A(s) = p_D + p_A F_A(s) - c_A s = \frac{1}{2}(\pi^* p_A + p_D) = Eu_A(0)$$

for all $s \in (\tilde{s}_A, \bar{s}_A]$. A rearrangement of terms gives

$$\begin{aligned} F_A(s) &= \pi^* + \frac{1}{p_A}\left(c_A s - \frac{1}{2}(\pi^* p_A + p_D)\right) \\ &= \pi^* + \frac{c_A}{p_A}(s - \tilde{s}_A), \end{aligned}$$

as claimed. Finally, setting $F_A(\bar{s}_A) = 1$ gives $\bar{s}_A = \tilde{s}_A + p_A(1 - \pi^*)/c_A$, as claimed. $\square$

Proposition 3. *If $p_D/2c_A < \alpha < 1/2c_A p_D$, then there is an equilibrium (σ^*, μ^*) that is essentially unique under D1 in which Disadvantaged candidates spend 0 for certain and Advantaged candidates* *employ a mixed strategy whose CDF is*

$$F_A^*(s) = \begin{cases} 0 & s < 0, \\ \pi^* & 0 \le s \le \tilde{s}_A^*, \\ \pi^* + c_A(s - \tilde{s}_A^*)/p_A & \tilde{s}_A^* < s < \bar{s}_A^*, \\ 1 & s \ge \bar{s}_A^*, \end{cases} \tag{14}$$

where $\pi^* = (\sqrt{2\alpha c_A p_D} - p_D)/p_A$, $\tilde{s}_A^* = (\pi^* p_A + p_D)/2c_A$, *and* $\bar{s}_A^* = \tilde{s}_A^* + p_A(1-\pi^*)/c_A$. *The electorate's beliefs are* $\mu^*(0) = \pi^* p_A/(\pi^* p_A + p_D)$ *and* $\mu^*(s) = 1$ *for all* $s > 0$.

Proof. As in the proofs of the previous propositions, we must confirm that there are no profitable deviations available and that the given beliefs are consistent with the candidates' strategies. In the proposed equilibrium, a candidate who spends 0 ties with probability $\pi^* p_A + p_D$ and loses otherwise, so each type's utility is $U_A = U_D = \lambda(0) = (\pi^* p_A + p_D)/2$. A candidate who deviated to an off-the-path amount $s \in (0, \tilde{s}_A^*)$ would not defeat one who spent 0, as

$$Eu_m(s) = s - \alpha \le \tilde{s}_A^* - \alpha = -\frac{\pi^* p_A}{\pi^* p_A + p_D}\alpha = Eu_m(0),$$

so such a deviation cannot be profitable. The median voter is indifferent between a candidate who spends 0 and one who spends $\tilde{s}_A^*$. The only sharing rule that makes the candidates' expected utility functions upper semicontinuous in s is for the median voter to elect a candidate who spends $\tilde{s}_A^*$ over one who spends 0. Then, the probability of victory for a candidate who spends $s \in [\tilde{s}_A^*, \bar{s}_A^*]$ is $\lambda(s) = p_A F_A^*(s) + p_D$. The payoff to an Advantaged type for spending such an amount is

$$\begin{aligned} Eu_A(s) &= p_A F_A^*(s) + p_D - s \\ &= \pi^* p_A + p_D - c_A \tilde{s}_A^* \\ &= \frac{\pi^* p_A + p_D}{2} \\ &= U_A, \end{aligned}$$

confirming the indifference condition for the Advantaged types. This also proves that Disadvantaged types have no incentive to deviate to an amount in this range, since $Eu_A \ge Eu_D$. Finally, neither type has an incentive to deviate to spending $s > \bar{s}_A^*$, as doing so yields the same chance of victory as spending $\bar{s}_A^*$ at strictly greater cost. It is obvious that the beliefs are consistent with Bayes' rule for spending amounts on the path, $s \in \{0\} \cup [\tilde{s}_A^*, \bar{s}_A^*]$, so the proposed assessment is an equilibrium. In addition, the cutpoint for beliefs under D1 is $\hat{s} = (U_A - U_D)/(c_D - c_A) = 0$, so the equilibrium survives D1, per Lemma 6. Its essential uniqueness under D1 follows from Lemmas 12 and 13. □

Lemma 14. *If* $0 < \alpha < p_D/2c_A$, *then in any equilibrium that survives D1, the CDF of the Disadvantaged type's mixed strategy is given by* (15)*, and the CDF of the Advantaged type's mixed strategy is given by* (16).

