Signaling Policy Intentions in Fundraising Contests*

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Abstract

I develop a theory of fundraising competition between candidates whose ability to raise money depends on the policy they will enact if elected. Unlike in existing models of money in politics, there is no exchange of policy favors for donations; donors merely have better information than the electorate about candidates' policy intentions. Even in the absence of *quid pro quo* exchanges between candidates and donors, I find that fundraising competition may skew policy outcomes away from the center. High spending may signal policy extremity to the median voter, but candidates sympathetic to special interests either spend enough to overcome the bad signal or conceal their intentions by not raising money at all. Consequently, when extremists can raise money most easily, there is a tradeoff between information disclosure and policy representativeness: the chance of electing a centrist is highest when candidates' spending decisions do not reveal their policy intentions. A key policy implication of the model is that campaign finance reforms that raise the opportunity cost of fundraising, such as individual contribution caps, never reduce (and sometimes increase) the chance of electing a centrist, even when the fundraising advantage is associated with centrism rather than extremism.

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Electoral campaigns are expensive. In 2016, candidates for the United States Congress spent more than $4 billion on their campaigns (Center for Responsive Politics 2017). Political activists and public interest groups often identify the campaign finance system as a corrupting influence on policy outcomes. Reflecting these concerns, various studies in political economy have sought to explain how an implicit or explicit agreement between candidates and special interest–aligned donors might result in policy distortions, even when some or all of the electorate is aware of the *quid pro quo* (Austen-Smith 1987; Baron 1994; Grossman and Helpman 1996; Prat 2002*a*,*b*; Ashworth 2006).

Of course, well-financed candidates resist the characterization of campaign donations as payments for promises. For example, when asked at a forum sponsored by Charles and David Koch whether billionaires have too much influence on public policy, U.S. Senator Marco Rubio said, "I think I can speak for my colleagues when I say we run for office and people buy into our agenda. [...] Most of the people that support us support us because they agree with what we're doing, not because we agree with what they're doing" (ABC News 2015). The claim is that there is no *quid pro quo*—the policies come first and the donors come second. Similarly, when challenged by primary opponent Bernie Sanders on taking donations from Wall Street sources, U.S. presidential candidate Hillary Clinton said "there is no example" of a decision she made differently because of donor influence (CNN 2016).

If it were true that politicians do not trade policy favors for campaign donations, should popular concerns about money in politics dissipate? To answer this question, I model fundraising competition between candidates who are pre-committed to the policies they would enact once elected. These policy intentions are fixed and cannot be affected by donors or voters. What donors have over the public at large is an informational advantage: each candidate's donor pool knows her policy intentions, which means a candidate's ability to raise money is related to what she will do if elected. However, the electorate has incomplete information about the candidates' policy intentions, as in Banks (1990). Therefore, in addition to directly increasing voters' perceptions of a candidate, campaign spending shapes the electorate's beliefs about what a candidate would do if elected. I focus on the case in which special interests dominate donations—i.e., the candidates who can raise money more easily have relatively extreme policy intentions—but also consider the reverse.

Despite ruling out *quid pro quo* agreements and assuming a rational Bayesian electorate, I show that fundraising competition may lead to policy outcomes skewed away from the median voter's ideal point. Candidates with a fundraising advantage always have a weak (and sometimes strict) electoral advantage, even when the fundraising advantage is associated with relatively extreme policy intentions. This finding is surprising in light of a rational electorate—we might expect high spending to cause the median voter to revise his beliefs downward, as the voter infers that the candidate's policy intentions favor special interests. Indeed, that is exactly what happens in the model. But because fundraising is endogenous, a candidate will never lose the election because of raising too *much* money. Instead, one possibility is that special interest–backed candidates simply spend enough to make up the negative policy inference that the median voter draws, in which case they have a strict electoral advantage over centrist-leaning candidates. Otherwise, if this strategy is not profitable, fundraising-advantaged candidates mimic the low spending of their less-advantaged counterparts, giving them even odds in the election as the electorate learns nothing from spending choices. Therefore, even in an environment that reflects politicians' claims of no donor influence on policy positions, the strategic imperatives of campaign fundraising may lead to skewed policy outcomes.

As noted above, many previous studies in political economy have shown that money in politics may distort policy outcomes. This study shows how the same conclusion holds in a distinct strategic environment, in which candidates are pre-committed to a policy and therefore cannot make policy promises to special interests or voters. Beyond this broad similarity—and perhaps of more interest to political economy scholars—I show how this strategic setting leads to distinct implications about the shape of spending competition that emerges in equilibrium, the relationship between voter knowledge and electoral outcomes, and the consequences of campaign finance reform.

In models where candidates can make binding policy commitments (e.g., Austen-Smith 1987; Baron 1994), these announcements give them a smooth means to balance the demands of donors with the need to appeal to the median voter. In the environment I consider, by contrast, candidates cannot strike this balance and therefore face a discrete electoral penalty for revealing that their pre-existing policy intentions are far from the median voter's ideal point. As a result, if the candidates who can raise money more easily are far from the center, equilibrium spending behavior ends up at one of two extremes. When the fundraising advantage for these candidates is too small, or their distance from the median voter too large, the electoral equilibrium entails no spending by either candidate. In this case, a candidate would rather settle for a tied race and save herself the high cost of raising enough to compensate for the median voter's negative inference about her policy intentions. It is surprising that a no-spending equilibrium is sustainable, given the all-pay structure of electoral competition—usually each candidate would have an incentive to deviate to spending some small amount to defeat her opponent (Meirowitz 2008). In this setting, however, voters infer that a candidate who spends has relatively extreme policy intentions, making a deviation electorally unprofitable.[1]

On the other hand, if the fundraising advantage is large enough or the policy intentions of the candidates who have it are not too far off center, there is positive spending in equilibrium. It is still true here that the electorate infers from high spending that a candidate's policy intentions are relatively extreme. In this case, however, special interest–backed candidates simply spend enough to overcome the negative inference that the median voter draws on the policy dimension. The result is an electoral equilibrium with a discrete gap between the spending of candidates with relatively extreme policy intentions and those closer to the center. Candidates face a real cost for having to compete on spending instead of by moderating their policy platforms. Although the more extreme candidates have a greater electoral advantage in this case than when the equilibrium entails no spending, the cost of raising enough to make up the gap in the median voter's approval sometimes leaves them worse off in expected utility terms. Candidates

[1]As I discuss further below, off-path beliefs are subject to the D1 refinement (Banks and Sobel 1987) throughout the analysis.

with off-center policy intentions are best off when their fundraising advantage is too small to justify revealing themselves or so overwhelming that the electoral costs of doing so are trivial.

In both types of equilibrium, there is a point at which spending slightly more would actually decrease a candidate's chance of victory, due to the inference the median voter draws about the candidate's policy intentions. The idea that spending more could hurt one's chance of victory may appear inconsistent with the empirical literature finding a positive, though sometimes small, correlation between spending and vote share (Jacobson 1978; Gerber 1998; Erikson and Palfrey 2000). However, these counterproductive levels of spending are never observed on the equilibrium path, as candidates strictly prefer to spend less time fundraising if doing so yields the same or greater chance of victory. Therefore, among *observed* spending levels, more spending is always associated with a greater probability of victory.

The analysis also yields novel implications about the relationship between voter knowledge and electoral outcomes. In the model, all voters are rational Bayesians who update their beliefs about the candidates' policy intentions after observing their spending. Candidates who can raise funds more easily may nonetheless conceal that fact by mimicking the spending strategy of those who lack the fundraising advantage. Therefore, the level of voter information is endogenous. When candidates with relatively extreme policy intentions have an advantage in fundraising, the greatest chance of electing a centrist politician is the extreme case in which candidates never spend and the electorate ends up uninformed. The tension between voter information and the eventual policy outcome arises because a candidate cannot signal via spending that she is at a disadvantage in fundraising (and thus has centrist policy intentions). A candidate who has a hard time raising money has no option but to spend little; one who can raise money easily can spend freely or refrain, depending on what is electorally advantageous. If a candidate spends enough that the electorate infers her extremist policy intentions, it must be because she is more likely to win the election by doing so. Therefore, in equilibrium, a more informed electorate is more likely to elect a candidate with extreme policy intentions.

An important question for both scholars and policymakers is how campaign finance reform affects spending competition and electoral outcomes. I investigate the effect of campaign finance reforms that seek to reduce disparities in candidates' ability to raise funds, finding that these never decrease (and often strictly increase) the chance of electing a candidate with centrist policy intentions. When candidates with extreme policy intentions have the fundraising advantage, a decrease in the magnitude of that advantage effectively raises the cost of spending enough to win the median voter's support after revealing oneself as the more extreme type. At the margin, this pushes candidates with extreme policy intentions to conceal their types and tie the election instead of spending enough to win. The result is an increase in the probability of electing a candidate with centrist policy intentions. I also consider a clean-elections public finance scheme, in which candidates may forego all private fundraising in order to receive a free lump sum, and find that the effects are similar.

Importantly, the negative effect of campaign finance reform on the electoral fortunes of advantaged candidates does not carry over to the case in which candidates with more centrist policy intentions can raise money more easily. In this case, even if the fundraising advantage of centrist candidates is rather small, there is full separation in equilibrium: the electorate infers each candidate's policy intentions with certainty, and a candidate with centrist policy intentions always defeats an opponent with more extreme commitments. An equalizing reform in this case changes the distribution of campaign spending, but not the probability of electing a centrist. Altogether, the results of the analysis suggest that campaign finance reform can move policy outcomes toward the median voter's ideal point on average even if candidates cannot make *quid pro quo* agreements with interest groups—and even if those with centrist policy intentions have the fundraising advantage in the vast majority of races.

Related Literature

This paper contributes to a long tradition of research in political economy on the relationship between campaign finance, candidate competition, and policy choices. In this section, I compare the strategic environment I study to those of closely related analyses, documenting the key theoretical differences and how these lead to different conclusions. For more comprehensive reviews of the voluminous literature on money in politics, see Morton and Cameron (1992) and Dawood (2015).

In the canonical models of campaign finance and special interest influence, candidates can exchange policy favors for campaign donations. The exact structure of this exchange varies across models. In some, there is an explicit *quid pro quo*. Baron (1989) assumes candidates make a binding offer of favors for a chosen level of contributions, which the interest group may accept or reject. Grossman and Helpman (1996) instead place the initiative with interest groups, who commit to a level of contributions for each platform the candidate might choose. Eschewing these sequential approaches, Denzau and Munger (1986) model policy choices and campaign contributions as emerging from a general equilibrium in a market of politicians, special interests, and voters. In other models, there is no explicit bargaining between candidates and special interests, as candidates commit to platforms before donors choose how much to contribute (Austen-Smith 1987; Baron 1994; Cameron and Enelow 1992). Even in these settings, however, candidates' equilibrium expectations about donor responses shape their platform choices, causing their policies to diverge away from the median voter's ideal point.

The setting I consider is distinct from these canonical models insofar as there is no mechanism, direct or indirect, by which special interests can change the policy a candidate would enact if elected. In the language of Kartik and McAfee (2007), all candidates in this model have character.[2] At most, special interests have an informational advantage over the electorate at large, reflected in the relationship between a candidate's policy intentions and her ability to fundraise. The policy distortions that occur in equilibrium are attributable to this informational advantage, not to *quid pro quo* exchanges. That said, what constitutes a distortion in this environment is different than in models where candidates can bind themselves to policy platforms. In the canonical models, the natural baseline is Downsian convergence to the median voter's

[2]Consequently, unlike in Kartik and McAfee's analysis, there is no electoral bonus for character in itself.

ideal point. In the present analysis, with candidates who have policy pre-commitments that are private information, a candidate with extreme leanings would sometimes be elected even if spending were banned. Even accounting for this, I find distortionary effects of money in politics in an environment without *quid pro quo* transactions.

As noted in the introduction, this analysis also differs with foundational models of campaign finance in its treatment of voter information. Both Baron (1994) and Grossman and Helpman (1996) model separate classes of informed and uninformed voters, where this distinction is exogenous. Informed voters care only about a candidate's policy platform, whereas uninformed voters care only about a candidate's spending. In these models, platforms become more extreme in equilibrium as the proportion of uninformed voters increases, making spending more important. By contrast, in the model I present below, all voters care about both policy and spending. Whether voters are informed is endogenous; as noted above, in equilibrium voter information is positively associated with the chance of electing an extremist.[3] That said, the electoral advantage for special interest–backed candidates decreases with the weight voters place on policy, reflecting the results in Baron (1994) and Grossman and Helpman (1996).

This paper also draws from and innovates on the literature concerned with campaign spending as a signal. As Stratmann (2003) asks, if high spending signals one's association with special interests, should we not expect voters to punish candidates who spend highly? Prat (2002*a*,*b*) and Wittman (2007) each assume candidates have an exogenous valence that is observable to special interests but not voters. In equilibrium, special interests select on valence when choosing whom to donate to, so campaign spending allows a candidate to signal her high valence. Doing so is electorally worthwhile even though the effects are partially offset by voters' inference that the candidate has promised policy favors to donors. In similar analyses, Coate (2004) and Ashworth (2006) do not assume special interests directly observe candidate valence, but that advertisements cannot transmit false information about quality. Once again,

[3] Campaign spending also technically plays an informational role in the model of Austen-Smith (1987), as it reduces voters' uncertainty about candidates' positions. However, as voters are risk-averse and each candidate's average position is common knowledge, the effect is that spending always raises all voters' estimation of the candidate, unlike in the context I consider.

my analysis is distinct from these signaling models insofar as I rule out candidates making policy promises to special interests. In addition, candidates do not have a fixed valence that advertising allows them to signal to the electorate—spending always directly raises voter perceptions, while indirectly sending a positive or negative signal about the candidate's policy intentions depending on which type of candidate can raise money more easily.

Another innovation over the existing signaling literature is that I incorporate a considerably richer environment of spending competition, drawn from the literature on endogenous valence. Meirowitz (2008) establishes the workhorse model of campaign spending as an all-pay contest (Baye, Kovenock and de Vries 1996), in which each candidate exerts costly effort to raise her standing in the eyes of voters. I extend Meirowitz's model to give candidates privately known policy intentions, which in turn are correlated with their marginal cost of exerting effort to raise campaign funds. Perhaps surprisingly, this extension yields a cleaner comparative static on a key question of public interest—the effects of campaign finance reform. Whereas Meirowitz finds that the electoral effects of an equalizing reform are ambiguous, I find that such a reform never decreases the chance of electing the more centrist type of candidate.

Further research on endogenous valence has focused on the interaction between candidates' policy platforms and their choice of effort (Wiseman 2006; Zakharov 2008; Ashworth and Bueno de Mesquita 2009; Serra 2010; Morton and Myerson 2012). A common finding in this literature is that more spending is associated with more polarization: as the marginal effectiveness of spending increases, the equilibrium platforms move further from the median in order to reduce the costs associated with fierce fundraising competition. In the environment I consider, with exogenous private policy intentions, two parameters plausibly capture polarization—the policy distance between the two types of candidate and the prior probability of the more extreme type. Interestingly, these do not always have the same effect on the overall probability of electing an extremist when such types have an advantage in fundraising. As the policy distance increases, so does the cost of making up for the median voter's inference that one is an extremist, pushing the more extreme types to conceal themselves and tie the election rather than outspend and win. On the other hand, as the prior probability of an extreme candidate approaches one, the probability that such candidates conceal their types in equilibrium goes to zero. However, short of this limit, the marginal effect of the prior probability on equilibrium concealment may be positive or negative.

The Model

There is an election in which two candidates, labeled 1 and 2, are competing for support from a set N consisting of n voters (n odd).[4] The game consists of two stages. In the first stage, each candidate chooses how much to spend, $s_i \geq 0$. In the second stage, each voter selects a candidate, $v_j \in \{1, 2\}$.

Candidates build valence by spending, but it is costly to do so. A candidate's cost of spending depends on her type $t_i \in T = \{A, D\}$, where the type names stand for *A*dvantaged and *D*isadvantaged. It is easier for Advantaged candidates to spend than it is for Disadvantaged candidates: the marginal cost of spending is c_A for Advantaged candidates and $c_D > c_A$ for Disadvantaged candidates. A candidate's type also determines the policy she will implement if elected, denoted x_{t_i}, which lies in the policy space $X \subseteq \mathbb{R}$. Candidate types are drawn independently by Nature at the start of the game. It is common knowledge that the prior probability each candidate is Advantaged is $p_A \in (0, 1)$, with $p_D = 1 - p_A$ denoting the prior probability that a candidate is Disadvantaged. However, the realization of each candidate's type is private information, unknown to the other candidate and to the electorate. Therefore, a candidate cannot base her spending decisions on the other candidate's type, and voters must infer candidates' policy intentions from their spending decisions.

Each candidate's objective is to win office. The expected utility function for a candidate, informally expressed, is

$$Eu_i(s_i \mid t_i) = \Pr(i \text{ wins} \mid s_i) - c_{t_i} s_i. \tag{1}$$

[4] Generic candidates are labeled i and take female pronouns; generic voters are labeled j and take male pronouns.

Although candidates may have distinct policy intentions, their payoff is not a function of the winning candidate's policy choice—they are office-motivated, not policy-motivated (see Calvert 1985). This framework draws from existing theories of candidates who have privately known policy intentions but are office-motivated (Banks 1990; Callander and Wilkie 2007).[5]

The game is symmetric from the perspective of the two candidates, who have identical type distributions, action spaces, and utility functions.[6] To simplify the analysis, I restrict attention to symmetric strategy profiles, in which the spending strategy of type t of candidate 1 is the same as that of type t of candidate 2. A mixed strategy profile is hence a pair $\sigma = (\sigma_A, \sigma_D)$, where each σ_t is a probability measure with support on $\mathbb{R}_+$. For any mixed strategy σ_t, let F_t denote the corresponding cumulative distribution function, and let $\operatorname{supp} \sigma_t$ denote its support.

Voters' preferences depend on the candidates' valence and policy intentions. All voters value valence identically, but each voter has a distinct policy ideal point $x_j \in X$. A voter's payoff from candidate i winning, given that she spent s_i, is

$$u_j(i \mid s_i) = s_i - \beta |x_j - x_{t_i}|, \tag{2}$$

where $\beta > 0$ is the relative weight voters place on policy. Because candidates' types are private information, voters do not know their exact payoff from a particular candidate winning. Instead, voters must base their expectations on observable information—namely, the amount a candidate spends. As the candidates do not know each others' types when making their spending decisions, the inference a voter makes about each candidate is solely a function of that candidate's spending. For every $s \geq 0$, let $\mu(s)$ denote the electorate's belief that a candidate who spends s is Advantaged.[7] A voter's expected utility from victory by a candidate who spends s

[5] Sawaki (2017) employs a similar informational framework, but with mixed motivations for the candidates.

[6] The results of the analysis would be the same if there were a known Left and Right candidate, as long as the policy intention of the Advantaged (resp. Disadvantaged) type of Left candidate were the same distance from the median voter's ideal point as that of the Advantaged (resp. Disadvantaged) type of Right candidate. I thank an anonymous reviewer for highlighting this.

