

Social Conflict and the Predatory State: When Does It Pay to Divide and Rule?*

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Abstract

Conventional wisdom and existing research suggest that a predatory state benefits from divide-and-rule politics, as competition among political factions distracts them from collective action against expropriation. Historically, however, predatory states vary in whether they seek to heighten or reduce internal tensions. Using a formal model, I develop a political economy theory of how social conflict affects the policy choices and overall revenue of a rent-seeking ruler. I show that the profitability of divide-and-rule politics depends critically on the nature of the state's revenue base. Internal conflict does not just reduce subjects' incentive to resist, but also to engage in economically productive activity. On the whole, a regime that taxes the products of the society's labor will profit from promoting social order. Conversely, a state whose objective is to control a fixed stock of wealth, such as natural resources, benefits from internal divisions.

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The object of a predatory state, such as a colonial empire or a kleptocratic dictatorship, is to profit from power. Collective resistance by the subject population against a predatory regime or its extractive policies threatens the profitability of rule. Therefore, conventional wisdom at least since the Roman empire has held that a predatory state benefits from a policy of *divide et impera*, or divide and rule. When ethnic groups, religious factions, or other social subgroups are in conflict with each other, they have less time and manpower left over for collective resistance against government predation. Even in the absence of open conflict, the incentives for collective action may be weak in fractionalized societies (see Habyarimana et al. 2007), meaning ruling regimes face a relatively low internal threat to their position. Indeed, divide and rule has been a guiding policy for predatory regimes across time and space: military conquerors like Hernán Cortés in Mexico (Burkholder and Johnson 2015), global empires like the British in India (Banerjee, Iyer and Somanathan 2005), and contemporary kleptocrats as in the post-Soviet republics (Driscoll 2015).

Yet not all predatory regimes exploit internal divisions. In fact, some do the opposite, promoting internal order over conflict. For example, when the Dutch East India Company arrived in the northern Sulawesi region of present-day Indonesia in the 17th century, the region was beset with raiding and other violence, largely between neighboring rival villages (Schouten 1998). Instead of encouraging and exploiting these conflicts, as a divide-and-rule theory of predatory regimes would predict, the Dutch sought to reduce looting and protect property rights. Warring parties regularly called on the Dutch to arbitrate their disputes, making the Company a kind of “stranger king” in Sulawesi society. And the Dutch found it in their interest to do so, as “any conflict quickly tended to interfere with the production and supply of the Minahasan rice which . . . formed the Company’s main economic interest in the area” (Henley 2004, 105). In other words, at least in Sulawesi, it did not pay to divide and rule.

In this paper, I develop a theory to explain why predatory governments sometimes benefit from social order and other times prefer internal conflict. I develop the theory through a formal model of the *predatory rule of a divided society*. By predatory rule, I mean the government’s sole concern is

the economic rent it can extract for its own purposes; it cares only about internal conflict insofar as such conflict affects its profits. Not all governments are predatory, but the assumption is reasonable for many empires and autocracies, including the colonial states most often associated with divide-and-rule politics. By divided society, I mean that the polity contains distinct political groupings that may engage in inefficient competition with one another, despite their common interest in resisting expropriation by the government. A key parameter in the analysis is just how divided the society is—the number of different groups and the strength of their incentives to compete with each other rather than cooperate against predation.

The model presents a simple political economy of the predatory rule of a divided society. First, the government chooses a tax rate. Then, each separate political faction must divide its labor between economically productive activity, collective resistance against government expropriation, and internal competition (or conflict) that alters the distribution of output among the factions. Each actor, including the government, consumes whatever it receives from this competitive process. The strategic tradeoff for the government is a kind of political Laffer curve: a tax rate that is too high is self-defeating, insofar as it gives the population an incentive to divert its labor into resistance against predation. Meanwhile, the tradeoff for the factions is between the private benefit of internal competition and the collective benefits of economic production and resistance against expropriation. In order for the government to profit from social divisions, the tradeoff between internal competition and collective resistance must outweigh the tradeoff between internal competition and economically productive activity.