Proof. Assume $0 < \alpha < p_D/2c_A$, and let (σ, μ) be an equilibrium that survives D1. We know from Lemma 11 that the Disadvantaged type's strategy places probability $\rho > 0$ on $\hat{s}$. If the Advantaged type's strategy contained a mass point, that would entail placing probability $\pi^* > 0$ on 0, as shown in the proof of Lemma 12. But $\alpha < p_D/2c_A$ implies $\pi^* < 0$, so σ_A must not have any mass points. We therefore have $\operatorname{supp} \sigma_A = [\underline{s}_A, \bar{s}_A]$, where $\bar{s}_A > \underline{s}_A > \hat{s}$. Bayes' rule then gives $\mu(\underline{s}_A) = 1$ and $\mu(\hat{s}) = 0$. The median voter must be indifferent between candidates spending $\underline{s}_A$ and $\hat{s}$, per Lemma 9, so we have $\underline{s}_A = \hat{s} + \alpha$.

I begin by ruling out the possibility that the Disadvantaged type employs a pure strategy. For a proof by contradiction, suppose $\rho = 1$. Then the chance of victory by a candidate who spends $\hat{s}$ is $p_D/2$, and the Disadvantaged type's equilibrium payoff is $U_D = p_D/2 - c_D\hat{s}$. Since $Eu_m(\underline{s}_A) = Eu_m(\hat{s})$, any candidate spending more than $\underline{s}_A$ defeats all Disadvantaged candidates. The Advantaged type's equilibrium utility is thus

$$U_A = \lim_{s \to \underline{s}_A^+} Eu_A(s) = p_D - c_A(\hat{s} + \alpha).$$

Substituting each type's equilibrium utility into (20), the definition of $\hat{s}$, gives

$$\begin{aligned}
\hat{s} &= \frac{U_A - U_D}{c_D - c_A} \\
&= \frac{p_D - c_A\hat{s} - c_A\alpha - p_D/2 + c_D\hat{s}}{c_S - c_A} \\
&> \frac{p_D - c_A\hat{s} - p_D + c_D\hat{s}}{c_S - c_A} \\
&= \hat{s},
\end{aligned}$$

where the inequality follows from $\alpha < p_D/2c_A$. This is a contradiction, so we conclude that $\rho < 1$.

Next, I characterize the Disadvantaged type's mixed strategy. Since the strategy places probability $\rho \in (0, 1)$ on $\hat{s}$, we may write its support as $\operatorname{supp} \sigma_D = [\underline{s}_D, \tilde{s}_D] \cup \{\hat{s}\}$, by Lemma 8. Under D1, we have $\mu(s) = 0$ for all $s \in [0, \hat{s}]$, by Lemma 6. Then, since the Advantaged type's strategy does not contain any mass points, the probability of victory λ is continuous on $[0, \hat{s})$. This implies $\underline{s}_D = 0$, as otherwise we have

$$U_D = Eu_D(\underline{s}_D) = -c_D\underline{s}_D < 0 = Eu_D(0),$$

contradicting the assumption of equilibrium. As a result, $U_D = Eu_D(0) = 0$. The Advantaged type's equilibrium utility is

$$U_A = \lim_{s \to \underline{s}_A^+} Eu_A(s) = p_D - c_A(\hat{s} + \alpha),$$

so (20), the definition of $\hat{s}$, gives

$$\hat{s} = \frac{U_A - U_D}{c_D - c_A} = \frac{p_D - c_A(\hat{s} + \alpha)}{c_D - c_A}.$$

Rearranging terms yields $\hat{s} = (p_D - c_A\alpha)/c_D$. Substituting this into the Disadvantaged type's expected utility from spending $\hat{s}$ gives

$$Eu_D(\hat{s}) = \left(1 - \frac{\rho}{2}\right)p_D - c_D\hat{s} = c_A\alpha - \frac{\rho p_D}{2}.$$

By the indifference condition of equilibrium, $Eu_D(\hat{s}) = Eu_D(0) = 0$, so the above implies $\rho = 2c_A\alpha/p_D$. (The conditions on α imply $0 < \rho < 1$, as required.) Because candidates spending $\hat{s}$ tie with positive probability, there is a discrete upward jump in the Disadvantaged type's expected utility at $\hat{s}$. Therefore, by the assumption of equilibrium, $\hat{s} > \tilde{s}_D$; otherwise, it would be profitable to deviate from spending just less than $\tilde{s}_D$. Consequently, for $s \in [0, \tilde{s}_D]$, the indifference condition of equilibrium gives

$$Eu_D(s) = p_D F_D(s) - c_D s = 0 = Eu_D(0),$$

and thereby $F_D(s) = c_D s/p_D$. Lastly, setting $F_D(\tilde{s}_D) = 1 - \rho$ gives $\tilde{s}_D = (p_D - 2c_A\alpha)/c_D$. We therefore yield (15) as the expression for F_D.