[7] This notation reflects the implicit assumption that (1) all voters use the same updating rule and (2) the rule is the same for both candidates, even for spending decisions off the equilibrium path.

is

$$Eu_j(s) = s - \beta\mu(s)|x_j - x_A| - \beta(1-\mu(s))|x_j - x_D|. \tag{3}$$

As the number of voters is odd, there is a median voter $m \in N$; without loss of generality, I assume $x_m = 0$. The policy distance from the median voter to an Advantaged candidate is thus $|x_A|$, while that to a Disadvantaged candidate is $|x_D|$. Let $\alpha = \beta(|x_A| - |x_D|)$ denote the difference in distance from the median across types, weighted by the relative importance of policy.[8] We can thus write the median voter's expected utility from a candidate who spends s as[9]

$$Eu_m(s) = s - \mu(s)\alpha. \tag{4}$$

I say Advantaged candidates are centrist if $\alpha \leq 0$ and non-centrist, or extremist, if $\alpha > 0$. Comparative statics on α reflect the effects of the policy difference between the two types of candidate and the relative importance of policy to the electorate.

As usual in models of electoral competition, there are numerous trivial equilibria in the voting stage in which no voter is pivotal. To eliminate these, I employ the usual assumption that no voter employs a weakly dominated strategy (e.g., Besley and Coate 1997). As voters have single-peaked preferences and identical belief systems, this assumption implies that the median voter's preference determines electoral outcomes (see Groseclose 2001, Appendix C). Let the median voter's strategy be $\xi : \mathbb{R}_+^2 \to [0,1]$, where $\xi(s,s')$ denotes his probability of electing a candidate who spends s over one who spends s'. In equilibrium, $Eu_m(s) > Eu_m(s')$ implies $\xi(s,s') = 1$. In line with the restriction to symmetric strategy profiles, I assume $\xi(s,s) = 1/2$ for all $s \in \mathbb{R}_+$. For cases in which $s \neq s'$ but $Eu_m(s) = Eu_m(s')$, I let the median voter's choice rule be determined endogenously (Simon and Zame 1990). Typically this results in the median voter, when indifferent, choosing the candidate who spends more.[10]

[8]The baseline model of Meirowitz (2008) is the limiting case of this one where $\alpha = 0$ and $c_D = c_A$.

[9]For ease of exposition, Equation 4 drops the constant $-\beta|x_D|$ that results when $x_j = 0$ is substituted into Equation 3.

[10]In the main model, ties between candidates spending different amounts occur with probability zero, so the choice of sharing rule is not critical.

A candidate's expected utility for spending a particular amount depends on her type and the median voter's strategy. Under the strategy profile σ, the *ex ante* distribution of an arbitrary candidate's spending s_i is given by the probability measure $\tilde{\sigma} = p_A\sigma_A + p_D\sigma_D$. The *ex ante* probability of victory for a candidate who spends s is therefore

$$\lambda(s) = \int \xi(s, s_i)\, d\tilde{\sigma}(s_i). \tag{5}$$

The expected utility of spending s for a candidate of type t, holding fixed her opponent's mixed strategy and the strategies of the voters, is therefore

$$Eu_t(s) = \lambda(s) - c_t s. \tag{6}$$

The *ex ante* expected utility of a candidate of type t under the mixed strategy profile σ is

$$U_t = \int Eu_t(s)\, d\sigma_t(s). \tag{7}$$

This is a two-stage game of incomplete information with a finite type space and an infinite action space, so I employ a solution concept akin to perfect Bayesian equilibrium (Fudenberg and Tirole 1991, pp. 331–333). An assessment (σ, μ) is an *equilibrium* if it meets the following requirements. First, the median voter's strategy is a best response given his beliefs. Second, each candidate's strategy is a best response given her type, the strategy of the other candidate, and the median voter's strategy. Under the restriction to symmetric strategy profiles, the optimality requirement is satisfied if and only if

$$U_t \geq Eu_t(s) \qquad \text{for all } t \in T \text{ and } s \in \mathbb{R}_+. \tag{8}$$

An important consequence of this requirement is the indifference condition of mixed-strategy equilibrium, which is that a candidate of type t must be indifferent across almost all spending choices in the support of her mixed strategy, σ_t. Third, the electorate's beliefs must be updated in accordance with Bayes' rule whenever possible. Specifically, I use the following criteria for consistency. If s is a mass point of either type's mixed strategy, the electorate applies Bayes' rule:

$$\mu(s) = \frac{p_A \sigma_A(\{s\})}{p_A \sigma_A(\{s\}) + p_D \sigma_D(\{s\})}. \tag{9}$$

If $s \in \operatorname{supp} \sigma_A \setminus \operatorname{supp} \sigma_D$, then $\mu(s) = 1$. Similarly, if $s \in \operatorname{supp} \sigma_D \setminus \operatorname{supp} \sigma_A$, then $\mu(s) = 0$. If $s \in \operatorname{supp} \sigma_A \cap \operatorname{supp} \sigma_D$ but is not a mass point of either, the electorate's belief is the weighted ratio of densities if they exist:

$$\mu(s) = \frac{p_A F_A'(s)}{p_A F_A'(s) + p_D F_D'(s)}, \tag{10}$$

where F_t is the cumulative distribution function corresponding to σ_t. I place no restriction in case $s \in \operatorname{supp} \sigma_A \cap \operatorname{supp} \sigma_D$ but $F_A'(s)$ or $F_D'(s)$ fails to exist.[11] Finally, beliefs for $s \notin \operatorname{supp} \sigma_A \cup \operatorname{supp} \sigma_D$ are unrestricted.

To rule out equilibria that are supported by implausible off-the-path beliefs, I employ the D1 refinement (Banks and Sobel 1987). D1 requires, in essence, that voters ascribe any off-the-path spending choice to the type of candidate that could potentially benefit most from making it. Other models of electoral competition with privately known policy intentions use similar refinements (e.g., Banks 1990; Callander and Wilkie 2007). For this model, the restriction that D1 places on beliefs is reasonable. I show in the Appendix that in any equilibrium, there is a cutpoint $\hat{s}$ such that no Disadvantaged candidate spends more than $\hat{s}$ and no Advantaged candidate spends less (Lemma 5). D1 imposes the natural requirement that off-the-path deviations below this cutpoint be ascribed to Disadvantaged candidates, and those above it to Advantaged candidates (Lemma 6). I say that an equilibrium (σ, μ) is *essentially unique* under

[11] The lack of restriction is immaterial in the baseline model. Only when Advantaged candidates are centrist (Proposition 1) does the equilibrium under D1 entail overlap of the types' mixed strategies on a non-mass point. In that case, the median voter's expected utility would still be strictly increasing in s regardless of his beliefs at the overlap point, implying the same best responses for the candidates.

D1 if any other equilibrium that survives D1 entails the same spending strategies.

Results

In this section, I characterize an equilibrium of the model across each region of the parameter space. In each case, the equilibrium described is essentially unique under D1; others may exist, but they involve implausible conjectures about off-the-path beliefs (i.e., ascribing high spending to Disadvantaged types).

The form of the equilibrium depends primarily on whether Advantaged candidates are centrist or non-centrist. In the former case, good things go together: the beneficial signal sent to the median voter by high spending complements the positive direct effect of spending. In equilibrium, Advantaged candidates always spend enough to separate themselves and defeat any Disadvantaged rival. The form of the equilibrium is more complicated when Advantaged candidates are non-centrist. In this case, the signaling effect of high spending offsets the direct effect, reducing the median voter's opinion of a candidate. Depending on how much it costs to overcome the median voter's aversion to a non-centrist candidate, the equilibrium entails no spending by either type, positive spending by both types, or a mixture in which Disadvantaged candidates never spend and Advantaged candidates mix between no spending and high spending.

General Remarks

I begin with some general observations about the shape of the equilibrium, regardless of whether Advantaged candidates are centrist or non-centrist. These are stated formally and proved in a sequence of lemmas in the Appendix. The first observation concerns the relationship between a candidate's spending and her electoral fortunes.

Remark 1. *On the equilibrium path, spending more implies a greater probability of victory.*

This result holds because campaign spending is costly to the candidate. Because candidates are office-motivated, a candidate would never incur an additional cost of fundraising if there were not an electoral payoff. However, we should be careful in how we interpret this result. It refers only to spending decisions that are made on the equilibrium path—not to all possible counterfactual spending decisions. In particular, when Advantaged candidates are non-centrist, it is conceivable that spending more might lead the median voter to update his beliefs negatively about the candidate's policy intentions, making him less likely to vote for that candidate. But no candidate would ever choose to do so, and hence any negative effect of greater spending is purely counterfactual, not observed on the equilibrium path.

The second observation is that it is better to be Advantaged at fundraising, even if Advantaged candidates' policy intentions are relatively far from the median voter's ideal point.

Remark 2. *In equilibrium, Advantaged types are weakly better off:* $U_A \geq U_D$.

It is obvious why this is true when centrists are Advantaged, as then there is no downside to exercising the fundraising advantage. What is more curious is that there is no utility disadvantage even when Advantaged candidates are very far on policy from the median voter (i.e., α is very high). What drives the result is that Advantaged candidates always have the option of mimicking their Disadvantaged counterparts. A candidate may, in essence, pretend to be Disadvantaged by following the spending strategy of Disadvantaged candidates. Doing so yields the same probability of victory at the same (or lower) cost of fundraising, and thus $U_A \geq U_D$. In other words, an Advantaged candidate can choose to reveal herself only when doing so is beneficial; otherwise, she may conceal her type by not exploiting her fundraising advantage.

The next baseline result is that Advantaged candidates spend at least as much as Disadvantaged candidates.

Remark 3. *In equilibrium, Disadvantaged (Advantaged) types never spend more (less) than* $\hat{s} = (U_A - U_D)/(c_D - c_A)$.

This result follows from the positive relationship between spending and the probability of victory summarized in Remark 1. Because $c_A < c_D$, if the increase in the chance of victory due to spending s outweighs the cost for a Disadvantaged candidate, then it must also outweigh the cost for an Advantaged candidate. In equilibrium, therefore, no Advantaged candidate will spend less than the most that a Disadvantaged candidate is willing to spend. The two types' mixed strategies may overlap at one point at most, namely $\hat{s}$. On the equilibrium path, if a candidate spends any amount besides $\hat{s}$, voters infer her type with certainty.

Two features of $\hat{s}$, the cutpoint identified in Remark 3, merit additional attention. First, as I show in the Appendix, this is also the cutpoint for off-the-path beliefs under the D1 refinement. In order for an equilibrium to survive D1, an off-the-path deviation must be attributed to a Disadvantaged type if it is below $\hat{s}$ and to an Advantaged type if it is above $\hat{s}$. This restriction is natural in light of Remark 3. Second, the value of the cutpoint depends on the equilibrium payoffs. This means it cannot be used to solve for equilibrium strategies, though the result is useful in demonstrating uniqueness.

The final baseline result is that a fundraising advantage confers an electoral advantage. Under no circumstances are winners disproportionately Disadvantaged, even when Advantaged candidates' policy intentions are rather extreme.

Remark 4. *In equilibrium, the probability that the winner is Advantaged is no less than the prior probability p_A.*

The logic of Remark 4 follows from the previous remarks. We have seen that greater spending is associated with a greater chance of victory along the equilibrium path (Remark 1) and that Advantaged candidates spend more in equilibrium (Remark 3). It then follows that Advantaged candidates have a greater chance of winning. Again, it is surprising that this result holds even if Advantaged candidates are non-centrist, meaning high spending sends a negative policy signal to the median voter. The key, as before, is that Advantaged candidates have the option to conceal their type by mimicking the spending strategy of Disadvantaged types. If candidates' types were exogenously revealed before spending decisions were made, then there would be parameters of the model under which Advantaged candidates were electorally disadvantaged (see Meirowitz 2008).

This final remark also illuminates the connection between electoral strategies and policy outcomes. When Advantaged candidates are centrist, the policy outcome disproportionately favors the median voter—the winner is always a centrist unless both candidates happen to be extreme. However, Remark 4 shows that this bias toward centrism is impossible when Advantaged candidates are non-centrist. In this case, the most centrist feasible policy outcome is for the distribution among winners to equal the prior distribution—i.e., what would happen if the winner were selected at random. This reflects the dilemma between endogenous disclosure and policy outcomes, wherein non-centrists are more likely to be elected the more voters know about the candidates' policy intentions.

Equilibrium with Advantaged Centrists

I begin with the case in which $\alpha \leq 0$, meaning Advantaged candidates' policy intentions are relatively close to the median voter's ideal point. Because candidates' spending decisions signal their policy intentions, the fundraising advantage is doubly potent in this case. Spending a high amount has both the direct benefit of raising one's perceived valence and the indirect benefit of signaling one's centrist policy intentions. Naturally, then, in equilibrium Advantaged candidates always spend enough to separate themselves from Disadvantaged types.

The separating equilibrium in this case entails both types of candidate mixing over a continuum. The logic of mixed strategies here is the same as in Meirowitz (2008), as well as all-pay contests more generally (Baye, Kovenock and de Vries 1996). If the equilibrium strategy of Advantaged candidates were to spend s for certain, then it would be profitable for Advantaged candidates to deviate to $s+\epsilon$ for some small $\epsilon > 0$, so as to defeat rather than tie an Advantaged opponent. An analogous consideration applies to Disadvantaged candidates. As I show below, the same logic does not necessarily apply when Advantaged candidates are non-centrist, as then spending slightly more might shift the median voter's assessment downward due to the policy inference.

The following proposition formally characterizes the equilibrium, which entails Disadvantaged candidates mixing uniformly over $[0, p_D/c_D]$ and Advantaged candidates mixing uniformly over $[p_D/c_D, p_D/c_D + p_A/c_A]$. Figure 1 illustrates the equilibrium.

Proposition 1. *If Advantaged candidates are centrist, then there is an equilibrium (σ^*, μ^*) that is essentially unique under D1 in which Disadvantaged candidates employ a mixed strategy whose CDF is*

$$F_D^*(s) = \begin{cases} 0 & s < 0, \\ c_D s/p_D & 0 \leq s \leq \bar{s}_D^*, \\ 1 & s > \bar{s}_D^*, \end{cases} \tag{11}$$

and Advantaged candidates employ a mixed strategy whose CDF is

$$F_A^*(s) = \begin{cases} 0 & s < \bar{s}_D^*, \\ c_A(s - \bar{s}_D^*)/p_A & \bar{s}_D^* \leq s \leq \bar{s}_A^*, \\ 1 & s > \bar{s}_A^*, \end{cases} \tag{12}$$

where $\bar{s}_D^* = p_D/c_D$ and $\bar{s}_A^* = \bar{s}_D^* + p_A/c_A$. The electorate's beliefs are

$$\mu^*(s) = \begin{cases} 0 & s < \bar{s}_D^*, \\ p_A & s = \bar{s}_D^*, \\ 1 & s > \bar{s}_D^*. \end{cases} \tag{13}$$

The most striking feature of the equilibrium is that in an election between one candidate of each type, the Advantaged candidate is sure to prevail.[12] Therefore, in terms of representing

[12] A tie between an Advantaged candidate and a Disadvantaged one is possible if both candidates spend p_D/c_D, but this is a zero-probability event.

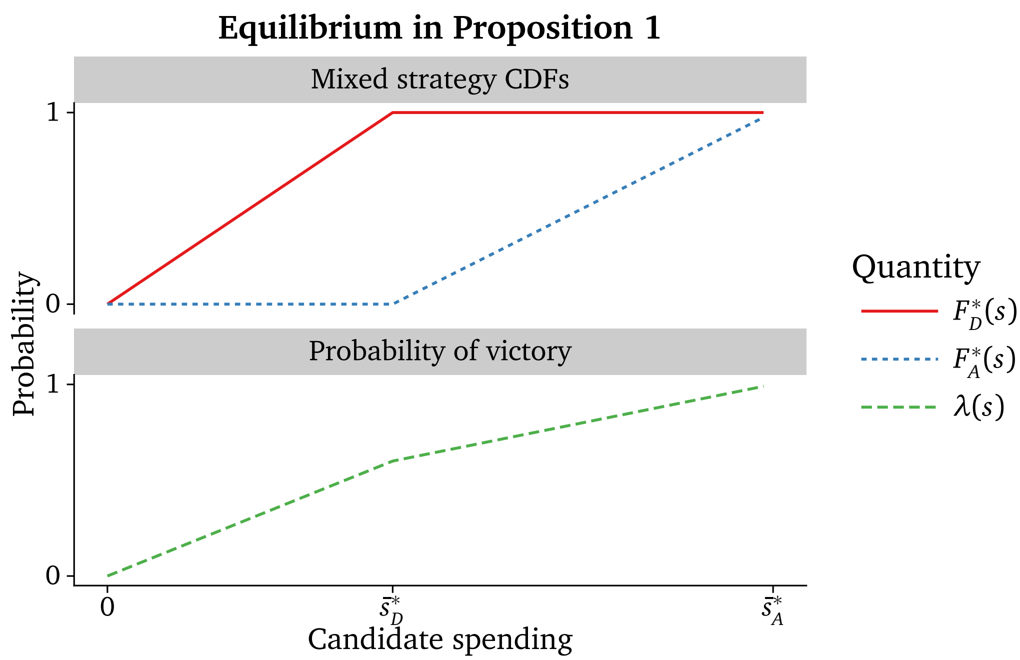


Figure 1. CDFs and probability of victory by spending in equilibrium when Advantaged candidates are centrist, as given in Proposition 1.

the median voter's policy preferences, the electoral outcome of this equilibrium is a first-best. Because the signaling effect of high spending works in favor of Advantaged candidates when they are centrist, in equilibrium they exploit that advantage to its fullest.

An important feature of the equilibrium is that the marginal electoral effect of spending is lower for Advantaged than for Disadvantaged candidates. This follows from the typical equilibrium condition that marginal benefit equal marginal cost. As there is no gap between the spending strategies of Advantaged and Disadvantaged candidates, if the marginal benefit of spending were greater for Advantaged candidates, then it would be profitable for Disadvantaged candidates to deviate to spending more and defeating Advantaged candidates with positive probability.

Another interesting property of the equilibrium when Advantaged candidates are centrist is that the strategies do not depend on α, the magnitude of the policy difference between types of candidates. The exact shape of the strategies depends only on the relative frequency of each type of candidate (p_A and p_D) and their respective marginal costs of fundraising (c_A and c_D).

In fact, the equilibrium takes the form described in Proposition 1 even if there is no policy difference between types ($\alpha = 0$). From the candidates' perspective as they make their spending decisions, the magnitude of the policy difference is only important to the non-centrist type. If a candidate reveals herself as non-centrist, she must spend α more than the other type to equalize herself in the eyes of the median voter. But in this case, where Advantaged candidates are centrist, this is a price that their Disadvantaged counterparts are unwilling to pay, as we have just seen. By contrast, the magnitude of the policy difference plays a critical role in the analysis below of equilibrium with Advantaged non-centrists.

Equilibrium with Advantaged Non-Centrists

I now characterize spending equilibria in case $\alpha > 0$, meaning candidates with a fundraising advantage have relatively extreme policy intentions. The key strategic question here is whether Advantaged candidates conceal their type by spending little, or spend enough to overcome the loss of revealing that they will enact non-centrist policies. Whether (and how often) they conceal their type so depends on the magnitude of α, the difference between Advantaged and Disadvantaged candidates in their policy distance from the median voter. The farther Advantaged candidates are from the median voter relative to their Disadvantaged counterparts, the more likely they are to refrain from spending.

To see why the magnitude of the policy distance determines the likelihood of concealment, imagine a pooling equilibrium in which all types spend the same amount, $\tilde{s}$. This represents the limiting case of total concealment by Advantaged types.[13] The median voter, updating his beliefs according to Bayes' rule, infers that a candidate who spends $\tilde{s}$ is Advantaged with probability p_A, same as the prior. The median voter's utility from electing a candidate who spends $\tilde{s}$ is therefore $Eu_m(\tilde{s}) = \tilde{s} - p_A\alpha$, per Equation 4. The candidates' strategies form an equilibrium only if it is not profitable for either type to deviate to spending a different amount. Under the D1 restriction on off-the-path beliefs, any deviation to $s > \tilde{s}$ will be ascribed to an

[13] Remark 3 implies that any fully pooling equilibrium must be in pure strategies.

Advantaged type, giving the median voter a payoff of $Eu_m(s) = s - \alpha$. A candidate who makes such a deviation will win the election only if $Eu_m(s) \geq Eu_m(\tilde{s})$, which is equivalent to

$$s \geq \tilde{s} + (1 - p_A)\alpha = \tilde{s} + p_D\alpha.$$

In other words, a candidate must spend an extra $p_D\alpha$ in order to increase her chance of victory by $1/2$ (as the imagined equilibrium always ends in a tie). The deviation is unprofitable for an Advantaged candidate only if the cost of the additional fundraising exceeds the electoral benefit: $c_A p_D \alpha \geq 1/2$. Therefore, in order for Advantaged candidates to fully conceal their types in equilibrium, the policy difference between Advantaged types and the median voter must be sufficiently large: $\alpha \geq 1/2c_A p_D$.

If this condition holds and there is a pooling equilibrium, it must entail zero spending. Imagine that both types pool on $\tilde{s} > 0$, and consider a deviation to $s = \tilde{s} - \epsilon$ for some small $\epsilon > 0$. Under the D1 restriction on off-the-path beliefs, a low deviation like this one is ascribed to a Disadvantaged type. The deviation reduces the valence component of the median voter's utility from electing the candidate by ϵ but increases the policy component by $p_A\alpha$. For any $\epsilon < p_A\alpha$, then, a candidate who deviated would win the election rather than tying. Because the deviation yields a better electoral outcome at lower cost, it is profitable for either type of candidate. Consequently, the only possible pooling equilibrium entails $\tilde{s} = 0$, in which case there is no downward deviation of this type available.

The foregoing arguments have shown that total concealment by Advantaged types is possible only if $\alpha \geq 1/2c_A p_D$ and that this kind of pooling equilibrium, if it exists, involves zero spending. In fact, if the condition holds, this is the unique equilibrium outcome under D1, as the following proposition states.

Proposition 2. *If $\alpha \geq 1/2c_A p_D$, then there is an equilibrium (σ^*, μ^*) that is essentially unique under D1 in which both types of candidates spend 0 for certain. The electorate's beliefs are $\mu^*(0) = p_A$ and $\mu^*(s) = 1$ for all $s > 0$.*

When the policy distance between Advantaged types and the median voter is not great enough for there to be total concealment in equilibrium, there may still be partial concealment. In an equilibrium with partial concealment, Disadvantaged candidates spend nothing, and Advantaged candidates employ a mixed strategy. With a certain probability, call it π, an Advantaged type spends nothing, in essence mimicking the Disadvantaged type. On the other hand, with probability $1-\pi$, an Advantaged type separates herself by spending enough to defeat candidates who spend nothing. Under such an equilibrium, the median voter infers that a candidate who spends 0 is Advantaged with probability $\mu(0) = \pi p_A/(\pi p_A + p_D)$, making his expected utility from electing such a candidate

$$Eu_m(0) = -\frac{\pi p_A}{\pi p_A + p_D}\alpha.$$

As before, any spending $s > 0$ is attributed to an Advantaged candidate under D1, resulting in $Eu_m(s) = s - \alpha$. Therefore, an Advantaged candidate who spends a positive amount must spend

$$s \geq \frac{p_D}{\pi p_A + p_D}\alpha,$$

or else she would be better off electorally spending nothing.

As explained above in the case of Advantaged centrists, any equilibrium with positive spending must entail a mixture over a continuum of spending choices. The following proposition gives the exact form of the mixture in this case, and the equilibrium is illustrated in Figure 2.

Proposition 3. *If $p_D/2c_A < \alpha < 1/2c_A p_D$, then there is an equilibrium (σ^*, μ^*) that is essentially unique under D1 in which Disadvantaged candidates spend 0 for certain and Advantaged candidates*

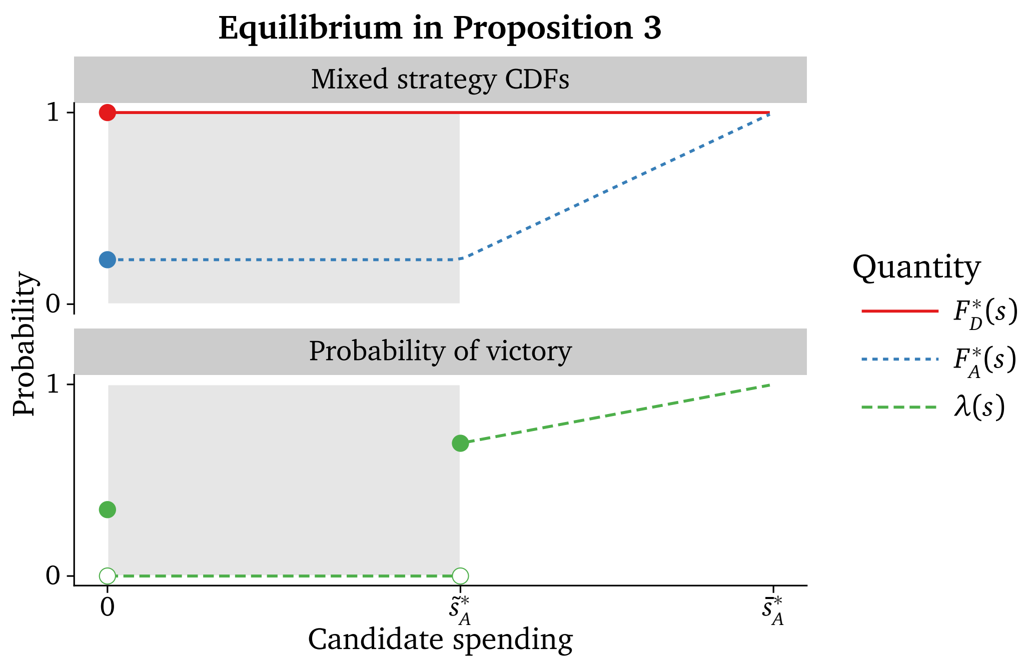


Figure 2. CDFs and probability of victory by spending in an equilibrium with partial concealment when non-centrists are Advantaged, as given in Proposition 3. Shaded regions represent spending levels off the equilibrium path.

employ a mixed strategy whose CDF is

$$F_A^*(s) = \begin{cases} 0 & s < 0, \\ \pi^* & 0 \leq s \leq \tilde{s}_A^*, \\ \pi^* + c_A(s - \tilde{s}_A^*)/p_A & \tilde{s}_A^* < s < \bar{s}_A^*, \\ 1 & s \geq \bar{s}_A^*, \end{cases} \tag{14}$$

where $\pi^* = (\sqrt{2\alpha c_A p_D} - p_D)/p_A$*,* $\tilde{s}_A^* = (\pi^* p_A + p_D)/2c_A$*, and* $\bar{s}_A^* = \tilde{s}_A^* + p_A(1 - \pi^*)/c_A$*. The electorate's beliefs are* $\mu^*(0) = \pi^* p_A/(\pi^* p_A + p_D)$ *and* $\mu^*(s) = 1$ *for all* $s > 0$*.*

Unlike in the previous case of full concealment, here the election does not always end in a tie. Consider an election between one Advantaged candidate and one Disadvantaged candidate. With probability

$$\pi^* = \frac{\sqrt{2\alpha c_A p_D} - p_D}{p_A},$$

the Advantaged candidate spends zero, concealing her type, and the election ends in a tie. Otherwise, with probability $1-\pi^*$, the Advantaged candidate spends more, revealing her type. Because she spends enough to overcome the median voter's downward shift in policy utility, in this contingency the election ends in victory for the Advantaged candidate.

Some important comparative statics emerge from the above expression for the probability of concealment. First, the chance that an Advantaged candidate conceals her type increases with α, the difference between Advantaged and Disadvantaged types' policy distance from the median voter. In fact, it is easy to check that this probability approaches one at the top of the range on α for which partial concealment is an equilibrium ($\alpha \to 1/2c_A p_D$) and zero at the bottom ($\alpha \to p_D/2c_A$). Second, despite also being associated with policy polarization in the candidate pool, the prior probability of an extremist, p_A, may decrease the chance of concealment. Specifically, as p_A approaches one, π^* approaches zero; there is little incentive for extremists to conceal their type when there is little chance of facing a centrist opponent. However, the marginal effect of p_A on the chance of concealment may be positive at values short of this limit, depending on the specific parameters. Finally, the probability of concealment increases with c_A, the marginal cost of fundraising for an Advantaged type. The more effort that an Advantaged type must expend to raise the same amount of funds, the less she will be willing to pay the extra cost it takes to defeat rather than tie an opponent who spends nothing.

An interesting feature of the partial concealment equilibrium is that the probability of victory as a function of spending is non-monotonic. Figure 2 shows that $\lambda(s)$ jumps downward between 0 and $\tilde{s}_A^*$, the lower bound of the positive portion of Advantaged types' mixed strategy. Because all positive spending is attributed to Advantaged candidates, the median voter would rather elect a candidate who spends nothing—who has a chance of being Disadvantaged and thus would implement policies strictly closer to his ideal point—than one who spends a bit more. This non-monotonicity does not contradict Remark 1, however, as the values where the probability dips are off the equilibrium path. Although it would be possible for a candidate to win less by spending more, such outcomes are strictly counterfactual.

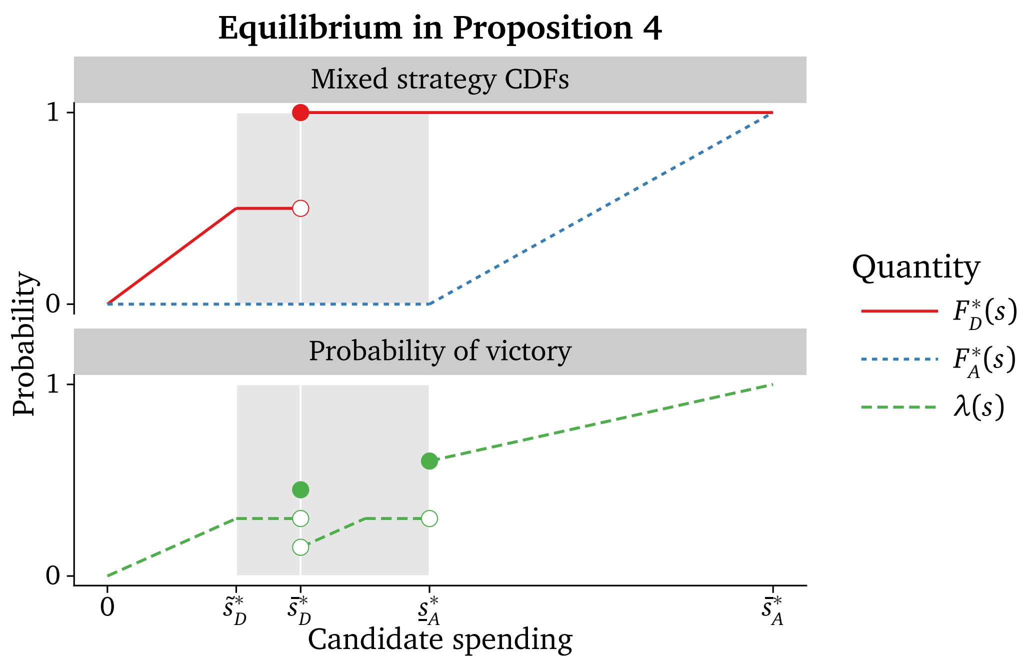


Figure 3. CDFs and probability of victory by spending in a fully separating equilibrium when non-centrists are Advantaged, as given in Proposition 4. Shaded regions represent spending levels off the equilibrium path.

When the difference between the two types' policy distance from the median voter is small enough, the equilibrium is fully separating. If both types are known, then α is the extra amount an Advantaged candidate must spend to defeat a Disadvantaged opponent. Therefore, when α is sufficiently small, even partial concealment is no longer sustainable in equilibrium, as Advantaged candidates would rather pay the extra cost to win rather than tie. However, the fully separating equilibrium here is not a mirror image of the one when centrists are Advantaged, as described in Proposition 1. First, there is a gap between the supports of the Advantaged and Disadvantaged types' mixed strategies. The width of the gap is α, the extra amount an Advantaged candidate must pay to make the median voter indifferent between herself and a Disadvantaged opponent. Second, the Disadvantaged type's strategy is not continuous; it places positive mass on its upper bound. The following proposition formally describes the equilibrium in this case, and Figure 3 illustrates.

Proposition 4. *If $0 < \alpha \leq p_D/2c_A$, then there is an equilibrium (σ^*, μ^*) that survives D1 in which*

Disadvantaged candidates employ a mixed strategy whose CDF is

$$F_D^*(s) = \begin{cases} 0 & s < 0, \\ c_D s / p_D & 0 \le s \le \tilde{s}_D^*, \\ c_D \tilde{s}_D^* / p_D & \tilde{s}_D^* < s < \bar{s}_D^*, \\ 1 & s \ge \bar{s}_D^*, \end{cases} \tag{15}$$

and Advantaged candidates employ a mixed strategy whose CDF is

$$F_A^*(s) = \begin{cases} 0 & s < \underline{s}_A^*, \\ c_A(s - \underline{s}_A^*)/p_A & \underline{s}_A^* \le s \le \bar{s}_A^*, \\ 1 & s > \bar{s}_A^*, \end{cases} \tag{16}$$

where $\tilde{s}_D^* = (p_D - 2c_A\alpha)/c_D$, $\bar{s}_D^* = (p_D - c_A\alpha)/c_D$, $\underline{s}_A^* = \bar{s}_D^* + \alpha$, *and* $\bar{s}_A^* = \underline{s}_A^* + p_A/c_A$. *The electorate's beliefs are* $\mu^*(s) = 0$ *for all* $s \le \bar{s}_D^*$ *and* $\mu^*(s) = 1$ *for all* $s > \bar{s}_D^*$. *If* $0 < \alpha < p_D/2c_A$, *this equilibrium is essentially unique under D1.*

It is intuitive that there will be full separation when the difference in policy distances between the two types of candidate is sufficiently small. The form that the Disadvantaged type's strategy takes under full separation is less intuitive. In the equilibrium described here, Disadvantaged candidates randomize between drawing from a uniform mixture over $[0, \tilde{s}_D^*]$ and spending the mass point $\bar{s}_D^*$. To see why the mass point is necessary, imagine a strategy profile in which each type mixed over a continuum. At the low point of the Advantaged type's mixed strategy, at best she would defeat almost every Disadvantaged opponent and lose to almost every Advantaged opponent. She could get the same electoral outcome at strictly less cost by deviating to the high point of the Disadvantaged type's mixed strategy. Therefore, the strategy profile cannot be an equilibrium. When the high point of the Disadvantaged type's mixed strategy is a mass point, such a deviation becomes less attractive, as it results in a positive probability of a tie with a

Disadvantaged opponent. Hence the mass point in the equilibrium here.

Equalizing Reforms

Imagine a reform that works to equalize candidates' chances by reducing the gap in fundraising ability between Advantaged and Disadvantaged candidates. For example, caps on individual donations to a candidate raise the marginal cost of fundraising by making candidates contact more individuals to raise the same amount of money (Meirowitz 2008, 690). If some candidates have a differential advantage in fundraising because they appeal to sympathetic interest groups, then such policies ought to raise the marginal cost of spending proportionally more for Advantaged candidates than for Disadvantaged ones. In this section, I consider the electoral consequences of such equalizing reforms for candidate behavior and electoral outcomes in the equilibrium of the model. I focus on how the probability of electing an Advantaged candidate changes with c_A, the marginal cost of fundraising for Advantaged types.

One potential concern for equalizing reforms is that they might sometimes be beneficial (e.g., when non-centrists are Advantaged) and other times harmful (e.g., when centrists are Advantaged). If campaign finance regulations were tailored to each individual race, this would not be a concern; policymakers could enact reforms when beneficial and prevent them otherwise. However, if the same campaign finance rules are set at the federal or even state level, they are bound to cover races with wide variation in the relevant parameters of the model—each type's prior probability, policy distance from the median voter, and marginal cost of fundraising. To what extent do the benefits, if any, of equalizing reforms depend on the distribution of these parameters across races?

Perhaps surprisingly, an across-the-board equalizing reform *never* decreases the chance of electing the type of candidate closest to the median voter. To see why, consider the *ex ante* probability of electing the closer candidate in equilibrium across each portion of parameter space, using the equilibria characterized in the previous section. When centrists are Advantaged, an

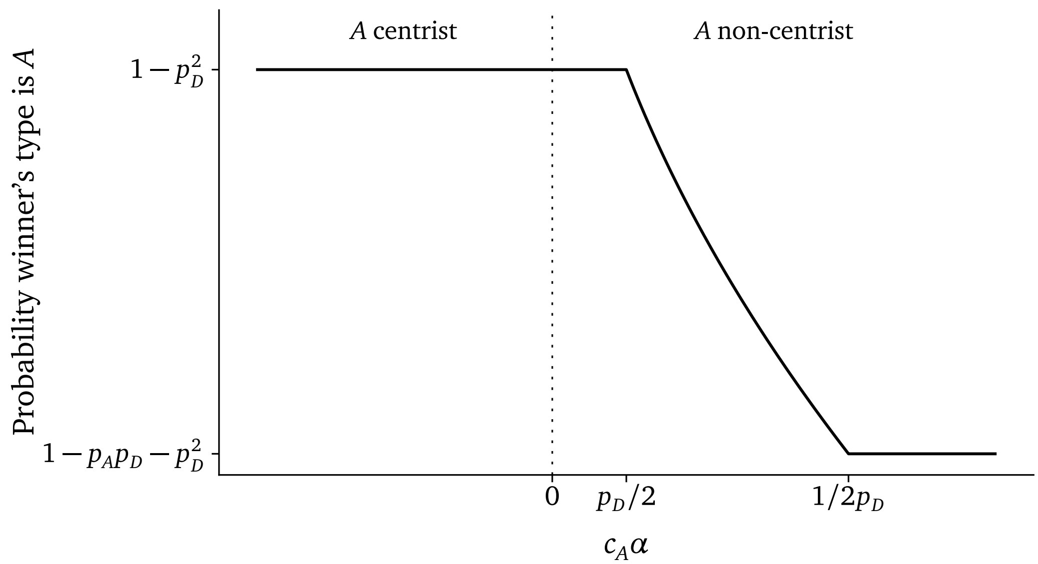


Figure 4. Relationship between $c_A\alpha$, the cost an Advantaged type must pay to make up the policy gap with Disadvantaged types, and the *ex ante* probability of electing an Advantaged type.