To conclude the proof, we must derive the CDF of the Advantaged type's mixed strategy. We already have that

$$\underline{s}_A = \hat{s} + \alpha = \frac{p_D + (c_D - c_A)\alpha}{c_D}$$

and that

$$U_A = \lim_{s \to \underline{s}_A^+} Eu_A(s) = p_D - c_A\underline{s}_A.$$

By continuity of λ on $(\underline{s}_A, \bar{s}_A]$ and the indifference condition of equilibrium, for $s \in (\underline{s}_A, \bar{s}_A]$ we have

$$Eu_A(s) = p_D + p_A F_A(s) - c_A s = p_D - c_A\underline{s}_A = U_A,$$

and therefore

$$F_A(s) = \frac{c_A(s - \underline{s}_A)}{p_A}.$$

Setting $F_A(\bar{s}_A) = 1$ gives

$$\bar{s}_A = \underline{s}_A + \frac{p_A}{c_A}.$$

We therefore yield equation (16) as the expression for F_A. □

Proposition 4. *If $0 < \alpha \le p_D/2c_A$, then there is an equilibrium (σ^*, μ^*) that survives D1 in which Disadvantaged candidates employ a mixed strategy whose CDF is*

$$F_D^*(s) = \begin{cases} 0 & s < 0, \\ c_D s/p_D & 0 \le s \le \tilde{s}_D^*, \\ c_D\tilde{s}_D^*/p_D & \tilde{s}_D^* < s < \bar{s}_D^*, \\ 1 & s \ge \bar{s}_D^*, \end{cases} \tag{15}$$

and Advantaged candidates employ a mixed strategy whose CDF is

$$F_A^*(s) = \begin{cases} 0 & s < \underline{s}_A^*, \\ c_A(s - \underline{s}_A^*)/p_A & \underline{s}_A^* \leq s \leq \bar{s}_A^*, \\ 1 & s > \bar{s}_A^*, \end{cases} \tag{16}$$

where $\tilde{s}_D^* = (p_D - 2c_A\alpha)/c_D$, $\bar{s}_D^* = (p_D - c_A\alpha)/c_D$, $\underline{s}_A^* = \bar{s}_D^* + \alpha$, *and* $\bar{s}_A^* = \underline{s}_A^* + p_A/c_A$. *The electorate's beliefs are* $\mu^*(s) = 0$ *for all* $s \leq \bar{s}_D^*$ *and* $\mu^*(s) = 1$ *for all* $s > \bar{s}_D^*$. *If* $0 < \alpha < p_D/2c_A$, *this equilibrium is essentially unique under D1.*

Proof. As in the proofs of the previous propositions, we must confirm that there are no profitable deviations available and that the given beliefs are consistent with the candidates' strategies. To begin, I will characterize the probability of victory for each potential level of spending. The median voter is indifferent between a candidate who spends $\bar{s}_D^*$ and one who spends $\underline{s}_A^*$, as

$$Eu_m(\bar{s}_D^*) = \bar{s}_D^* = \underline{s}_A^* - \alpha = Eu_m(\underline{s}_A^*).$$

The only sharing rule that makes the candidates' expected utility functions upper semicontinuous in s is for the median voter to elect a candidate who spends $\underline{s}_A^*$ over one who spends $\bar{s}_D^*$. We therefore have $\lambda(s) = p_D F_D^*(s)$ for all $s < \tilde{s}_D^*$ and $\lambda(s) = p_D + p_A F_A^*(s)$ for all $s \geq \underline{s}_A^*$. For the off-the-path values $s \in (\bar{s}_D^*, \underline{s}_A^*)$, we have $\lambda(s) = \lambda(\max\{0, s - \alpha\})$. Finally, because Disadvantaged candidates spend $\bar{s}_D^*$ with probability

$$\rho = 1 - F_D^*(\tilde{s}_D^*) = \frac{2c_A\alpha}{p_D},$$

we have $\lambda(\bar{s}_D) = p_D(1 - \rho/2) = p_D - c_A\alpha$.