Advantaged candidate always defeats a Disadvantaged opponent, per Proposition 1. Therefore, a marginal increase in c_A does not affect the *ex ante* probability that the election produces an Advantaged winner. When non-centrists are Advantaged, the probability that a Disadvantaged candidate defeats an Advantaged opponent is

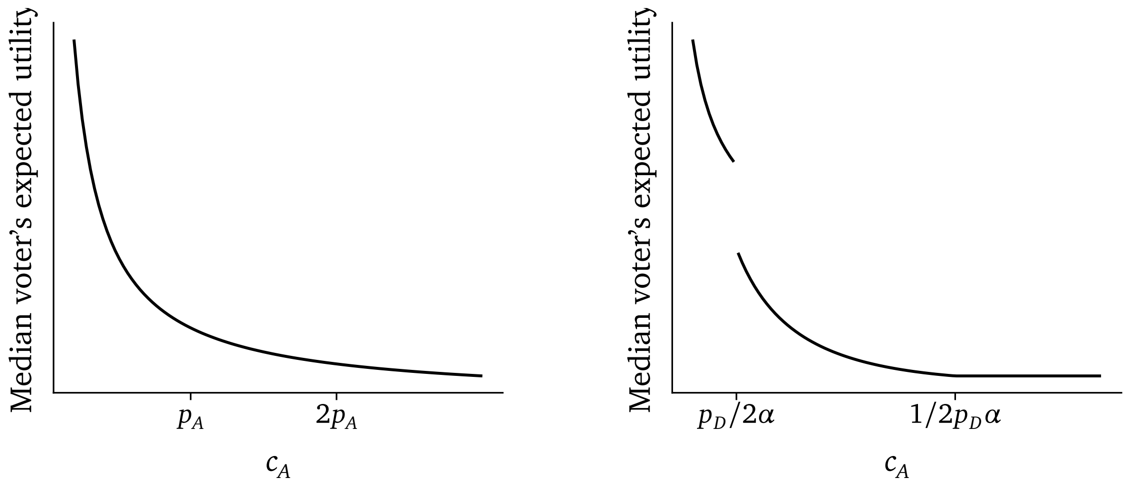
$$\iint \xi^*(s_D, s_A)\, d\sigma_D^*(s_D)\, d\sigma_A^*(s_A) = \begin{cases} 0 & \alpha \leq p_D/2c_A \text{ (Proposition 4),} \\ \pi^*/2 & p_D/2c_A < \alpha < 1/2c_Ap_D \text{ (Proposition 3),} \\ 1/2 & \alpha \geq 1/2c_Ap_D \text{ (Proposition 2).} \end{cases} \tag{17}$$

From Proposition 3 we have that π^* increases with c_A, so a marginal increase in c_A either increases or does not affect the chance of electing a Disadvantaged candidate.

Consequently, the asymmetry in the form of the equilibrium between the two cases (centrists or non-centrists Advantaged) turns out to be substantively important for the electoral consequences of reform. When centrists are Advantaged, increasing c_A causes them to spend less, but still enough to defeat a Disadvantaged opponent. Conversely, when non-centrists are Advantaged, increasing c_A makes it more attractive for such candidates to conceal their type—and thus to tie with, rather than defeat, Disadvantaged opponents who are closer to the median voter. In no context does an increase in c_A make it less likely that the eventual winner will enact policy relatively close to the median voter. Even in the context considered here, in which politicians' positions are fixed rather than the result of *quid pro quo* arrangements with special interests, campaign finance reform can bring expected policy outcomes closer to the center.

Another implication of the equilibrium results is that the means of equalization matters. As I have just shown, raising the marginal cost of fundraising for Advantaged types may increase the chance of electing a candidate with relatively centrist policy intentions. However, decreasing the marginal cost of fundraising for Disadvantaged types does not have a symmetric effect—in fact, it has no effect on the *ex ante* distribution over which type is elected. Mechanisms like matching funds for small donations, which decrease time costs for candidates without special interest backing, might thus be less effective than those like individual donation caps. Nevertheless, I show in an extension below that a public financing scheme that provides a fixed benefit to candidates rather than lowering their marginal costs may promote the electoral chances of centrist candidates.

I have shown that a particular campaign finance reform can move expected policy outcomes toward the center, but this alone does not settle the question of voter welfare. As indicated in Equation 2, voters' utility is a function not only of policy outcomes, but also of spending. Surprisingly, despite moving expected policy outcomes toward the center, an equalizing reform that raises the marginal cost of fundraising for Advantaged types has a negative effect on the median voter's overall expected utility in equilibrium. Figure 5 illustrates this result, which is proven formally in the Appendix. It is clear why this kind of reform reduces the median voter's welfare when centrists are Advantaged, as it reduces spending without changing the distribution over the winner's policy intentions. What is more surprising is that the negative effects on spending outweigh the policy benefits when non-centrists are Advantaged. However, Advantaged candidates could not get elected in the first place if their spending did not outweigh



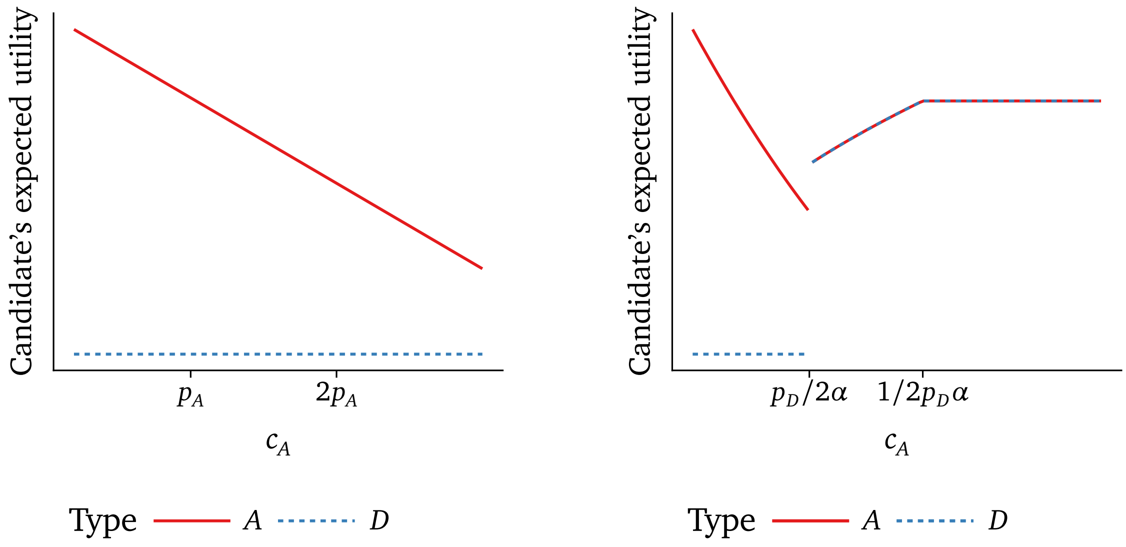
(a) Centrists are Advantaged. **(b)** Non-centrists are Advantaged.

Figure 5. The median voter's *ex ante* expected utility as a function of the Advantaged type's marginal cost of fundraising.

the policy loss to the median voter. The reduction in the intensity of competition between Advantaged candidates does more to reduce voters' estimation of the eventual winner than the centrist policy nudge increases it.

As Meirowitz (2008, 690) notes in a similar comparative statics analysis, some caution is necessary in the interpretation of voter utility here. One way to interpret the voters' utility from spending is that the funds candidates spend really do increase their quality, or valence, allowing them to better serve voters once elected. For example, if candidates spend campaign funds to travel their district (or other constituency) and build connections that will allow them to better perform constituent service once elected, a literal interpretation is appropriate—equalizing reform decreases voter welfare. However, one could also interpret the role of spending in voter utility as merely a behavioral assumption that spending increases votes, all else equal (see the discussion in Morton and Myerson 2012). For example, campaigns might spend on advertisements that simply raise the candidate's name recognition, with no real connection to the substantive well-being of voters. Under this interpretation, the effects of campaign finance reform on policy are of more interest than the effects on voter expected utility.

Finally, I consider the effects of an equalizing reform on the candidates' welfare, as illustrated



(a) Centrists are Advantaged.

(b) Non-centrists are Advantaged.

Figure 6. Each type of candidate's expected utility as a function of the Advantaged type's marginal cost of fundraising.

in Figure 6. When centrists are Advantaged, an increase in c_A has no effect on Disadvantaged types' expected utility, which is always zero. An increase in c_A strictly decreases Advantaged types' expected utility, as the downward shift in the distribution of spending is more than offset by the additional cost of the spending that remains. However, the effects of equalizing reform on Advantaged candidates are non-monotonic when non-centrists are Advantaged. When c_A is low enough that the equilibrium is fully separating, the same logic applies as when centrists are Advantaged, and a marginal increase in c_A reduces Advantaged candidates' welfare. In the partial concealment equilibrium, by contrast, Advantaged candidates' expected utility *increases* with their cost of fundraising. The savings on the cost of competition here are greater, since c_A not only shifts downward the continuum over which Advantaged types mix, but also increases the probability that they do not spend at all. Surprisingly, when full separation is not sustainable, non-centrist Advantaged types are best off under full concealment in equilibrium. Endogenous candidate entry is beyond the scope of the present analysis, but these results suggest that equalizing reforms may have mixed effects on entry in a context where candidates' policy intentions are fixed. An increase in c_A only makes the equilibrium outcome less attractive for non-centrists in a narrow range of the parameter space in which such candidates have a large advantage and are heavily favored to win the election.

Public Financing

An alternative kind of equalizing reform is public financing. I extend the model to consider a clean-election public finance scheme, under which candidates may receive a lump sum for their campaign in exchange for foregoing private fundraising.[14] The timing is as follows:

1. Candidates learn their types, $t_i \in T$.
2. Each candidate simultaneously chooses whether to take public financing, $P_i \in \{0, 1\}$.
3. These choices are observed by the candidates and the electorate, who update their beliefs accordingly.
4. Each candidate who chose $P_i = 0$ simultaneously spends $s_i \in \mathbb{R}_+$, at marginal cost c_{t_i}. Each candidate who chose $P_i = 0$ spends $\ell \geq 0$ at no cost.
5. The electorate observes the spending choices and updates its beliefs.
6. The median voter selects a winner.

I assume that the effect of the publicly financed spending on the spending component of the electorate's utility is the same as the equivalent amount of privately raised money. However, the electorate may make a different inference about the policy intentions of a candidate who privately raises ℓ than one who takes public financing. As before, I focus on symmetric equilibria, so that the electorate's belief system is symmetric across the two candidates. Because the candidates learn about each other from the choice of whether or not to take the lump sum, the equilibria of the spending subgames may be different than that of the main model.

[14] This is simpler than existing clean election laws in states like Maine, which require candidates to receive a particular number of small donations from the public in order to qualify (National Conference of State Legislatures 2017). The extension here implicitly assumes that the costs of this qualification requirement are negligible and do not vary by candidate type.

The first result is that if the amount provided by the clean election scheme is sufficiently generous, then there is a pooling equilibrium in which all candidates take it.

Proposition 5. *If $\ell \geq 1/2c_A - p_D\alpha$, there is an equilibrium of the game with public finance in which all candidates select public finance.*

In such an equilibrium, the race is always a tie. If non-centrists are Advantaged, this is the best feasible scenario for the median voter in terms of policy. Notice that the condition for the pooling equilibrium to hold is equivalent to

$$\alpha \geq \frac{1}{2c_A p_D} - \frac{\ell}{p_D},$$

which, if $\alpha > 0$, is strictly weaker than the condition for Proposition 2 to hold. In other words, even a relatively stingy clean elections law can push a race just below the threshold for full concealment into this range.

However, that does not necessarily imply that clean elections always pull policy outcomes to the center. Does the availability of a free pot of money give Disadvantaged candidates an electoral boost when centrists are Advantaged? For small ℓ, I find that the original equilibrium when centrists are Advantaged holds up, as stated in the following proposition.

Proposition 6. *If $\alpha \leq 0$ and $\ell \leq 1/c_D - p_A\alpha$, there is an equilibrium of the game with public finance that is outcome-equivalent to the equilibrium in Proposition 1, with no candidate selecting public finance.*

Taken together, these two results imply that a small clean elections scheme promotes centrist policy outcomes, just like marginal increases to c_A in the baseline model.

The type of clean election program considered here is fairly rare; only four U.S. states (Arizona, Connecticut, Maine, New Mexico) had such a program as of the 2015–2016 election cycle (National Conference of State Legislatures 2017). Matching funds schemes, in which candidates receive public matching for small private donations if they agree to adhere to a spending cap, are more popular. As noted in the discussion of equalizing reforms above, insofar as matching funds merely reduce the marginal cost of fundraising for Disadvantaged candidates, they have no effect on policy outcomes in equilibrium. With endogenous entry, they might promote entry by Disadvantaged candidates by lowering their costs of competition. The effects of the interaction with spending caps are harder to parse in the present context, as spending caps introduce equilibrium existence problems.[15] Finally, a few states give lump sums directly to political parties. A similar unconditional transfer to candidates would have no effect in the model here, as voters' utility is linear in spending. If spending instead had diminishing marginal returns on voters' assessments of candidates, then an unconditional transfer would be equivalent to increasing the marginal cost of private fundraising for both types in the model here. Such a change would move policy outcomes toward the center on average, as described in the previous section.

Correlated Types

As a final robustness check, I consider an extension of the baseline model (without public financing) in which the candidates' types are not independent. Specifically, I consider the case in which a race between candidates of the same type is relatively unlikely. Many of the equilibrium properties of spending competition in the main model stem from Advantaged candidates' anticipation of competition with a fellow Advantaged candidate; here I consider the case in which an Advantaged candidate is disproportionately likely to consider a Disadvantaged opponent (and vice versa). As the simplest model of this possibility, called the *game with correlated types*, I assume the prior probability of each type is $1/2$, and that the interim probability

[15] This is easiest to see in the case of unconditional spending caps, like the ones Meirowitz (2008) considers. If the spending cap is low enough and the effectiveness of spending great enough, there is no equilibrium in which candidates spend less than the cap. But if non-centrists are Advantaged and all candidates spend exactly the cap, then it is profitable to deviate to spending just less than the cap, as then under D1 the voter will infer for sure that one is Disadvantaged.

of facing a candidate the same type as oneself is $q \in (0, 1/2)$. Table 1 gives the joint distribution of types under this assumption.

	$t_2 = A$	$t_2 = D$
$t_1 = A$	$q/2$	$(1-q)/2$
$t_1 = D$	$(1-q)/2$	$q/2$

Table 1. Joint probability distribution of types in the game with correlated types.

In the limiting case $q = 1/2$, we have the original model with independent types and $p_A = p_D = 1/2$. In the other limiting case, $q = 0$, every election involves exactly one Advantaged and one Disadvantaged candidate.

As before, I focus on symmetric equilibria. Even so, the median voter's belief system is more complex in the original model. When a candidate learns her own type, she non-trivially updates her beliefs about her opponent's type. Therefore, a candidate's spending choice signals not only her own type, but also that of her opponent.

I focus on the case when centrists are Advantaged, to examine under what conditions c_A still has no effect on the probability of victory by an Advantaged candidate, as in the baseline model. It is clear that the equilibrium cannot take the form of Proposition 1 if an Advantaged candidate is nearly certain to face a Disadvantaged opponent. To see why, imagine an equilibrium where an Advantaged candidate defeats a Disadvantaged opponent for sure, as in the independent case. Then a Disadvantaged candidate, nearly certain her opponent is Advantaged, must spend nothing. This in turn means the Advantaged candidate's best response must be to spend barely more than nothing. Yet then it would be profitable for a Disadvantaged candidate to spend enough to beat an Advantaged opponent—meaning this cannot be an equilibrium. So if interdependence of types reduces the electoral dominance of Advantaged candidates, does it also eliminate the insensitivity of that dominance to c_A?

The following proposition characterizes equilibria across the range of $q \in (0, 1/2)$ when Advantaged candidates are in the majority. For sufficiently small violations of the independence condition, the basic form of the equilibrium remains as before. Disadvantaged and Advantaged types mix over separate intervals, and an Advantaged candidate always prevails over a Disadvantaged opponent. However, the form of the equilibrium differs once the negative correlation between types is large. Disadvantaged candidates mix over $[0, 1/c_D]$ and Advantaged candidates mix over some upper portion of this range. Advantaged candidates are still relatively likely to win, but no longer certain, and the probability of their victory decreases with c_A.

Proposition 7. *Let $\alpha \leq 0$.*

(a) If $q \geq c_A/(c_A + c_D)$, there exists an equilibrium of the game with correlated types in which an Advantaged candidate defeats a Disadvantaged opponent with probability one.

(b) If $q < c_A/(c_A + c_D)$, there exists an equilibrium of the game with correlated types in which an Advantaged candidate defeats a Disadvantaged opponent with probability

$$\frac{1}{2}\left(1 + \frac{c_D - c_A}{(1-q)c_D - qc_A}\right), \tag{18}$$

which is decreasing in c_A.

As types become more negatively correlated, Disadvantaged candidates focus more on remaining competitive against an Advantaged opponent. The concomitant increase in spending by Disadvantaged types makes it more expensive for Advantaged types to assure victory against a Disadvantaged opponent. If their marginal cost of fundraising is high enough, assured victory is not worth the cost. Through this mechanism of increased competition from Disadvantaged candidates, equalizing reforms now reduce the equilibrium probability of electing an Advantaged candidate.

Conclusion

I have modeled an electoral environment in which candidates are privately pre-committed to the policies they will implement once elected, and these policy intentions determine their ability to raise campaign funds. Unlike in existing models of money in politics, candidates cannot change their policy commitments to meet the desires of donors—or, for that matter, the electorate. Even though voters rationally infer candidates' policy intentions from their spending, policy outcomes may be biased in favor of special interests, as sympathetic candidates can exert their fundraising advantage when doing so is beneficial and conceal it otherwise. When special interest–backed candidates can raise money more easily, on the equilibrium path such candidates are more likely to win when voters are more informed. Regardless of which type of candidate has the fundraising advantage, an increase in the marginal cost of fundraising for advantaged types promotes centrist policy outcomes in the baseline model.

Candidates who benefit from unrestricted fundraising prefer to maintain that their principles dictate their policy positions, as illustrated in the introduction. I have shown that even if we take these claims at face value, it does not follow logically that electoral outcomes are immune to special interest influence. If special interests with extreme policy preferences can tell better than the public which candidates are sympathetic, then fundraising competition produces winners whose policy intentions disproportionately favor special interests. Insofar as the aim of campaign finance reform is to move policy outcomes toward the median voter, the idea that candidates are principled in their policy positions does not undercut the argument for reform. However, such reforms may be hard to sustain politically, as the associated reduction in the intensity of competition reduces the median voter's average perception of electoral winners, even accounting for the shift in the distribution of policy outcomes. More surprisingly, the model shows how campaign finance reform might be in the personal interest of special interest–backed candidates, as it can reduce the intensity of costly fundraising competition between them at a relatively small electoral cost.

Despite their agreement on the question of policy distortions due to fundraising from special interests, there are also important differences between the model here and existing models of money in politics. These differences may be useful for empiricists in identifying which kind of model better explains fundraising competition in various electoral contexts. The first is the connection between voter knowledge and eventual policy outcomes. The equilibrium relationship here—that more extreme policy outcomes are correlated with a more informed electorate, as long as special interest–backed candidates can raise money more easily—is the opposite of that in canonical models (Baron 1994; Grossman and Helpman 1996). The second is the distribution of candidate spending. A distinguishing feature of the model here is that *ex ante* identical candidates (i.e., without an incumbency or partisan advantage) may spend vastly different amounts in equilibrium, as candidates who reveal themselves to have relatively extreme policy intentions need to spend extra to make it up. Moreover, that extra spending may result in only a slight advantage in the final vote share. Finally, there is a distinction between the signaling environment here in which donors know candidates' policy intentions and existing signaling models in which they know candidates' valence (Prat 2002*a*,*b*; Wittman 2007). If raising money from special interests sends a signal to voters that a candidate has high innate quality, as in existing models, then candidates should be eager to disclose the sources of their funding. Resisting disclosure makes more sense in an environment where special interests know candidates' policy intentions but not their non-policy qualities.

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Supplemental Appendix

Properties of Equilibrium

Lemma 1. *Let (σ, μ) be an equilibrium. For each type $t \in T$, $Eu_t = U_t$ σ_t-almost everywhere.*

Proof. Take either type t, and suppose there exists a set $S \subseteq \mathbb{R}_+$ such that $\sigma_t(S) > 0$ and $U_t \neq Eu_t(s)$ for all $s \in S$. If $U_t < Eu_t(s')$ for some $s' \in S$, then it would be profitable for a candidate of type t to deviate to a strategy that places probability one on s', contradicting the assumption of equilibrium. But if $U_t > Eu_t(s)$ for all $s \in S$, then a candidate of type t could strictly benefit by deviating to a strategy that places probability zero on S, which again violates the assumption of equilibrium. □

Lemma 2. *Let (σ, μ) be an equilibrium. For each type $t \in T$ and each $s \in \operatorname{supp} \sigma_t$, if σ_t places positive probability on s or λ is continuous at s, then $Eu_t(s) = U_t$.*

Proof. Take either type t. The first part of the claim, concerning mass points of σ_t, is immediate from Lemma 1. To prove the second part, take any $s \in \operatorname{supp} \sigma_t$ such that λ is continuous at s, and suppose $Eu_t(s) < U_t$. It is apparent from Equation 6, the definition of Eu_t, that continuity of λ at s implies continuity of Eu_t at s. Therefore, there exists $\epsilon > 0$ such that $Eu_t(s') < U_t$ for all s' in an ϵ-neighborhood of s, contradicting the indifference condition of equilibrium.[16] □

Lemma 3. *Let (σ, μ) be an equilibrium. For each type $t \in T$ and each $s \in \mathbb{R}_+$ such that $Eu_t(s) = U_t$,*

$$\begin{aligned} &\lambda(s) > \lambda(s') && \text{for all } s' < s, \\ &Eu_m(s) > Eu_m(s') && \text{for all } s' < s. \end{aligned} \tag{19}$$

Proof. Take either type t, and take any s such that $Eu_t(s) = U_t$. In equilibrium, then, we have $Eu_t(s) \geq Eu_t(s')$ for all s'. For $s' < s$, this implies $\lambda(s) > \lambda(s')$, proving the first claim. The second claim is then immediate from the fact that $\lambda(s) > \lambda(s')$ only if $Eu_m(s) > Eu_m(s')$. □

Lemma 4. *In any equilibrium (σ, μ), $U_A \geq U_D$.*

Proof. The assumption $c_A < c_D$ implies $Eu_A \geq Eu_D$. Therefore, the optimality condition of equilibrium gives

$$U_A = \max_{s \in \mathbb{R}_+} Eu_A(s) \geq \max_{s \in \mathbb{R}_+} Eu_D(s) = U_D.$$ □

Lemma 5. *In any equilibrium (σ, μ), $\max \operatorname{supp} \sigma_D \leq \hat{s} \leq \min \operatorname{supp} \sigma_A$, where*

$$\hat{s} = \frac{U_A - U_D}{c_D - c_A}. \tag{20}$$

[16] The same logic applies if λ is right-continuous at s, as long as $\sigma_t((s, s+\epsilon)) > 0$ for all $\epsilon > 0$.

Proof. Let (σ, μ) be an equilibrium. For all $s > \hat{s}$,

$$(c_D - c_A)s > U_A - U_D \geq \lambda(s) - c_A s - U_D.$$

A rearrangement of terms yields

$$U_D > \lambda(s) - c_D s = Eu_D(s),$$

so a Disadvantaged candidate's mixed strategy may not place positive probability on $(\hat{s}, \infty)$. An analogous argument establishes that $\min \operatorname{supp} \sigma_A \geq \hat{s}$. $\square$

Equilibria Surviving D1

Lemma 6. *An equilibrium (σ, μ) survives the D1 refinement if and only if*

$$\begin{aligned} s < \hat{s} \quad &\Rightarrow \quad \mu(s) = 0, \\ s > \hat{s} \quad &\Rightarrow \quad \mu(s) = 1, \end{aligned}$$

for all $s \leq \max_{t \in T}\{(1 - U_t)/c_t\}$.

Proof. The claim is true for all $s \neq \hat{s}$ on the equilibrium path by Bayes' rule and Lemma 5, so now consider off-the-path values of s. As in Banks (1990) and Callander and Wilkie (2007), beliefs under D1 depend on the probability of victory that would be necessary to give a candidate an incentive to deviate to an off-the-path spending choice. For each $s \geq 0$ and each type t, let $q_t(s) = U_t + c_t s$ denote the minimal probability of victory that would give a candidate of type t a weak incentive to deviate to spending s. Notice that

$$\hat{s} - s = \frac{U_A - U_D - s(c_D - c_A)}{c_D - c_A} = \frac{q_A(s) - q_D(s)}{c_D - c_A}.$$

Therefore, if $\hat{s} < s \leq (1 - U_A)/c_A$, then $q_A(s) < q_D(s)$ and $q_A(s) \leq 1$, so D1 requires that a deviation to s be ascribed to an Advantaged candidate. Similarly, if $s < \hat{s}$ and $s \leq (1 - U_D)/c_D$, then $q_D(s) < q_A(s)$ and $q_D(s) \leq 1$, so D1 requires that a deviation to s be ascribed to a Disadvantaged candidate. Finally, if $s > \max_{t \in T}\{(1 - U_t)/c_t\}$, then $q_A(s) > 1$ and $q_D(s) > 1$, so D1 places no restriction on beliefs. $\square$

Lemma 7. *Let (σ, μ) be an equilibrium that survives D1.*

(a) Neither type of candidate's strategy contains any mass points besides $\hat{s}$.

(b) If centrists are Advantaged, then neither type's strategy contains any mass points.

(c) If non-centrists are Advantaged and the Advantaged type's strategy places positive probability on $\hat{s}$, then $\hat{s} = 0$ and the Disadvantaged type spends 0 for certain.

Proof. I begin by proving a necessary intermediate claim, namely that no $s \geq (1 - U_t)/c_t$ can be a mass point for a candidate of type t. Suppose not, so $\sigma_t(\{s'\}) > 0$ with $s' \geq (1 - U_t)/c_t$. We then have, by Lemma 2,

$$U_t = \lambda(s') - c_t s' \leq \lambda(s') - (1 - U_t).$$

Rearranging terms gives $\lambda(s') \geq 1$. This is a contradiction, as a candidate who spends s' has a positive probability of tying and thus cannot win the election for certain.

To prove claim (a), suppose some type t places probability $\pi > 0$ on $s' \neq \hat{s}$. Because $s' < (1-U_t)/c_t$ and $s' \neq \hat{s}$, we have from Lemma 6 that the electorate's beliefs are constant in an ϵ-neighborhood of s'. A candidate who spent $s \in (s', s'+\epsilon)$ would thus defeat any candidate who spent s'. Therefore, by spending infinitesimally more than s' and thereby defeating rather than tying those who spend s', a candidate would raise her chance of victory by at least $\pi p_t/2 > 0$. This contradicts the assumption of equilibrium.

To prove claim (b), suppose that centrists are Advantaged and that some type t places positive probability on $\hat{s}$. By Lemma 6, there exists $s' > \hat{s}$ such that $\mu(s) = 1$ for all $s \in (\hat{s}, s')$. This implies that Eu_m is strictly increasing on $[\hat{s}, s')$, as centrists are Advantaged, so any candidate who spent $s \in (\hat{s}, s')$ would defeat a candidate who spent $\hat{s}$. As in the proof of the last claim, a sufficiently small deviation would therefore be profitable, violating the assumption of equilibrium.

To prove claim (c), suppose that non-centrists are Advantaged and that their mixed strategy places positive probability on $\hat{s} > 0$. By Bayes' rule, then, the electorate's beliefs are $\mu(\hat{s}) > 0$. However, under D1, we have $\mu(s) = 0$ for all $s \in [0, \hat{s})$, per Lemma 6. Because non-centrists are Advantaged, this means there exists $s' < \hat{s}$ such that $Eu_m(s) > Eu_m(\hat{s})$ for all $s \in (s', \hat{s})$. We have $U_A = Eu_A(\hat{s})$, as Advantaged candidates spend $\hat{s}$ with positive probability, so this contradicts Lemma 3. □

Lemma 8. *Let (σ, μ) be an equilibrium that survives D1. For each type $t \in T$, $\operatorname{supp} \sigma_t \setminus \{\hat{s}\}$ is convex.*

Proof. Take either type $t \in T$, and suppose $\operatorname{supp} \sigma_t \setminus \{\hat{s}\}$ is not convex, so there exist $s', s'' \in \operatorname{supp} \sigma_t \setminus \{\hat{s}\}$ such that $s' < s''$ and $\sigma_t((s', s'')) = 0$. Because $s' \neq \hat{s}$ and $s'' \neq \hat{s}$, neither of these is a mass point of σ_t, per Lemma 7(a). Therefore, there exists $\delta > 0$ such that $[s' - \delta, s'] \cup [s'', s'' + \delta] \subseteq \operatorname{supp} \sigma_t \setminus \{\hat{s}\}$. Let $S = [s' - \delta, s'' + \delta]$. By Lemma 6, the electorate's beliefs μ are constant on S, which in turn implies the median voter's expected payoff Eu_m is continuous and strictly increasing on S. Consequently, the set of $s \notin S$ such that $Eu_m(s) \in Eu_m(S)$ has $\tilde{\sigma}$-measure zero, per Lemma 3. Two implications follow from this claim. First, because there is not positive mass on any s such that $Eu_m(s) \in Eu_m(S)$, the probability of victory λ is continuous on S. Second, because the set of s such that $Eu_m(s) \in Eu_m((s', s''))$ has $\tilde{\sigma}$-measure zero, $\lambda(s') = \lambda(s'')$. We therefore have $Eu_t(s') > Eu_t(s'')$. But, by Lemma 2, continuity of λ implies $Eu_t(s') = Eu_t(s'') = U_t$, a contradiction. □

Lemma 9. *In any equilibrium (σ, μ) that survives D1, $Eu_m(\max \operatorname{supp} \sigma_D) = Eu_m(\min \operatorname{supp} \sigma_A)$.*

Proof. Let $\bar{s}_D = \max \operatorname{supp} \sigma_D$, and let $\underline{s}_A = \min \operatorname{supp} \sigma_A$. The claim is trivial if $\bar{s}_D = \underline{s}_A$, so suppose $\bar{s}_D < \underline{s}_A$ and $Eu_m(\bar{s}_D) \neq Eu_m(\underline{s}_A)$. The first step of the proof is to establish that $U_A = Eu_A(\underline{s}_A)$. We know that $\underline{s}_A$ is not a mass point of σ_A, as the only amount on which an Advantaged candidate's strategy may place positive mass is 0, by Lemma 7. Therefore, there exists $s' > \underline{s}_A$ such that

$[\underline{s}_A, s'] \subseteq \operatorname{supp} \sigma_A$. By Bayes' rule, the electorate's beliefs are $\mu(s) = 1$ for all $s \in [\underline{s}_A, s']$, so Eu_m is continuous and strictly increasing on this interval. Because $\bar{s}_D$ is the only possible mass point of either type's strategy and $Eu_m(\bar{s}_D) \neq Eu_m(\underline{s}_A)$, this in turn implies λ is right-continuous at $\underline{s}_A$. Then, by Lemma 2 (see note 16), $U_A = Eu_A(\underline{s}_A)$.

We are now prepared to show that $Eu_m(\bar{s}_D) = Eu_m(\underline{s}_A)$. Suppose $Eu_m(\bar{s}_D) > Eu_m(\underline{s}_A)$. By definition, then, $\lambda(\bar{s}_D) \geq \lambda(\underline{s}_A)$. Because $\bar{s}_D < \underline{s}_A$ and $U_A = Eu_A(\underline{s}_A)$, this contradicts the optimality requirement of equilibrium, per Lemma 3. On the other hand, suppose $Eu_m(\bar{s}_D) < Eu_m(\underline{s}_A)$. If $\bar{s}_D$ is not a mass point of a Disadvantaged candidate's mixed strategy, then we have $\lambda(\bar{s}_D) = \lambda(\underline{s}_A)$, again contradicting the optimality requirement of equilibrium. Otherwise, we have $\hat{s} = \bar{s}_D$, so the electorate's beliefs μ are constant on $(\bar{s}_D, \underline{s}_A]$ under D1, per Lemma 6. The median voter's expected utility function Eu_m is thereby continuous and strictly increasing on $(\bar{s}_D, \underline{s}_A]$, so there exists $s'' \in (\bar{s}_D, \underline{s}_A)$ such that $Eu_m(s'') > Eu_m(\bar{s}_D)$. Because the set of s such that $Eu_m(s'') \leq Eu_m(s) \leq Eu_m(\underline{s}_A)$ has $\tilde{\sigma}$-measure zero, we have $\lambda(\underline{s}_A) = \lambda(s'')$, once again contradicting the optimality requirement of equilibrium. Consequently, we must have $Eu_m(\bar{s}_D) = Eu_m(\underline{s}_A)$. $\square$

Equilibrium with Advantaged Centrists

Lemma 10. *If centrists are Advantaged, then in any equilibrium (σ, μ) that survives D1, the CDF of the Disadvantaged type's mixed strategy is given by Equation 11, and the CDF of the Advantaged type's mixed strategy is given by Equation 12.*

Proof. Assume $\alpha \leq 0$, and let (σ, μ) be an equilibrium that survives D1. Because centrists are Advantaged, we have from Lemma 7(b) that neither type's strategy contains any mass points. Consequently, the support of each type t's mixed strategy is a closed interval, $\operatorname{supp} \sigma_t = [\underline{s}_t, \bar{s}_t]$, per Lemma 8. Moreover, we have $0 \leq \underline{s}_D < \bar{s}_D \leq \hat{s} \leq \underline{s}_A < \bar{s}_A$, per Lemma 5. Let $S = [0, \bar{s}_A]$. Because centrists are Advantaged, the median voter's expected utility Eu_m is strictly increasing on S. Then, as neither type's mixed strategy contains any mass points, the probability of victory function λ is continuous and non-decreasing on S. Therefore, each type's expected utility function Eu_t is continuous on $[0, \bar{s}_A]$, and we have $U_t = Eu_t(s)$ for all $s \in \operatorname{supp} \sigma_t$, per Lemma 2.

I begin by characterizing the Disadvantaged type's mixed strategy. First, as $\lambda(\underline{s}_D) = 0$ and $U_D = Eu_D(\underline{s}_D)$, we must have $\underline{s}_D = 0$, or else it would be profitable for Disadvantaged candidates to deviate to spending 0. It follows immediately that $U_D = 0$. Next, to derive the expression for Disadvantaged candidates' mixed strategy, take any $s \in \operatorname{supp} \sigma_D$. Because Eu_m is strictly increasing on S and neither type's strategy contains any mass points, we have $\lambda(s) = p_D F_D(s)$. Moreover, by continuity of Eu_D at s and the indifference condition of equilibrium, we have

$$Eu_D(s) = p_D F_D(s) - c_D s = 0 = Eu_D(0).$$

Rearranging terms gives $F_D(s) = c_D s / p_D$. Then, from $F_D(\bar{s}_D) = 1$, we yield $\bar{s}_D = p_D / c_D$. Therefore, the Disadvantaged type's mixed strategy must satisfy Equation 11.

The characterization of the Advantaged type's mixed strategy is similar. Recall from Lemma 9 that $Eu_m(\bar{s}_D) = Eu_m(\underline{s}_A)$. As Eu_m is strictly increasing on S, this implies $\underline{s}_A = \bar{s}_D = p_D / c_D$. Now, to derive the expression for Advantaged candidates' mixed strategy, take any $s \in \operatorname{supp} \sigma_A$. We have $\lambda(s) = p_D + p_A F_A(s)$, again because Eu_m is strictly increasing on S and neither type's mixed strategy contains any mass points. By continuity of Eu_A at s and the indifference condition of equilibrium, we have

$$Eu_A(s) = p_D + p_A F_A(s) - c_A s = p_D - c_A \frac{p_D}{c_D} = Eu_A(\underline{s}_A).$$

Rearranging terms gives

$$F_A(s) = \frac{c_A}{p_A}\left[s - \frac{p_D}{c_D}\right].$$

Then, from $F_A(\bar{s}_A) = 1$, we yield $\bar{s}_A = p_D/c_D + p_A/c_A$. Therefore, the Advantaged type's mixed strategy must satisfy Equation 12. □

Proposition 1. *If Advantaged candidates are centrist, then there is an equilibrium (σ^*, μ^*) that is essentially unique under D1 in which Disadvantaged candidates employ a mixed strategy whose CDF is*

$$F_D^*(s) = \begin{cases} 0 & s < 0, \\ c_D s/p_D & 0 \le s \le \bar{s}_D^*, \\ 1 & s > \bar{s}_D^*, \end{cases} \tag{11}$$

and Advantaged candidates employ a mixed strategy whose CDF is

$$F_A^*(s) = \begin{cases} 0 & s < \bar{s}_D^*, \\ c_A(s - \bar{s}_D^*)/p_A & \bar{s}_D^* \le s \le \bar{s}_A^*, \\ 1 & s > \bar{s}_A^*, \end{cases} \tag{12}$$

where $\bar{s}_D^* = p_D/c_D$ and $\bar{s}_A^* = \bar{s}_D^* + p_A/c_A$. The electorate's beliefs are

$$\mu^*(s) = \begin{cases} 0 & s < \bar{s}_D^*, \\ p_A & s = \bar{s}_D^*, \\ 1 & s > \bar{s}_D^*. \end{cases} \tag{13}$$

Proof. The first task is to confirm that (σ^*, μ^*) is an equilibrium, which requires confirming that there are no profitable deviations available and that the given beliefs are consistent with the candidates' strategies. Because centrists are Advantaged and μ^* is weakly increasing, the median voter's expected utility Eu_m is strictly increasing. Consequently, as neither type's strategy contains any mass points, the probability of victory by a candidate who spends s is $\lambda(s) = p_A F_A^*(s) + p_D F_D^*(s)$. Each type's expected utility is continuous in s regardless of the median voter's behavior when indifferent, so the choice of sharing rule is immaterial.