To rule out a profitable deviation for Disadvantaged types, notice that their expected utility from spending $s \in [0, \tilde{s}_D^*]$ is

$$Eu_D(s) = p_D F_D^*(s) - c_D s = 0,$$

so their equilibrium payoff is $U_D = 0$. At the mass point $\bar{s}_D^*$, we have

$$Eu_D(\bar{s}_D^*) = (p_D - c_A\alpha) - c_D\bar{s}_D^* = 0,$$

confirming the Disadvantaged type's indifference condition. To mimic an Advantaged candidate by spending $s \in [\underline{s}_A^*, \bar{s}_A^*]$ would yield a payoff of

$$\begin{aligned} Eu_D(s) &= p_D + p_A F_A^*(s) - c_D s \\ &= p_D - c_A\underline{s}_A^* + (c_A - c_D)s \\ &< p_D - c_D\underline{s}_A^* \\ &= (c_A - c_D)\alpha \\ &< 0, \end{aligned}$$

and would thus be unprofitable. Finally, it is obviously unprofitable to deviate to any value $s \in (\tilde{s}_D^*, \bar{s}_D^*) \cup (\bar{s}_D^*, \underline{s}_A^*) \cup (\bar{s}_A^*, \infty)$, since for any such value it is possible to attain the same chance of victory at strictly less cost.

To rule out a profitable deviation for Advantaged types, notice that their expected utility from spending $s \in [\underline{s}_A^*, \bar{s}_A^*]$ is

$$Eu_A(s) = p_D + p_A F_A^*(s) - c_A s = p_D - c_A \underline{s}_A^*,$$

so their equilibrium payoff is

$$U_A = p_D - c_A \underline{s}_A^* = (c_D - c_A)\bar{s}_D^*.$$

To mimic a Disadvantaged candidate by spending $s \in [0, \tilde{s}_D^*]$ would yield a payoff of

$$\begin{aligned} Eu_A(s) &= p_D F_D^*(s) - c_A s \\ &= (c_D - c_A)s \\ &< (c_D - c_A)\bar{s}_D^* \\ &= U_A, \end{aligned}$$

so such a deviation would be unprofitable. Similarly, deviating to the mass point $\bar{s}_D^*$ would yield a payoff of

$$Eu_A(\bar{s}_D^*) = p_D - \alpha - c_A \bar{s}_D^* = (c_D - c_A)\bar{s}_D^* = U_A,$$

so it is also unprofitable. Finally, just as with Disadvantaged candidates, there cannot be an incentive for an Advantaged candidate to deviate to any $s \in (\tilde{s}_D^*, \bar{s}_D^*) \cup (\bar{s}_D^*, \underline{s}_A^*) \cup (\bar{s}_A^*, \infty)$.

Since the assessment is fully separating, it is obvious that the beliefs on the path are consistent with Bayes' rule, so the assessment is an equilibrium. To confirm that it survives D1, per Lemma 6, notice that the cutpoint for off-the-path beliefs is

$$\hat{s} = \frac{U_A - U_D}{c_D - c_A} = \frac{(c_D - c_A)\bar{s}_D^* - 0}{c_D - c_A} = \bar{s}_D^*.$$

Lastly, essential uniqueness when $\alpha < p_D/2c_A$ follows from Lemma 14. □

A.5 Extensions

Let $\mu(\emptyset)$ denote the electorate's updated belief about a candidate who chooses public finance, and let $Eu_m(\emptyset) = \ell - \mu(\emptyset)\alpha$ denote his utility from electing such a candidate.

Proposition 5. *If $\ell \geq 1/2c_A - p_D\alpha$, there is an equilibrium of the game with public finance in which all candidates select public finance.*

Proof. Suppose $\ell \geq 1/2c_A - p_D\alpha$, and consider the following assessment.

- Along the path of play, both candidates (regardless of type) choose public finance. The median voter infers that each candidate is Advantaged with probability p_A and randomizes uniformly between them.

- If either candidate deviates by foregoing public finance, the median voter and the other candidate infer she is Advantaged with probability one. Since $Eu_m(\emptyset) = \ell - p_A\alpha$, a deviant must spend $s' = \max\{0, \ell + p_D\alpha\}$ to make the median voter indifferent. The only sharing rule that averts an open-set problem in this subgame is for the median voter to elect a deviant who spends s' with probability one over a publicly financed opponent.

- In the subgame where one candidate deviates, she employs a pure strategy drawn from $\text{argmax}_{s\in\{0,s'\}} \left\{\mathbf{1}\{s = s'\} - c_{t_i} s\right\}$, which is sequentially rational by construction.

- In the subgame where both deviate, the median voter believes both are Advantaged for sure and consequently elects whichever spends most. Advantaged candidates mix uniformly over $[0, 1/c_A]$, which is sequentially rational given the median voter's strategy and the fact that each candidate believes the other is Advantaged (see Meirowitz 2008, Proposition 2). Consequently, the best response for a Disadvantaged candidate in this subgame is to spend nothing.

Under this assessment, $U_A = U_D = 1/2$. Consider a unilateral deviation by an Advantaged candidate. If the Advantaged type's strategy in the consequent subgame is to spend 0 and $0 < s'$, then she loses the election and receives a payoff of 0, so the deviation is not profitable. If her strategy is to spend s', thereby winning the election, her utility from the deviation is

$$\begin{aligned}
1 - c_A s' &\leq 1 - c_A(\ell + p_D\alpha) \\
&\leq 1 - c_A\left(\frac{1}{2c_A} - p_D\alpha + p_D\alpha\right) \\
&= \frac{1}{2} \\
&= U_A,
\end{aligned}$$

so the deviation is not profitable. Nor would such a deviation be profitable for a Disadvantaged type, whose marginal cost of fundraising is even greater. Therefore, since beliefs along the path of play are consistent with the application of Bayes' rule, this assessment is an equilibrium. □

Proposition 6. *If $\alpha \leq 0$ and $\ell \leq 1/c_D - p_A\alpha$, there is an equilibrium of the game with public finance that is outcome-equivalent to the equilibrium in Proposition 1, with no candidate selecting public finance.*

Proof. Suppose $\alpha \leq 0$ and $\ell \leq 1/c_D - p_A\alpha$, and consider the following assessment.

- Along the path of play, both candidates (regardless of type) forego public finance. After a candidate selects to forego public finance, the electorate and the other candidate infer that she is Advantaged with probability p_A. The candidates then employ the same spending strategies, and the electorate updates its beliefs according to the same system, as in Proposition 1. This constitutes an equilibrium of the subgame, per Proposition 1.

- If either candidate deviates by selecting public finance, the electorate infers she is Disadvantaged with probability one. The median voter's utility from such a deviant is $Eu_m(\emptyset) = \ell$. The sharing rule must be the same as in Proposition 5, with the median voter selecting the non-publicly funded candidate when indifferent.

- In the subgame where one candidate deviates to public financing, her opponent spends $s' = \max\{0, \ell + p_A\alpha\}$ regardless of type. The median voter infers that the non-deviant is Advantaged with probability p_A regardless of her spending choice. The median voter is thus indifferent; consequently, under the sharing rule above, the non-deviant wins the election. The non-deviant's payoff in this subgame is

$$\begin{aligned}
1 - c_{t_i}s' &\geq 1 - c_D(\ell + p_A\alpha) \\
&\geq 1 - c_D\left(\frac{1}{c_D} - p_A\alpha + p_A\alpha\right) \\
&\geq 0,
\end{aligned}$$

  so her choice of s' is sequentially rational.

- In the subgame where both candidates deviate to public financing, the median voter randomizes uniformly between them.

Since the strategies in each subgame are sequentially rational and the electorate's beliefs are consistent with the application of Bayes' rule whenever possible, all that remains is to confirm that neither candidate has an incentive to deviate to taking public financing. A candidate who does so loses the election for sure, receiving a payoff of zero. But we have $U_A \geq U_D = 0$ along the path of play, so such a deviation is not profitable for either type. □

Proposition 7. *If $\alpha \leq 0$ and $c_A \leq c_D q/(1-q)$, there exists an equilibrium of the game with correlated types in which an Advantaged candidate defeats a Disadvantaged opponent with probability one.*

Proof. Suppose $\alpha \leq 0$ and $c_A \leq c_D q/(1-q)$. I claim that the following assessment constitutes an equilibrium. The mixed strategy profile is given by the CDFs

$$F_D(s) = \begin{cases} 0 & s < 0, \\ c_D s/q & 0 \leq s \leq \bar{s}_D, \\ 1 & s > \bar{s}_D, \end{cases}$$

$$F_A(s) = \begin{cases} 0 & s < \bar{s}_D, \\ c_A(s - \bar{s}_D)/q & \bar{s}_D \leq s \leq \bar{s}_A, \\ 1 & s > \bar{s}_A, \end{cases}$$

where $\bar{s}_D = q/c_D$ and $\bar{s}_A = \bar{s}_D + q/c_A$. The electorate's updated beliefs are

$$\mu_{DD}(s_1, s_2) = \begin{cases} 1 & s_1 \leq \bar{s}_D, s_2 \leq \bar{s}_D, \\ 0 & \text{otherwise}, \end{cases}$$
$$\mu_{DA}(s_1, s_2) = \begin{cases} 1 & s_1 \leq \bar{s}_D, s_2 > \bar{s}_D, \\ 0 & \text{otherwise}, \end{cases}$$
$$\mu_{AD}(s_1, s_2) = \begin{cases} 1 & s_1 > \bar{s}_D, s_2 \leq \bar{s}_D, \\ 0 & \text{otherwise}, \end{cases}$$
$$\mu_{AA}(s_1, s_2) = 1 - \mu_{DD}(s_1, s_2) - \mu_{DA}(s_1, s_2) - \mu_{AD}(s_1, s_2).$$