I begin by checking for profitable deviations. For every point in the support of a Disadvantaged type's mixed strategy, $s \in [0, \bar{s}_D^*]$, we have

$$Eu_D(s) = p_D F_D^*(s) - c_D s = 0,$$

confirming the indifference condition for Disadvantaged types. It is not profitable for a Disadvantaged candidate to mimic an Advantaged one, because for any $s \in (\bar{s}_D^*, \bar{s}_A^*]$ we have

$$Eu_D(s) = p_D + p_A F_A^*(s) - c_D s = (c_A - c_D)(s - \bar{s}_D^*) < 0.$$

Similarly, for an Advantaged candidate, for all $s \in [\bar{s}_D^*, \bar{s}_A^*]$, we have

$$Eu_A(s) = p_D + p_A F_A^*(s) - c_A s = p_D - c_A \bar{s}_D^*,$$

confirming the indifference condition for Advantaged types. It is not profitable for an Advantaged candidate to deviate to spending less, because for any $s < \bar{s}_D^*$ we have

$$Eu_A(s) = p_D F_D^*(s) - s = (c_D - c_A)s < p_D - c_A \bar{s}_D^*.$$

Finally, it is not profitable for either type to deviate to spending $s > \bar{s}_A^*$, because doing so yields the same chance of victory as spending $\bar{s}_A^*$ but at greater cost.

Next, I confirm that the electorate's beliefs are consistent with the application of Bayes' rule on the path of play and that the off-the-path beliefs survive D1. It is obvious that the on-the-path beliefs are consistent with Bayes' rule. Then, notice that the cutpoint for beliefs under D1 is $\hat{s} = (U_A - U_D)/(c_D - c_A) = p_D/c_D = \bar{s}_D^*$, so the equilibrium survives D1, per Lemma 6. The final claim of the proposition, that the equilibrium is essentially unique under D1, follows from Lemma 10. $\square$

Equilibrium with Advantaged Non-Centrists

Lemma 11. *Let (σ, μ) be an equilibrium that survives D1. If non-centrists are Advantaged, then the Disadvantaged type's strategy places positive probability on $\hat{s}$, and there exists $\delta > 0$ such that neither type's strategy places positive probability on $(\hat{s}, \hat{s} + \delta)$.*

Proof. Suppose $\alpha > 0$, so non-centrists are Advantaged. I begin by showing that the Disadvantaged type's strategy places positive probability on $\hat{s}$. For the sake of contradiction, suppose not, so $\sigma_D(\{\hat{s}\}) = 0$. Because $\hat{s}$ is the only possible mass point of either type's strategy, per Lemma 7(a), this means σ_D contains no mass points. Moreover, as non-centrists are Advantaged, the lack of a mass point in σ_D implies there is none in σ_A either, per Lemma 7(c). Because neither type t's mixed strategy contains a mass point, the support of each is an interval $[\underline{s}_t, \bar{s}_t]$, per Lemma 8. From Lemma 5, we have $\bar{s}_D \leq \hat{s} \leq \underline{s}_A$.

To show that the lack of a mass point in σ_D yields a contradiction, there are two cases to consider. First, suppose $\bar{s}_D = \underline{s}_A = \hat{s}$. By Bayes' rule, $\mu(s) = 0$ for all $s \in [\underline{s}_D, \hat{s})$ and $\mu(s) = 1$ for all $s \in (\hat{s}, \bar{s}_A]$. Taking the left- and right-hand limits of the median voter's expected utility at $\hat{s}$ gives

$$\lim_{s \to \hat{s}^+} Eu_m(s) = \hat{s} - \alpha < \hat{s} = \lim_{s \to \hat{s}^-} Eu_m(s).$$

Therefore, it would be profitable for an Advantaged candidate to deviate to spending slightly less than $\hat{s}$, contradicting the assumption of equilibrium. Second, suppose $\bar{s}_D < \underline{s}_A$. In this case, we have $\mu(s) = 1$ for all $s \in [\underline{s}_A, \bar{s}_A]$. In addition, because neither type's mixed strategy contains any mass points, the probability of victory λ is continuous on this interval. Therefore, by Lemma 2, $U_A = Eu_A(\underline{s}_A)$. However, recall that $Eu_m(\underline{s}_A) = Eu_m(\bar{s}_D)$ in any equilibrium that survives D1, per Lemma 9. Because $\bar{s}_D < \underline{s}_A$, it would thus be profitable for the Advantaged type to deviate to spending $\bar{s}_D$, again contradicting the assumption of equilibrium. As both cases yield a contradiction, we can conclude that σ_D places positive mass on $\hat{s}$.

The last step is to prove that there exists $\delta > 0$ such that $(\hat{s}, \hat{s} + \delta)$ lies outside the support of both types' strategies. Because the Disadvantaged type's mixed strategy places positive mass on $\hat{s}$, Bayes' rule implies $\mu(\hat{s}) < 1$. Because the equilibrium survives D1, we have $\mu(s) = 1$ for all $s \in (\hat{s}, \bar{s}_A]$, per Lemma 6. Similar to before, taking the right-hand limit of the median voter's utility at $\hat{s}$ gives

$$\lim_{s \to \hat{s}^+} Eu_m(s) = \hat{s} - \alpha < \hat{s} - \mu(\hat{s})\alpha = Eu_m(\hat{s}).$$

Any candidate would be strictly better off spending $\hat{s}$ than any amount just above it, so there must exist $\delta > 0$ such that $\sigma_t((\hat{s}, \hat{s} + \delta)) = 0$ for each type t. □

Lemma 12. *Suppose $\alpha > p_D/2c_A$. In any equilibrium that survives D1,*

(a) Disadvantaged types spend 0 for certain.

(b) Advantaged types spend 0 with probability $\pi^* > 0$, where

$$\pi^* = \min\left\{ \frac{\sqrt{2\alpha c_A p_D} - p_D}{p_A}, 1 \right\}. \tag{21}$$

Proof. Throughout the proof, let π denote the probability that an Advantaged candidate spends 0, so $\pi = \sigma_A(\{0\})$.

To prove claim (a), it will suffice to show that $\pi > 0$, as then Lemma 7(c) gives $\sigma_D(\{0\}) = 1$. For a proof by contradiction, suppose there is an equilibrium (σ, μ) that survives D1 in which $\pi = 0$. We know from Lemma 7(c) that the Advantaged type's strategy cannot contain any mass point besides 0. Consequently, the support of the Advantaged type's strategy is an interval, $\operatorname{supp} \sigma_A = [\underline{s}_A, \bar{s}_A]$, per Lemma 8. On the other hand, as non-centrists are Advantaged, we have from Lemma 11 that the Disadvantaged type's strategy places positive mass on $\hat{s} < \underline{s}_A$. We then have from Lemma 9 that $Eu_m(\underline{s}_A) = Eu_m(\hat{s})$. Bayes' rule gives $\mu(\underline{s}_A) = 1$ and $\mu(\bar{s}_D) = 0$, so we have $\underline{s}_A = \hat{s} + \alpha$. Because the Advantaged type's strategy contains no mass points, $\lambda(s) \to p_D$ as $s \to \underline{s}_A$ from the right. In addition, because a candidate who spends $\hat{s}$ either defeats or ties any Disadvantaged opponent, $\lambda(\hat{s}) \geq p_D/2$. Combining these with the fact that $\alpha > p_D/2c_A$ gives

$$\begin{aligned}
U_A &= \lim_{s \to \underline{s}_A^+} Eu_A(s) \\
&= p_D - c_A(\hat{s} + \alpha) \\
&< \frac{p_D}{2} - c_A\hat{s} \\
&\leq Eu_A(\hat{s}).
\end{aligned}$$

Therefore, it would be profitable for the Advantaged type to deviate to spending $\hat{s}$, contradicting the assumption of equilibrium. We conclude that $\pi > 0$ in any equilibrium that survives D1, which in turn implies that Disadvantaged candidates spend 0 for certain.

Before moving on to the proof of claim (b), it is worth noting two implications of the results just derived. First, the electorate's beliefs about a candidate who spends 0 are

$$\mu(0) = \frac{\pi p_A}{\pi p_A + p_D}. \tag{22}$$

Second, we know from Lemma 3 that on the equilibrium path, a candidate who spends nothing has zero probability of defeating a candidate who spends more. Consequently, the chance of victory for a candidate who spends 0 is

$$\lambda(0) = \frac{1}{2}\left(\pi p_A + p_D\right). \tag{23}$$

The proof of claim (b) consists of two steps. First, assume $\alpha \geq 1/2c_A p_D$, so $\pi^* = 1$. For a proof by contradiction, suppose there is an equilibrium (σ, μ) that survives D1 in which $\pi < 1$. We can then write the support of the Advantaged type's strategy as $\operatorname{supp} \sigma_A = \{0\} \cup [\tilde{s}_A, \bar{s}_A]$, where $0 < \tilde{s}_A < \bar{s}_A$. Because off-the-path beliefs must satisfy D1, Lemma 6 gives $\mu(s) = 1$ for all $s \in (0, \bar{s}_A]$. Then, applying the same line of argument as in the proof of Lemma 9, we must have $Eu_m(\tilde{s}_A) = Eu_m(0)$, which implies $\tilde{s}_A = (1 - \mu(0))\alpha$. A candidate who spends slightly more than $\tilde{s}_A$ thereby defeats those who spend 0 rather than tying. Taking the limit of the Advantaged type's utility as it approaches $\tilde{s}_A$ from the right gives

$$\begin{aligned}
\lim_{s \to \tilde{s}_A^+} Eu_A(s) &= \pi p_A + p_D - c_A \frac{p_D}{\pi p_A + p_D}\alpha \\
&\leq \pi p_A + p_D - \frac{1}{2}\frac{1}{\pi p_A + p_D} \\
&< \frac{1}{2}\left(\pi p_A + p_D\right) \\
&= Eu_A(0),
\end{aligned}$$

where the last inequality holds because $\pi < 1$ implies $\pi p_A + p_D < 1 < 1/(\pi p_A + p_D)$. Therefore, an Advantaged candidate is better off spending 0 than any amount just above $\tilde{s}_A$, which contradicts the assumption of equilibrium. We conclude that $\pi = 1$ in any equilibrium surviving D1 if $\alpha \geq 1/2c_A p_D$.

Second, assume $p_D/2c_A < \alpha < 1/2c_A p_D$, so $\pi^* < 1$. For a proof by contradiction, suppose there is an equilibrium (σ, μ) that survives D1 in which $\pi = 1$. Because both candidates spend 0 for certain, we have $U_A = U_D = \lambda(0) = 1/2$ and $\mu(0) = p_A$. A candidate could assure victory by spending $s > p_D \alpha$, as

$$Eu_m(s) > p_D \alpha - \alpha = -p_A \alpha = Eu_m(0).$$

Therefore, for any $s \in (p_D \alpha, 1/2c_A)$ (where $p_D \alpha < 1/2c_A$ because $\alpha < 1/2c_A p_D$), we have

$$Eu_A(s) = 1 - c_A s > \frac{1}{2} = U_A,$$

contradicting the assumption of equilibrium. We conclude that $\pi < 1$ in any equilibrium surviving D1 if $p_D/2c_A < \alpha < 1/2c_A p_D$. Moreover, as in the previous part of the proof, we have supp $\sigma_A = \{0\} \cup [\tilde{s}_A, \bar{s}_A]$, where $\tilde{s}_A = (1-\mu(0))\alpha > 0$. Taking limits as the Advantaged type's utility approaches $\tilde{s}_A$ from the right, the indifference condition of equilibrium gives

$$\lim_{s \to \tilde{s}_A^+} Eu_A(s) = \pi p_A + p_D - c_A \frac{p_D}{\pi p_A + p_D} \alpha = \frac{1}{2}(\pi p_A + p_D) = Eu_A(0).$$

Multiplying both sides by $\pi p_A + p_D$ and rearranging terms gives $\pi = (\sqrt{2\alpha c_A p_D} - p_D)/p_A$, as claimed. □

Proposition 2. *If $\alpha \geq 1/2c_A p_D$, then there is an equilibrium (σ^*, μ^*) that is essentially unique under D1 in which both types of candidates spend 0 for certain. The electorate's beliefs are $\mu^*(0) = p_A$ and $\mu^*(s) = 1$ for all $s > 0$.*

Proof. As in the proof of Proposition 1, we must confirm that there are no profitable deviations available and that the given beliefs are consistent with the candidates' strategies. In the proposed equilibrium, the election always ends in a tie with both candidates spending nothing, so each type's utility is $U_A = U_D = \lambda(0) = 1/2$. A candidate who spent $s \in (0, p_D\alpha)$ would lose to a candidate who spent 0, so neither type has an incentive to deviate to spending such an amount. Given his beliefs, the median voter is indifferent between a candidate who spends 0 and one who spends $p_D\alpha$. Regardless of the sharing rule employed in case of indifference, a candidate who spent $s \geq p_D\alpha$ would receive a payoff of

$$Eu_t(s) \leq 1 - c_A s \leq 1 - c_A p_D \alpha \leq \frac{1}{2},$$

so such a deviation would also be unprofitable. It is obvious that the on-the-path beliefs (namely, $\mu(0) = p_A$) are consistent with Bayes' rule, so the proposed assessment is an equilibrium. In addition, the cutpoint for beliefs under D1 is $\hat{s} = (U_A - U_D)/(c_D - c_A) = 0$, so the equilibrium survives D1, per Lemma 6. Its essential uniqueness under D1 follows from Lemma 12. □

Lemma 13. *If $p_D/2c_A < \alpha < 1/2c_A p_D$, then in any equilibrium that survives D1, the Disadvantaged type spends 0 for certain, and the CDF of the Advantaged type's mixed strategy is given by Equation 14.*

Proof. Assume $p_D/2c_A < \alpha < 1/2c_A p_D$, and let (σ, μ) be an equilibrium that survives D1. We already have from Lemma 12 that Disadvantaged candidates spend 0 for certain and that Advantaged candidates spend 0 with probability $\pi^* = (\sqrt{2\alpha c_A p_D} - p_D)/p_A > 0$. In addition, using the same logic as in the final part of the proof of Lemma 12, we can derive that the support of the Advantaged type's strategy is $\{0\} \cup [\tilde{s}_A, \bar{s}_A]$, where

$$\tilde{s}_A = \frac{p_D}{\pi^* p_A + p_D} \alpha = \frac{1}{2c_A}(\pi^* p_A + p_D).$$

All that remains is to derive $\bar{s}_A$ and the probability distribution of the Advantaged type's strategy on $[\tilde{s}_A, \bar{s}_A]$. Under D1, we have from Lemma 6 that $\mu(s) = 1$ for all $s \in (0, \bar{s}_A]$, so the median voter's expected utility is strictly increasing on this interval. Because $Eu_m(\tilde{s}_A) = Eu_m(0)$, this implies $\lambda(s) = p_D + p_A F_A(s)$ for all $s \in (\tilde{s}_A, \bar{s}_A]$. Moreover, because neither candidate's strategy contains any mass points besides 0, the probability of victory λ is continuous on $(\tilde{s}_A, \bar{s}_A]$. By the indifference condition of equilibrium,

$$Eu_A(s) = p_D + p_A F_A(s) - c_A s = \frac{1}{2}(\pi^* p_A + p_D) = Eu_A(0)$$

for all $s \in (\tilde{s}_A, \bar{s}_A]$. A rearrangement of terms gives

$$\begin{aligned} F_A(s) &= \pi^* + \frac{1}{p_A}\left(c_A s - \frac{1}{2}(\pi^* p_A + p_D)\right) \\ &= \pi^* + \frac{c_A}{p_A}(s - \tilde{s}_A), \end{aligned}$$

as claimed. Finally, setting $F_A(\bar{s}_A) = 1$ gives $\bar{s}_A = \tilde{s}_A + p_A(1 - \pi^*)/c_A$, as claimed. □

Proposition 3. *If $p_D/2c_A < \alpha < 1/2c_A p_D$, then there is an equilibrium (σ^*, μ^*) that is essentially unique under D1 in which Disadvantaged candidates spend 0 for certain and Advantaged candidates employ a mixed strategy whose CDF is*

$$F_A^*(s) = \begin{cases} 0 & s < 0, \\ \pi^* & 0 \le s \le \tilde{s}_A^*, \\ \pi^* + c_A(s - \tilde{s}_A^*)/p_A & \tilde{s}_A^* < s < \bar{s}_A^*, \\ 1 & s \ge \bar{s}_A^*, \end{cases} \tag{14}$$

where $\pi^* = (\sqrt{2\alpha c_A p_D} - p_D)/p_A$, $\tilde{s}_A^* = (\pi^* p_A + p_D)/2c_A$, and $\bar{s}_A^* = \tilde{s}_A^* + p_A(1-\pi^*)/c_A$. The electorate's beliefs are $\mu^*(0) = \pi^* p_A/(\pi^* p_A + p_D)$ and $\mu^*(s) = 1$ for all $s > 0$.