Given these beliefs and the fact that $\alpha \leq 0$, the median voter is never indifferent between two candidates who spend different amounts; he always strictly prefers whichever spends more. Since neither type's mixed strategy contains a mass point, spending the same amount as one's opponent is a zero-probability event, so the choice of sharing rule for these cases is immaterial. It follows that in this assessment, an Advantaged candidate defeats a Disadvantaged opponent with probability one.

I begin by proving there are no profitable deviations for Disadvantaged candidates. For any $s \in [0, \bar{s}_D]$, we have

$$Eu_D(s) = qF_D(s) - c_D s = 0,$$

which confirms D's indifference condition and implies $U_D = 0$. For any $s \in (\bar{s}_D, \bar{s}_A]$, we have

$$Eu_D(s) = q + (1-q)F_A(s) - c_D s$$

and thus

$$Eu'_D(s) = \frac{(1-q)c_A}{q} - c_D \leq 0;$$

therefore, $Eu_D(s) \leq Eu_D(\bar{s}_D) = U_D$. Finally, it cannot be profitable to deviate to $s > \bar{s}_A$, since doing so yields the same probability of victory as spending $\bar{s}_A$ at strictly greater cost.

To prove that there are no profitable deviations for Advantaged candidates, first observe that for any $s \in [\bar{s}_D, \bar{s}_A]$,

$$Eu_A(s) = (1-q) + qF_A(s) - c_A s = 1 - q - c_A \bar{s}_D.$$

This confirms A's indifference condition and implies $U_A = 1 - q - c_A \bar{s}_D$. For any $s \in [0, \bar{s}_D)$, we have

$$Eu_A(s) = (1-q)F_D(s) - c_A s$$

and thus

$$Eu'_A(s) = \frac{(1-q)c_D}{q} - c_A.$$

From $q \leq 1/2$ we have $(1-q)/q \geq 1$ and thus $Eu'_A(s) \geq c_D - c_A > 0$; therefore, $Eu_A(s) < Eu_A(\bar{s}_D) = U_A$. Finally, as before, it cannot be profitable to deviate to $s > \bar{s}_A$.

Given the candidates' strategies, the median voter's beliefs are consistent with the application of Bayes' rule wherever possible. Therefore, the assessment constitutes an equilibrium. □

Proposition 8. *If $\alpha \leq 0$ and $c_A > c_D q/(1-q)$, there exists an equilibrium of the game with correlated types in which an Advantaged candidate defeats a Disadvantaged opponent with probability*

$$\frac{1}{2}\left(1+\frac{c_D - c_A}{(1-q)c_D - qc_A}\right), \tag{18}$$

which is decreasing in c_A.

Proof. Suppose $\alpha \leq 0$ and $c_A > c_D q/(1-q)$. I claim that the following assessment constitutes an equilibrium. The mixed strategy profile is given by the CDFs

$$F_D(s) = \begin{cases} 0 & s < 0, \\ c_D s/q & 0 \leq s \leq \underline{s}_A, \\ c_D \underline{s}_A/q + k_D(s-\underline{s}_A) & \underline{s}_A \leq s \leq \bar{s}, \\ 1 & s > \bar{s}, \end{cases}$$

$$F_A(s) = \begin{cases} 0 & s < \underline{s}_A, \\ k_A(s-\underline{s}_A) & \underline{s}_A \leq s \leq \bar{s}, \\ 1 & s > \bar{s}, \end{cases}$$

where

$$\begin{aligned} \underline{s}_A &= \frac{1-c_A/c_D}{(1-q)c_D/q - c_A}, \\ \bar{s} &= \frac{1}{c_D}, \\ k_A &= \frac{(1-q)c_D - qc_A}{1-2q}, \\ k_D &= \frac{(1-q)c_A - qc_D}{1-2q}. \end{aligned}$$

The electorate's updated beliefs are

$$\mu_{DD}(s_1,s_2) = \begin{cases} 1 & s_1 \leq \underline{s}_A, s_2 \leq \underline{s}_A, \\ (q-\Phi q)/(1-\Phi q) & s_1 \leq \underline{s}_A, s_2 > \underline{s}_A, \\ (q-\Phi q)/(1-\Phi q) & s_1 > \underline{s}_A, s_2 \leq \underline{s}_A, \\ (1-\Phi)^2 q/(2-2\Phi+\Phi^2 q) & s_1 > \underline{s}_A, s_2 > \underline{s}_A, \end{cases}$$

$$\mu_{DA}(s_1,s_2) = \begin{cases} 0 & s_2 \leq \underline{s}_A, \\ (1-q)/(1-\Phi q) & s_1 \leq \underline{s}_A, s_2 > \underline{s}_A, \\ (1-\Phi)(1-q)/(2-2\Phi+\Phi^2 q) & s_1 > \underline{s}_A, s_2 > \underline{s}_A, \end{cases}$$

$$\mu_{AD}(s_1,s_2)=\begin{cases}0 & s_1\le \underline{s}_A,\\ (1-q)/(1-\Phi q) & s_1>\underline{s}_A, s_2\le \underline{s}_A,\\ (1-\Phi)(1-q)/(2-2\Phi+\Phi^2 q) & s_1>\underline{s}_A, s_2>\underline{s}_A,\end{cases}$$

$$\mu_{AA}(s_1,s_2)=\begin{cases}q/(2-2\Phi+\Phi^2 q) & s_1>\underline{s}_A, s_2>\underline{s}_A,\\ 0 & \text{otherwise},\end{cases}$$

where $\Phi = F_D(\underline{s}_A)$. Given these beliefs and the fact that $\alpha \le 0$, the median voter is never indifferent between two candidates who spend different amounts; he always strictly prefers whichever spends more. (To confirm this, notice that μ_{AD} and μ_{AA} are increasing in s_1, and μ_{DA} and μ_{AA} are increasing in s_2.) Since neither type's mixed strategy contains a mass point, spending the same amount as one's opponent is a zero-probability event, so the choice of sharing rule for these cases is immaterial.

First I will confirm that there are no profitable deviations for Disadvantaged candidates. For $s \in [0, \underline{s}_A]$ we have

$$Eu_D(s) = qF_D(s) - c_D s = 0,$$

which implies $U_D = 0$ and confirms D's indifference across this range. For $s \in [\underline{s}_A, \bar{s}]$, we have

$$\begin{aligned}Eu_D(s) &= qF_D(s) + (1-q)F_A(s) - c_D s\\ &= c_D\underline{s}_A + [qk_D + (1-q)k_A](s-\underline{s}_A) - c_D s\\ &= c_D\underline{s}_A + c_D(s-\underline{s}_A) - c_D s\\ &= 0\\ &= U_D,\end{aligned}$$

confirming D's indifference condition for this range. It cannot be profitable for D to deviate to $s > \bar{s}$, since doing so yields the same probability of victory as spending $\bar{s}$ for strictly greater cost.

Next I will confirm that there are no profitable deviations for Advantaged candidates. For $s \in [\underline{s}_A, \bar{s}]$ we have

$$\begin{aligned}Eu_A(s) &= (1-q)F_D(s) + qF_A(s) - c_A s\\ &= \frac{(1-q)c_D\underline{s}_A}{q} + [(1-q)k_D + qk_A](s-\underline{s}_A) - c_A s\\ &= \frac{(1-q)c_D\underline{s}_A}{q} + c_A(s-\underline{s}_A) - c_A s\\ &= \left[\frac{(1-q)c_D}{q} - c_A\right]\underline{s}_A\\ &= 1 - \frac{c_A}{c_D},\end{aligned}$$

which implies $U_A = 1 - c_A/c_D$ and confirms A's indifference across this range. For $s \in [0, \underline{s}_A)$ we have

$$Eu_A'(s) = (1-q)F_D'(s) - c_A$$

$$
\begin{aligned}
&= \frac{(1-q)c_D}{q} - c_A \\
&\geq c_D - c_A \\
&> 0,
\end{aligned}
$$

so $Eu_A(s) < Eu_A(\underline{s}_A) = U_A$, meaning it is not profitable for A to deviate to such s. Finally, as before, it also cannot be profitable for A to deviate to $s > \bar{s}$.