Proof. As in the proofs of the previous propositions, we must confirm that there are no profitable deviations available and that the given beliefs are consistent with the candidates' strategies. In the proposed equilibrium, a candidate who spends 0 ties with probability $\pi^* p_A + p_D$ and loses otherwise, so each type's utility is $U_A = U_D = \lambda(0) = (\pi^* p_A + p_D)/2$. A candidate who deviated to an off-the-path amount $s \in (0, \tilde{s}_A^*)$ would not defeat one who spent 0, as

$$Eu_m(s) = s - \alpha \le \tilde{s}_A^* - \alpha = -\frac{\pi^* p_A}{\pi^* p_A + p_D}\alpha = Eu_m(0),$$

so such a deviation cannot be profitable. The median voter is indifferent between a candidate who spends 0 and one who spends $\tilde{s}_A^*$. The only sharing rule that makes the candidates' expected utility functions upper semicontinuous in s is for the median voter to elect a candidate who spends $\tilde{s}_A^*$ over one who spends 0. Then, the probability of victory for a candidate who spends $s \in [\tilde{s}_A^*, \bar{s}_A^*]$ is $\lambda(s) = p_A F_A^*(s) + p_D$. The payoff to an Advantaged type for spending such an amount is

$$
\begin{aligned}
Eu_A(s) &= p_A F_A^*(s) + p_D - s \\
&= \pi^* p_A + p_D - c_A \tilde{s}_A^* \\
&= \frac{\pi^* p_A + p_D}{2} \\
&= U_A,
\end{aligned}
$$

confirming the indifference condition for the Advantaged types. This also proves that Disadvantaged types have no incentive to deviate to an amount in this range, as $Eu_A \geq Eu_D$. Finally, neither type has an incentive to deviate to spending $s > \bar{s}_A^*$, as doing so yields the same chance of victory as spending $\bar{s}_A^*$ at strictly greater cost. It is obvious that the beliefs are consistent with Bayes' rule for spending amounts on the path, $s \in \{0\} \cup [\tilde{s}_A^*, \bar{s}_A^*]$, so the proposed assessment is an equilibrium. In addition, the cutpoint for beliefs under D1 is $\hat{s} = (U_A - U_D)/(c_D - c_A) = 0$, so the equilibrium survives D1, per Lemma 6. Its essential uniqueness under D1 follows from Lemmas 12 and 13. □

Lemma 14. *If $0 < \alpha < p_D/2c_A$, then in any equilibrium that survives D1, the CDF of the Disadvantaged type's mixed strategy is given by Equation 15, and the CDF of the Advantaged type's mixed strategy is given by Equation 16.*

Proof. Assume $0 < \alpha < p_D/2c_A$, and let (σ, μ) be an equilibrium that survives D1. We know from Lemma 11 that the Disadvantaged type's strategy places probability $\rho > 0$ on $\hat{s}$. If the Advantaged type's strategy contained a mass point, that would entail placing probability $\pi^* > 0$ on 0, as shown in the proof of Lemma 12. But $\alpha < p_D/2c_A$ implies $\pi^* < 0$, so σ_A must not have any mass points. We therefore have $\operatorname{supp} \sigma_A = [\underline{s}_A, \bar{s}_A]$, where $\bar{s}_A > \underline{s}_A > \hat{s}$. Bayes' rule then gives $\mu(\underline{s}_A) = 1$ and $\mu(\hat{s}) = 0$. The median voter must be indifferent between candidates spending $\underline{s}_A$ and $\hat{s}$, per Lemma 9, so we have $\underline{s}_A = \hat{s} + \alpha$.

I begin by ruling out the possibility that the Disadvantaged type employs a pure strategy. For a proof by contradiction, suppose $\rho = 1$. Then the chance of victory by a candidate who spends $\hat{s}$ is $p_D/2$, and the Disadvantaged type's equilibrium payoff is $U_D = p_D/2 - c_D\hat{s}$. Because $Eu_m(\underline{s}_A) = Eu_m(\hat{s})$, any candidate spending more than $\underline{s}_A$ defeats all Disadvantaged candidates. The Advantaged type's equilibrium utility is thus

$$
U_A = \lim_{s \to \underline{s}_A^+} Eu_A(s) = p_D - c_A(\hat{s} + \alpha).
$$

Substituting each type's equilibrium utility into Equation 20, the definition of $\hat{s}$, gives

$$\begin{aligned}\hat{s} &= \frac{U_A - U_D}{c_D - c_A} \\ &= \frac{p_D - c_A\hat{s} - c_A\alpha - p_D/2 + c_D\hat{s}}{c_S - c_A} \\ &> \frac{p_D - c_A\hat{s} - p_D + c_D\hat{s}}{c_S - c_A} \\ &= \hat{s},\end{aligned}$$

where the inequality follows from $\alpha < p_D/2c_A$. This is a contradiction, so we conclude that $\rho < 1$.

Next, I characterize the Disadvantaged type's mixed strategy. Because the strategy places probability $\rho \in (0,1)$ on $\hat{s}$, we may write its support as $\operatorname{supp} \sigma_D = [\underline{s}_D, \tilde{s}_D] \cup \{\hat{s}\}$, by Lemma 8. Under D1, we have $\mu(s) = 0$ for all $s \in [0, \hat{s}]$, by Lemma 6. Then, as the Advantaged type's strategy does not contain any mass points, the probability of victory λ is continuous on $[0, \hat{s})$. This implies $\underline{s}_D = 0$, as otherwise we have

$$U_D = Eu_D(\underline{s}_D) = -c_D\underline{s}_D < 0 = Eu_D(0),$$

contradicting the assumption of equilibrium. As a result, $U_D = Eu_D(0) = 0$. The Advantaged type's equilibrium utility is

$$U_A = \lim_{s \to \underline{s}_A^+} Eu_A(s) = p_D - c_A(\hat{s} + \alpha),$$

so Equation 20, the definition of $\hat{s}$, gives

$$\hat{s} = \frac{U_A - U_D}{c_D - c_A} = \frac{p_D - c_A(\hat{s} + \alpha)}{c_D - c_A}.$$

Rearranging terms yields $\hat{s} = (p_D - c_A\alpha)/c_D$. Substituting this into the Disadvantaged type's expected utility from spending $\hat{s}$ gives

$$Eu_D(\hat{s}) = \left(1 - \frac{\rho}{2}\right)p_D - c_D\hat{s} = c_A\alpha - \frac{\rho p_D}{2}.$$

By the indifference condition of equilibrium, $Eu_D(\hat{s}) = Eu_D(0) = 0$, so the above implies $\rho = 2c_A\alpha/p_D$. (The conditions on α imply $0 < \rho < 1$, as required.) Because candidates spending $\hat{s}$ tie with positive probability, there is a discrete upward jump in the Disadvantaged type's expected utility at $\hat{s}$. Therefore, by the assumption of equilibrium, $\hat{s} > \tilde{s}_D$; otherwise, it would be profitable to deviate from spending just less than $\tilde{s}_D$. Consequently, for $s \in [0, \tilde{s}_D]$, the indifference condition of equilibrium gives

$$Eu_D(s) = p_D F_D(s) - c_D s = 0 = Eu_D(0),$$

and thereby $F_D(s) = c_D s/p_D$. Lastly, setting $F_D(\tilde{s}_D) = 1 - \rho$ gives $\tilde{s}_D = (p_D - 2c_A\alpha)/c_D$. We therefore yield Equation 15 as the expression for F_D.

To conclude the proof, we must derive the CDF of the Advantaged type's mixed strategy. We already have that

$$\underline{s}_A = \hat{s} + \alpha = \frac{p_D + (c_D - c_A)\alpha}{c_D}$$

and that

$$U_A = \lim_{s \to \underline{s}_A^+} Eu_A(s) = p_D - c_A \underline{s}_A.$$

By continuity of λ on $(\underline{s}_A, \bar{s}_A]$ and the indifference condition of equilibrium, for $s \in (\underline{s}_A, \bar{s}_A]$ we have

$$Eu_A(s) = p_D + p_A F_A(s) - c_A s = p_D - c_A \underline{s}_A = U_A,$$

and therefore

$$F_A(s) = \frac{c_A(s - \underline{s}_A)}{p_A}.$$

Setting $F_A(\bar{s}_A) = 1$ gives

$$\bar{s}_A = \underline{s}_A + \frac{p_A}{c_A}.$$

We therefore yield Equation 16 as the expression for F_A. □

Proposition 4. *If $0 < \alpha \leq p_D/2c_A$, then there is an equilibrium (σ^*, μ^*) that survives D1 in which Disadvantaged candidates employ a mixed strategy whose CDF is*

$$F_D^*(s) = \begin{cases} 0 & s < 0, \\ c_D s/p_D & 0 \leq s \leq \tilde{s}_D^*, \\ c_D \tilde{s}_D^*/p_D & \tilde{s}_D^* < s < \bar{s}_D^*, \\ 1 & s \geq \bar{s}_D^*, \end{cases} \tag{15}$$

and Advantaged candidates employ a mixed strategy whose CDF is

$$F_A^*(s) = \begin{cases} 0 & s < \underline{s}_A^*, \\ c_A(s - \underline{s}_A^*)/p_A & \underline{s}_A^* \leq s \leq \bar{s}_A^*, \\ 1 & s > \bar{s}_A^*, \end{cases} \tag{16}$$

where $\tilde{s}_D^* = (p_D - 2c_A\alpha)/c_D$, $\bar{s}_D^* = (p_D - c_A\alpha)/c_D$, $\underline{s}_A^* = \bar{s}_D^* + \alpha$, and $\bar{s}_A^* = \underline{s}_A^* + p_A/c_A$. The electorate's beliefs are $\mu^*(s) = 0$ for all $s \leq \bar{s}_D^*$ and $\mu^*(s) = 1$ for all $s > \bar{s}_D^*$. If $0 < \alpha < p_D/2c_A$, this equilibrium is essentially unique under D1.

Proof. As in the proofs of the previous propositions, we must confirm that there are no profitable deviations available and that the given beliefs are consistent with the candidates' strategies. To begin, I will characterize the probability of victory for each potential level of spending. The median voter is indifferent between a candidate who spends $\bar{s}_D^*$ and one who spends $\underline{s}_A^*$, as

$$Eu_m(\bar{s}_D^*) = \bar{s}_D^* = \underline{s}_A^* - \alpha = Eu_m(\underline{s}_A^*).$$

The only sharing rule that makes the candidates' expected utility functions upper semicontinuous in s is for the median voter to elect a candidate who spends $\underline{s}_A^*$ over one who spends $\bar{s}_D^*$. We therefore have $\lambda(s) = p_D F_D^*(s)$ for all $s < \bar{s}_D^*$ and $\lambda(s) = p_D + p_A F_A^*(s)$ for all $s \geq \underline{s}_A^*$. For the off-the-path values $s \in (\bar{s}_D^*, \underline{s}_A^*)$, we have $\lambda(s) = \lambda(\max\{0, s - \alpha\})$. Finally, because Disadvantaged candidates spend $\bar{s}_D^*$ with probability

$$\rho = 1 - F_D^*(\tilde{s}_D^*) = \frac{2c_A\alpha}{p_D},$$

we have $\lambda(\bar{s}_D) = p_D(1 - \rho/2) = p_D - c_A\alpha$.

To rule out a profitable deviation for Disadvantaged types, notice that their expected utility from spending $s \in [0, \tilde{s}_D^*]$ is

$$Eu_D(s) = p_D F_D^*(s) - c_D s = 0,$$

so their equilibrium payoff is $U_D = 0$. At the mass point $\bar{s}_D^*$, we have

$$Eu_D(\bar{s}_D^*) = (p_D - c_A\alpha) - c_D\bar{s}_D^* = 0,$$

confirming the Disadvantaged type's indifference condition. To mimic an Advantaged candidate by spending $s \in [\underline{s}_A^*, \bar{s}_A^*]$ would yield a payoff of

$$\begin{aligned} Eu_D(s) &= p_D + p_A F_A^*(s) - c_D s \\ &= p_D - c_A\underline{s}_A^* + (c_A - c_D)s \\ &< p_D - c_D\underline{s}_A^* \\ &= (c_A - c_D)\alpha \\ &< 0, \end{aligned}$$

and would thus be unprofitable. Finally, it is obviously unprofitable to deviate to any value $s \in (\tilde{s}_D^*, \bar{s}_D^*) \cup (\bar{s}_D^*, \underline{s}_A^*) \cup (\bar{s}_A^*, \infty)$, as for any such value it is possible to attain the same chance of victory at strictly less cost.

To rule out a profitable deviation for Advantaged types, notice that their expected utility from spending $s \in [\underline{s}_A^*, \bar{s}_A^*]$ is

$$Eu_A(s) = p_D + p_A F_A^*(s) - c_A s = p_D - c_A\underline{s}_A^*,$$

so their equilibrium payoff is

$$U_A = p_D - c_A\underline{s}_A^* = (c_D - c_A)\bar{s}_D^*.$$

To mimic a Disadvantaged candidate by spending $s \in [0, \tilde{s}_D^*]$ would yield a payoff of

$$\begin{aligned} Eu_A(s) &= p_D F_D^*(s) - c_A s \\ &= (c_D - c_A)s \\ &< (c_D - c_A)\bar{s}_D^* \\ &= U_A, \end{aligned}$$

so such a deviation would be unprofitable. Similarly, deviating to the mass point $\bar{s}^*_D$ would yield a payoff of

$$Eu_A(\bar{s}^*_D) = p_D - \alpha - c_A\bar{s}^*_D = (c_D - c_A)\bar{s}^*_D = U_A,$$

so it is also unprofitable. Finally, just as with Disadvantaged candidates, there cannot be an incentive for an Advantaged candidate to deviate to any $s \in (\tilde{s}^*_D, \bar{s}^*_D) \cup (\bar{s}^*_D, \underline{s}^*_A) \cup (\bar{s}^*_A, \infty)$.

Because the assessment is fully separating, it is obvious that the beliefs on the path are consistent with Bayes' rule, so the assessment is an equilibrium. To confirm that it survives D1, per Lemma 6, notice that the cutpoint for off-the-path beliefs is

$$\hat{s} = \frac{U_A - U_D}{c_D - c_A} = \frac{(c_D - c_A)\bar{s}^*_D - 0}{c_D - c_A} = \bar{s}^*_D.$$

Lastly, essential uniqueness when $\alpha < p_D/2c_A$ follows from Lemma 14. □

Equalizing Reform

Here I outline the argument that a marginal increase in c_A has a weakly negative effect on the median voter's *ex ante* expected utility.

Centrists Advantaged. In the parameter region covered by Proposition 1, a marginal increase in c_A does not affect the distribution over the winning candidate's type, nor does it affect the distribution of Disadvantaged candidates' spending. Because Advantaged candidates mix uniformly over $[p_D/c_D + p_A/c_A]$, a marginal increase in c_A reduces spending by Advantaged candidates, thereby reducing the median voter's *ex ante* expected utility. Therefore, the overall effect of the increase is a reduction in the median voter's *ex ante* expected utility.

Non-Centrists Advantaged, Full Concealment. In the parameter region covered by Proposition 2, a marginal increase in c_A does not affect the distribution over the winning candidates' type or either types' spending strategy. Therefore, there is no effect on the median voter's *ex ante* expected utility.

Non-Centrists Advantaged, Partial Concealment. In the parameter region covered by Proposition 3, a marginal increase in c_A affects the distribution over the winning candidates' types and the Advantaged type's spending strategy. Since these effects may offset each other in the median voter's utility, I now explicitly demonstrate that the total effect is negative.

In case both candidates are Disadvantaged, the election ends with no spending and the median voter's expected utility is 0. If one candidate is Advantaged and the other is Disadvantaged, the median voter's expected utility is

$$\pi^*(c_A)\left[\frac{1}{2}(0) + \frac{1}{2}(-\alpha)\right] + (1 - \pi^*(c_A))\left[\frac{\tilde{s}^*_A + \bar{s}^*_A}{2} - \alpha\right] = \frac{1 - \pi^*(c_A)}{2c_A} - \alpha\left[1 - \frac{\pi^*(c_A)}{2}\right],$$

where π^* is written as a function of c_A because below we differentiate with respect to c_A. Finally, using the fact that the expected value of the maximum of two i.i.d. random variables distributed

$U[a, b]$ is $a + 2(b - a)/3$ (Casella and Berger 1990, 235), the median voter's expected utility in case both candidates are Advantaged is

$$(1-\pi^*(c_A)^2)\left[\tilde{s}_A^* + \frac{2}{3}\left(\bar{s}_A^* - \tilde{s}_A^*\right)\right] - \alpha = (1-\pi^*(c_A)^2)\frac{4 - p_D - \pi^*(c_A)p_A}{6c_A} - \alpha.$$

Altogether, the median voter's *ex ante* expected utility as a function of c_A is

$$\begin{aligned}
U_m(c_A) &= p_D^2[0] + 2p_A p_D\left[\frac{1-\pi^*(c_A)}{2c_A} - \alpha\left(1 - \frac{\pi^*(c_A)}{2}\right)\right] \\
&\quad + p_A^2\left[(1-\pi^*(c_A)^2)\frac{4 - p_D - \pi^*(c_A)p_A}{6c_A} - \alpha\right] \\
&= p_A\Bigg[-\alpha(2p_D + p_A) + \frac{4 + 2p_D - p_A p_D}{6c_A} \\
&\qquad + \pi^*(c_A)\left(\alpha p_D - \frac{6p_D + p_A^2}{6c_A}\right) - \pi^*(c_A)^2 p_A\left(\frac{4 - p_D - \pi^*(c_A)p_A}{6c_A}\right)\Bigg].
\end{aligned}$$

Differentiating, factoring, and substituting $p_A = 1 - p_D$ yields

$$\begin{aligned}
\frac{dU_m(c_A)}{dc_A} &= \frac{1}{12c_A^2}\big[(2p_A^2 p_D - 8p_A - 4p_A p_D) \\
&\qquad + [\upsilon(3\upsilon^2 + 6p_D + p_A^2) - 12p_D^2 - 2p_D p_A^2] + [\upsilon(\upsilon^2 - 8p_D - p_D^2) + 8p_D^2]\big] \\
&\propto -8 + 4p_D + \upsilon(4\upsilon^2 + 1 - 4p_D),
\end{aligned}$$

where $\upsilon = \sqrt{2\alpha c_A p_D}$. Under the conditions of Proposition 3, $p_D < \upsilon < 1$, so we have

$$\begin{aligned}
\frac{dU_m(c_A)}{dc_A} &\propto -8 + 4p_D + \upsilon(4\upsilon^2 + 1 - 4p_D) \\
&< -8 + 4p_D + \upsilon(5 - 4p_D) \\
&< -3.
\end{aligned}$$

Therefore, a marginal increase in c_A strictly decreases the median voter's expected utility.

Non-Centrists Advantaged, Full Separation. In the parameter region covered by Proposition 4, a marginal increase in c_A does not affect the distribution over the winning candidate's type. It unambiguously decreases spending by Advantaged candidates, leading to a reduction in the median voter's expected utility in case either candidate is Advantaged. For the Disadvantaged types, a marginal increase in c_A shrinks the continuum over which they mix and reduces the location of the mass point, but increases the mass placed on the mass point. We therefore must explicitly confirm that the total effect is negative when both candidates are Disadvantaged. The median voter's expected utility in this case is

$$(1-\rho)^2\left[\frac{2}{3}\tilde{s}_D^*\right] + (\rho^2 + 2\rho(1-\rho))\bar{s}_D^* = \frac{2}{3c_D p_D^2}\left[p_D^3 - 2\alpha^3 c_A^3\right],$$

which is strictly decreasing in c_A.

Public Financing

Let $\mu(\emptyset)$ denote the electorate's updated belief about a candidate who chooses public finance, and let $Eu_m(\emptyset) = \ell - \mu(\emptyset)\alpha$ denote his utility from electing such a candidate.

Proposition 5. *If $\ell \geq 1/2c_A - p_D\alpha$, there is an equilibrium of the game with public finance in which all candidates select public finance.*

Proof. Suppose $\ell \geq 1/2c_A - p_D\alpha$, and consider the following assessment.

- Along the path of play, both candidates (regardless of type) choose public finance. The median voter infers that each candidate is Advantaged with probability p_A and randomizes uniformly between them.
- If either candidate deviates by foregoing public finance, the median voter and the other candidate infer she is Advantaged with probability one. Because $Eu_m(\emptyset) = \ell - p_A\alpha$, a deviant must spend $s' = \max\{0, \ell + p_D\alpha\}$ to make the median voter indifferent. The only sharing rule that averts an open-set problem in this subgame is for the median voter to elect a deviant who spends s' with probability one over a publicly financed opponent.
- In the subgame where one candidate deviates, she employs a pure strategy drawn from $\operatorname{argmax}_{s\in\{0,s'\}} \left\{\mathbf{1}\{s = s'\} - c_{t_i}s\right\}$, which is sequentially rational by construction.
- In the subgame where both deviate, the median voter believes both are Advantaged for sure and consequently elects whichever spends most. Advantaged candidates mix uniformly over $[0, 1/c_A]$, which is sequentially rational given the median voter's strategy and the fact that each candidate believes the other is Advantaged (see Meirowitz 2008, Proposition 2). Consequently, the best response for a Disadvantaged candidate in this subgame is to spend nothing.

Under this assessment, $U_A = U_D = 1/2$. Consider a unilateral deviation by an Advantaged candidate. If the Advantaged type's strategy in the consequent subgame is to spend 0 and $0 < s'$, then she loses the election and receives a payoff of 0, so the deviation is not profitable. If her strategy is to spend s', thereby winning the election, her utility from the deviation is

$$\begin{aligned}
1 - c_A s' &\leq 1 - c_A(\ell + p_D\alpha) \\
&\leq 1 - c_A\left(\frac{1}{2c_A} - p_D\alpha + p_D\alpha\right) \\
&= \frac{1}{2} \\
&= U_A,
\end{aligned}$$

so the deviation is not profitable. Nor would such a deviation be profitable for a Disadvantaged type, whose marginal cost of fundraising is even greater. Therefore, because beliefs along the path of play are consistent with the application of Bayes' rule, this assessment is an equilibrium. □

Proposition 6. *If $\alpha \leq 0$ and $\ell \leq 1/c_D - p_A\alpha$, there is an equilibrium of the game with public finance that is outcome-equivalent to the equilibrium in Proposition 1, with no candidate selecting public finance.*

Proof. Suppose $\alpha \leq 0$ and $\ell \leq 1/c_D - p_A\alpha$, and consider the following assessment.

- Along the path of play, both candidates (regardless of type) forego public finance. After a candidate selects to forego public finance, the electorate and the other candidate infer that she is Advantaged with probability p_A. The candidates then employ the same spending strategies, and the electorate updates its beliefs according to the same system, as in Proposition 1. This constitutes an equilibrium of the subgame, per Proposition 1.

- If either candidate deviates by selecting public finance, the electorate infers she is Disadvantaged with probability one. The median voter's utility from such a deviant is $Eu_m(\emptyset) = \ell$. The sharing rule must be the same as in Proposition 5, with the median voter selecting the non-publicly funded candidate when indifferent.

- In the subgame where one candidate deviates to public financing, her opponent spends $s' = \max\{0, \ell + p_A\alpha\}$ regardless of type. The median voter infers that the non-deviant is Advantaged with probability p_A regardless of her spending choice. The median voter is thus indifferent; consequently, under the sharing rule above, the non-deviant wins the election. The non-deviant's payoff in this subgame is

$$\begin{aligned} 1 - c_{t_i}s' &\geq 1 - c_D(\ell + p_A\alpha) \\ &\geq 1 - c_D\left(\frac{1}{c_D} - p_A\alpha + p_A\alpha\right) \\ &\geq 0, \end{aligned}$$

  so her choice of s' is sequentially rational.

- In the subgame where both candidates deviate to public financing, the median voter randomizes uniformly between them.

Because the strategies in each subgame are sequentially rational and the electorate's beliefs are consistent with the application of Bayes' rule whenever possible, all that remains is to confirm that neither candidate has an incentive to deviate to taking public financing. A candidate who does so loses the election for sure, receiving a payoff of zero. But we have $U_A \geq U_D = 0$ along the path of play, so such a deviation is not profitable for either type. □

Correlated Types

For any pair of spending choices (s_1, s_2), denote the median voter's beliefs

$$\mu_{AA}(s_1, s_2) = \Pr(t_1 = A, t_2 = A \mid s_1, s_2),$$

and so on for the other possible type pairings.

Proposition 7. *Let $\alpha \leq 0$.*

(a) If $q \geq c_A/(c_A + c_D)$, there exists an equilibrium of the game with correlated types in which an Advantaged candidate defeats a Disadvantaged opponent with probability one.

(b) If $q < c_A/(c_A + c_D)$, there exists an equilibrium of the game with correlated types in which an Advantaged candidate defeats a Disadvantaged opponent with probability

$$\frac{1}{2}\left(1 + \frac{c_D - c_A}{(1-q)c_D - qc_A}\right), \tag{18}$$

which is decreasing in c_A.

Proof of part (a). Suppose $\alpha \leq 0$ and $q \geq c_A/(c_A + c_D)$. I claim that the following assessment constitutes an equilibrium. The mixed strategy profile is given by the CDFs

$$F_D(s) = \begin{cases} 0 & s < 0, \\ c_D s/q & 0 \leq s \leq \bar{s}_D, \\ 1 & s > \bar{s}_D, \end{cases}$$

$$F_A(s) = \begin{cases} 0 & s < \bar{s}_D, \\ c_A(s - \bar{s}_D)/q & \bar{s}_D \leq s \leq \bar{s}_A, \\ 1 & s > \bar{s}_A, \end{cases}$$

where $\bar{s}_D = q/c_D$ and $\bar{s}_A = \bar{s}_D + q/c_A$. The electorate's updated beliefs are

$$\mu_{DD}(s_1, s_2) = \begin{cases} 1 & s_1 \leq \bar{s}_D, s_2 \leq \bar{s}_D, \\ 0 & \text{otherwise}, \end{cases}$$

$$\mu_{DA}(s_1, s_2) = \begin{cases} 1 & s_1 \leq \bar{s}_D, s_2 > \bar{s}_D, \\ 0 & \text{otherwise}, \end{cases}$$

$$\mu_{AD}(s_1, s_2) = \begin{cases} 1 & s_1 > \bar{s}_D, s_2 \leq \bar{s}_D, \\ 0 & \text{otherwise}, \end{cases}$$

$$\mu_{AA}(s_1, s_2) = 1 - \mu_{DD}(s_1, s_2) - \mu_{DA}(s_1, s_2) - \mu_{AD}(s_1, s_2).$$

Given these beliefs and the fact that $\alpha \leq 0$, the median voter is never indifferent between two candidates who spend different amounts; he always strictly prefers whichever spends more. Because neither type's mixed strategy contains a mass point, spending the same amount as one's opponent is a zero-probability event, so the choice of sharing rule for these cases is immaterial. It follows that in this assessment, an Advantaged candidate defeats a Disadvantaged opponent with probability one.

I begin by proving there are no profitable deviations for Disadvantaged candidates. For any $s \in [0, \bar{s}_D]$, we have

$$Eu_D(s) = qF_D(s) - c_D s = 0,$$

which confirms D's indifference condition and implies $U_D = 0$. For any $s \in (\bar{s}_D, \bar{s}_A]$, we have

$$Eu_D(s) = q + (1-q)F_A(s) - c_D s$$

and thus

$$Eu'_D(s) = \frac{(1-q)c_A}{q} - c_D \leq 0;$$

therefore, $Eu_D(s) \leq Eu_D(\bar{s}_D) = U_D$. Finally, it cannot be profitable to deviate to $s > \bar{s}_A$, as doing so yields the same probability of victory as spending $\bar{s}_A$ at strictly greater cost.

To prove that there are no profitable deviations for Advantaged candidates, first observe that for any $s \in [\bar{s}_D, \bar{s}_A]$,

$$Eu_A(s) = (1-q) + qF_A(s) - c_A s = 1 - q - c_A \bar{s}_D.$$

This confirms A's indifference condition and implies $U_A = 1 - q - c_A \bar{s}_D$. For any $s \in [0, \bar{s}_D)$, we have

$$Eu_A(s) = (1-q)F_D(s) - c_A s$$

and thus

$$Eu'_A(s) = \frac{(1-q)c_D}{q} - c_A.$$

From $q \leq 1/2$ we have $(1-q)/q \geq 1$ and thus $Eu'_A(s) \geq c_D - c_A > 0$; therefore, $Eu_A(s) < Eu_A(\bar{s}_D) = U_A$. Finally, as before, it cannot be profitable to deviate to $s > \bar{s}_A$.

Given the candidates' strategies, the median voter's beliefs are consistent with the application of Bayes' rule wherever possible. Therefore, the assessment constitutes an equilibrium. □

Proof of part (b). Suppose $\alpha \leq 0$ and $q > c_A/(c_A + c_D)$. I claim that the following assessment constitutes an equilibrium. The mixed strategy profile is given by the CDFs

$$F_D(s) = \begin{cases} 0 & s < 0, \\ c_D s/q & 0 \leq s \leq \underline{s}_A, \\ c_D \underline{s}_A/q + k_D(s - \underline{s}_A) & \underline{s}_A \leq s \leq \bar{s}, \\ 1 & s > \bar{s}, \end{cases}$$

$$F_A(s) = \begin{cases} 0 & s < \underline{s}_A, \\ k_A(s - \underline{s}_A) & \underline{s}_A \leq s \leq \bar{s}, \\ 1 & s > \bar{s}, \end{cases}$$

where

$$\underline{s}_A = \frac{1 - c_A/c_D}{(1-q)c_D/q - c_A},$$

$$\bar{s} = \frac{1}{c_D},$$

$$k_A = \frac{(1-q)c_D - qc_A}{1-2q},$$

$$k_D = \frac{(1-q)c_A - qc_D}{1-2q}.$$

The electorate's updated beliefs are

$$\mu_{DD}(s_1, s_2) = \begin{cases} 1 & s_1 \leq \underline{s}_A, s_2 \leq \underline{s}_A, \\ (q - \Phi q)/(1 - \Phi q) & s_1 \leq \underline{s}_A, s_2 > \underline{s}_A, \\ (q - \Phi q)/(1 - \Phi q) & s_1 > \underline{s}_A, s_2 \leq \underline{s}_A, \\ (1-\Phi)^2 q/(2 - 2\Phi + \Phi^2 q) & s_1 > \underline{s}_A, s_2 > \underline{s}_A, \end{cases}$$

$$\mu_{DA}(s_1, s_2) = \begin{cases} 0 & s_2 \leq \underline{s}_A, \\ (1-q)/(1-\Phi q) & s_1 \leq \underline{s}_A, s_2 > \underline{s}_A, \\ (1-\Phi)(1-q)/(2 - 2\Phi + \Phi^2 q) & s_1 > \underline{s}_A, s_2 > \underline{s}_A, \end{cases}$$

$$\mu_{AD}(s_1, s_2) = \begin{cases} 0 & s_1 \leq \underline{s}_A, \\ (1-q)/(1-\Phi q) & s_1 > \underline{s}_A, s_2 \leq \underline{s}_A, \\ (1-\Phi)(1-q)/(2 - 2\Phi + \Phi^2 q) & s_1 > \underline{s}_A, s_2 > \underline{s}_A, \end{cases}$$

$$\mu_{AA}(s_1, s_2) = \begin{cases} q/(2 - 2\Phi + \Phi^2 q) & s_1 > \underline{s}_A, s_2 > \underline{s}_A, \\ 0 & \text{otherwise,} \end{cases}$$

where $\Phi = F_D(\underline{s}_A)$. Given these beliefs and the fact that $\alpha \leq 0$, the median voter is never indifferent between two candidates who spend different amounts; he always strictly prefers whichever spends more. (To confirm this, notice that μ_{AD} and μ_{AA} are increasing in s_1, and μ_{DA} and μ_{AA} are increasing in s_2.) Because neither type's mixed strategy contains a mass point, spending the same amount as one's opponent is a zero-probability event, so the choice of sharing rule for these cases is immaterial.

First I will confirm that there are no profitable deviations for Disadvantaged candidates. For $s \in [0, \underline{s}_A]$ we have

$$Eu_D(s) = qF_D(s) - c_D s = 0,$$

which implies $U_D = 0$ and confirms D's indifference across this range. For $s \in [\underline{s}_A, \bar{s}]$, we have

$$\begin{aligned} Eu_D(s) &= qF_D(s) + (1-q)F_A(s) - c_D s \\ &= c_D \underline{s}_A + [qk_D + (1-q)k_A](s - \underline{s}_A) - c_D s \\ &= c_D \underline{s}_A + c_D(s - \underline{s}_A) - c_D s \\ &= 0 \\ &= U_D, \end{aligned}$$

confirming D's indifference condition for this range. It cannot be profitable for D to deviate to $s > \bar{s}$, as doing so yields the same probability of victory as spending $\bar{s}$ for strictly greater cost.

Next I will confirm that there are no profitable deviations for Advantaged candidates. For $s \in [\underline{s}_A, \bar{s}]$ we have

$$Eu_A(s) = (1-q)F_D(s) + qF_A(s) - c_A s$$

$$
\begin{aligned}
&= \frac{(1-q)c_D \underline{s}_A}{q} + [(1-q)k_D + qk_A](s - \underline{s}_A) - c_A s \\
&= \frac{(1-q)c_D \underline{s}_A}{q} + c_A(s - \underline{s}_A) - c_A s \\
&= \left[\frac{(1-q)c_D}{q} - c_A\right]\underline{s}_A \\
&= 1 - \frac{c_A}{c_D},
\end{aligned}
$$

which implies $U_A = 1 - c_A/c_D$ and confirms A's indifference across this range. For $s \in [0, \underline{s}_A)$ we have

$$
\begin{aligned}
Eu_A'(s) &= (1-q)F_D'(s) - c_A \\
&= \frac{(1-q)c_D}{q} - c_A \\
&\geq c_D - c_A \\
&> 0,
\end{aligned}
$$

so $Eu_A(s) < Eu_A(\underline{s}_A) = U_A$, meaning it is not profitable for A to deviate to such s. Finally, as before, it also cannot be profitable for A to deviate to $s > \bar{s}$.

For the median voter's beliefs, observe that

$$
\begin{aligned}
\Pr(s_i \leq \underline{s}_A \,|\, t_i = D) &= \Phi, \\
\Pr(s_i > \underline{s}_A \,|\, t_i = D) &= 1 - \Phi, \\
\Pr(s_i \leq \underline{s}_A \,|\, t_i = A) &= 0, \\
\Pr(s_i > \underline{s}_A \,|\, t_i = A) &= 1,
\end{aligned}
$$

where I denote $\Phi = F_D(\underline{s}_A)$. Moreover, s_1 and s_2 are conditionally independent given (t_1, t_2). Therefore, in case both candidates spend no more than $\underline{s}_A$, we have

$$
\Pr(t_1 = D, t_2 = D \,|\, s_1 \leq \underline{s}_A, s_2 \leq \underline{s}_A) = 1.
$$

In case 1 spends no more than $\underline{s}_A$ and 2 spends more, the electorate infers for sure that $t_1 = D$, so we have

$$
\begin{aligned}
\Pr(t_1 = D, t_2 = D \,|\, s_1 \leq \underline{s}_A, s_2 > \underline{s}_A) &= \frac{\Phi(1-\Phi)q/2}{\Phi(1-\Phi)q/2 + \Phi(1-q)/2} \\
&= \frac{q - \Phi q}{1 - \Phi q}, \\
\Pr(t_1 = D, t_2 = A \,|\, s_1 \leq \underline{s}_A, s_2 > \underline{s}_A) &= 1 - \Pr(t_1 = D, t_2 = D \,|\, s_1 \leq \underline{s}_A, s_2 > \underline{s}_A) \\
&= \frac{1-q}{1 - \Phi q}.
\end{aligned}
$$

Then, by symmetry,

$$\Pr(t_1 = D, t_2 = D \,|\, s_1 > \underline{s}_A, s_2 \le \underline{s}_A) = \frac{q - \Phi q}{1 - \Phi q},$$
$$\Pr(t_1 = A, t_2 = D \,|\, s_1 > \underline{s}_A, s_2 \le \underline{s}_A) = \frac{1 - q}{1 - \Phi q}.$$

Finally, consider the case where both candidates spend more than $\underline{s}_A$. The probability that this occurs is

$$\begin{aligned}
\Pr(s_1 > \underline{s}_A, s_2 > \underline{s}_A) &= (1-\Phi)^2 \Pr(t_1 = D, t_2 = D) \\
&\quad + (1-\Phi)[\Pr(t_1 = D, t_2 = A) + \Pr(t_1 = A, t_2 = D)] \\
&\quad + \Pr(t_1 = A, t_2 = A) \\
&= \frac{(1-\Phi)^2 q}{2} + 2(1-\Phi)\left[\frac{1-q}{2}\right] + \frac{q}{2} \\
&= 1 - \Phi + \frac{\Phi^2 q}{2}.
\end{aligned}$$

Therefore, we have

$$\Pr(t_1 = D, t_2 = D \,|\, s_1 > \underline{s}_A, s_2 > \underline{s}_A) = \frac{(1-\Phi)^2 q/2}{1 - \Phi + \Phi^2 q/2},$$
$$\Pr(t_1 = D, t_2 = A | s_1 > \underline{s}_A, s_2 > \underline{s}_A) = \frac{(1-\Phi)(1-q)/2}{1 - \Phi + \Phi^2 q/2},$$
$$\Pr(t_1 = A, t_2 = D \,|\, s_1 > \underline{s}_A, s_2 > \underline{s}_A) = \frac{(1-\Phi)(1-q)/2}{1 - \Phi + \Phi^2 q/2},$$
$$\Pr(t_1 = A, t_2 = A | s_1 > \underline{s}_A, s_2 > \underline{s}_A) = \frac{q/2}{1 - \Phi + \Phi^2 q/2}.$$

The given beliefs are consistent with these conditional probabilities.

I have confirmed that the given assessment is an equilibrium. In equilibrium, in an election between an Advantaged and a Disadvantaged candidate, the Advantaged candidate is guaranteed to win if D spends $s < \underline{s}_A$ and wins with probability $1/2$ if D spends $s \in [\underline{s}_A, \bar{s}]$. Therefore, the probability of victory by an Advantaged candidate is

$$\begin{aligned}
\Phi + \frac{1-\Phi}{2} &= \frac{1+\Phi}{2} \\
&= \frac{1 + c_D \underline{s}_A / q}{2} \\
&= \frac{1}{2}\left(1 + \frac{c_D - c_A}{(1-q)c_D - q c_A}\right),
\end{aligned}$$

as claimed in Equation 18. Differentiating with respect to c_A confirms that this probability decreases with c_A. □