For the median voter's beliefs, observe that

$$
\begin{aligned}
\Pr(s_i \leq \underline{s}_A \mid t_i = D) &= \Phi, \\
\Pr(s_i > \underline{s}_A \mid t_i = D) &= 1 - \Phi, \\
\Pr(s_i \leq \underline{s}_A \mid t_i = A) &= 0, \\
\Pr(s_i > \underline{s}_A \mid t_i = A) &= 1,
\end{aligned}
$$

where I denote $\Phi = F_D(\underline{s}_A)$. Moreover, s_1 and s_2 are conditionally independent given (t_1, t_2). Therefore, in case both candidates spend no more than $\underline{s}_A$, we have

$$
\Pr(t_1 = D, t_2 = D \mid s_1 \leq \underline{s}_A, s_2 \leq \underline{s}_A) = 1.
$$

In case 1 spends no more than $\underline{s}_A$ and 2 spends more, the electorate infers for sure that $t_1 = D$, so we have

$$
\begin{aligned}
\Pr(t_1 = D, t_2 = D \mid s_1 \leq \underline{s}_A, s_2 > \underline{s}_A) &= \frac{\Phi(1-\Phi)q/2}{\Phi(1-\Phi)q/2 + \Phi(1-q)/2} \\
&= \frac{q - \Phi q}{1 - \Phi q}, \\
\Pr(t_1 = D, t_2 = A \mid s_1 \leq \underline{s}_A, s_2 > \underline{s}_A) &= 1 - \Pr(t_1 = D, t_2 = D \mid s_1 \leq \underline{s}_A, s_2 > \underline{s}_A) \\
&= \frac{1-q}{1 - \Phi q}.
\end{aligned}
$$

Then, by symmetry,

$$
\begin{aligned}
\Pr(t_1 = D, t_2 = D \mid s_1 > \underline{s}_A, s_2 \leq \underline{s}_A) &= \frac{q - \Phi q}{1 - \Phi q}, \\
\Pr(t_1 = A, t_2 = D \mid s_1 > \underline{s}_A, s_2 \leq \underline{s}_A) &= \frac{1-q}{1 - \Phi q}.
\end{aligned}
$$

Finally, consider the case where both candidates spend more than $\underline{s}_A$. The probability that this occurs is

$$
\begin{aligned}
\Pr(s_1 > \underline{s}_A, s_2 > \underline{s}_A) &= (1-\Phi)^2 \Pr(t_1 = D, t_2 = D) \\
&\quad + (1-\Phi)[\Pr(t_1 = D, t_2 = A) + \Pr(t_1 = A, t_2 = D)] \\
&\quad + \Pr(t_1 = A, t_2 = A) \\
&= \frac{(1-\Phi)^2 q}{2} + 2(1-\Phi)\left[\frac{1-q}{2}\right] + \frac{q}{2}
\end{aligned}
$$

$$= 1 - \Phi + \frac{\Phi^2 q}{2}.$$

Therefore, we have

$$\Pr(t_1 = D, t_2 = D \,|\, s_1 > \underline{s}_A, s_2 > \underline{s}_A) = \frac{(1-\Phi)^2 q/2}{1-\Phi+\Phi^2 q/2},$$
$$\Pr(t_1 = D, t_2 = A \,|\, s_1 > \underline{s}_A, s_2 > \underline{s}_A) = \frac{(1-\Phi)(1-q)/2}{1-\Phi+\Phi^2 q/2},$$
$$\Pr(t_1 = A, t_2 = D \,|\, s_1 > \underline{s}_A, s_2 > \underline{s}_A) = \frac{(1-\Phi)(1-q)/2}{1-\Phi+\Phi^2 q/2},$$
$$\Pr(t_1 = A, t_2 = A \,|\, s_1 > \underline{s}_A, s_2 > \underline{s}_A) = \frac{q/2}{1-\Phi+\Phi^2 q/2}.$$

The given beliefs are consistent with these conditional probabilities.

I have confirmed that the given assessment is an equilibrium. In equilibrium, in an election between an Advantaged and a Disadvantaged candidate, the Advantaged candidate is guaranteed to win if D spends $s < \underline{s}_A$ and wins with probability $1/2$ if D spends $s \in [\underline{s}_A, \bar{s}]$. Therefore, the probability of victory by an Advantaged candidate is

$$\begin{aligned}\Phi + \frac{1-\Phi}{2} &= \frac{1+\Phi}{2} \\ &= \frac{1 + c_D \underline{s}_A / q}{2} \\ &= \frac{1}{2}\left(1 + \frac{c_D - c_A}{(1-q)c_D - q c_A}\right),\end{aligned}$$

as claimed in (18). Differentiating with respect to c_A confirms that this probability decreases with c_A. □

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