The main insight from the model is that the government's preference for social order depends critically on the type of economic product the government seeks to extract. I begin with governments that profit from the productive output of the subjects under its rule, a form of extraction I refer to as *tribute*. Crucially, the less labor the society devotes to economic production, the less tribute there is for the government to extract. Common forms of tribute in this sense are government demands on agricultural output, including staple crops like coffee, cotton, and sugar; forced mining performed by the subject population; and taxation of trading activity. I contrast tribute-

seeking governments with those that seek to extract from a fixed stock of natural resources or other value, which I refer to as *plunder*. The archetypical example of plunder is capital-intensive natural resource extraction such as offshore oil drilling or kimberlite diamond mining, whose relatively small labor force is often imported (Le Billon 2013). Other examples of plunder include taking land for settlement and sheer theft of existing valuables, like the room full of gold demanded for the ransom of the Inca emperor Atahualpa (Elliott 2007, 88).

The most striking result of the analysis is that the usual divide-and-rule logic is reversed for tribute-seeking governments. When the object of predation is production by the subject population, internal conflict makes predatory rule less profitable. This surprising result arises because increased internal conflict has two countervailing effects. On one hand, as traditional theories of collective action (Olson 1965) or divide-and-rule politics (Acemoglu, Verdier and Robinson 2004) would predict, social fractionalization decreases the incentives for collective action against government predation. As the incentives for internal conflict increase, so does the proportional tax rate the government can impose without provoking resistance from the population. On the other hand, internal conflict also comes at a cost to economic productivity. The more labor that groups within the subject population expend fighting each other, the less they have left over to produce commodities that form a tribute-seeking government's revenue base. Therefore, as internal divisions and conflict increase, the government ends up with a slightly larger (proportionally) slice of a significantly smaller pie. The upshot is that fractionalization makes a tribute-seeking government worse off, and such a government prefers policies that minimize raiding and other forms of competition among social groups. This is exactly the dynamic illustrated in the case of the Dutch in Sulawesi.

However, divide-and-rule politics are profitable for a government whose goal is plunder—the extraction of some fixed source of value, rather than the variable output of the population's labor. Plunder-seeking governments are better off when the subject population is more fractionalized, and they prefer not to implement policies that promote social order, even if these policies are costless. The reason for the difference between tribute- and plunder-based political economies is

simple. The potential upside of internal conflict, namely that it distracts the subject population from coordinating to drive out the new government, is present in both settings. However, the downside for a tribute-seeking government—that internal conflict also reduces the society’s productivity—does not matter for the extraction of plunder, whose value is independent of the society’s labor decisions. For a ruler whose revenues come from is plunder, sowing internal discord among the conquered population is always beneficial.

In the course of establishing these results on the distinction between tribute and plunder, the analysis also highlights how casual observation might mislead us about when a ruler profits from internal divisions. For example, holding fixed the number of separate political factions, I find that the optimal tax rate for a tribute-seeking government leads to a relatively high amount of internal conflict among those factions. At a glance, this kind of interaction might lead us to believe the government benefits from internal division. But the important counterfactual question is whether the government would be better off if there were even more contending factions—to which the answer is no. If we think of fractionalization mainly as a fixed attribute of a society,¹ then counterfactuals like this would be hard to uncover from historical observation alone. The theory presented here therefore illuminates both challenges and future directions for empirical research on social division and government predation.

In addition to these results on predation by an established government, I also examine how internal discord affects the process of establishing control in the first place. I extend the baseline model to have a prior stage in which an outsider contends for power against existing political factions, who must divide their effort between repelling the outsider and competing amongst themselves. Regardless of whether the eventual rents to the ruler come from tribute or plunder, the chance that the outsider successfully takes control increases with the level of social fractionalization. The logic of this result mirrors that of the benefits of internal division for a plunder-seeking government: during the stage of contention for power, internal competition only comes at the expense of collective resistance against the outsider. Therefore, while I find only conditional support

¹For a model in which it is not, see Penn (2008).

for the idea of divide-and-rule, the benefits of divide-and-*conquer* are less ambiguous.

1 Related Literature

This paper contributes to a longstanding body of theory on the state as an extractive or predatory institution (North 1981; Tilly 1985; Olson 1993; Levi 1989). In particular, partially mirroring the analyses by Bates, Greif and Singh (2002) and North, Wallis and Weingast (2009), the theory presented in this paper concerns the relationship between predatory government, social violence, and social order. In this section, I highlight the most closely related strands of existing work and discuss how my analysis innovates on prior research.

Existing political economy models of divide-and-rule politics show how a policy of division might benefit an autocratic ruler. Acemoglu, Verdier and Robinson (2004) identify divide-and-rule politics as an explanation for the persistence of kleptocracies. In their model, a kleptocrat facing a divided opposition can credibly threaten to divert resources to a favored group in case of a challenge, thereby deterring a challenge in the first place. Debs (2007) introduces an informational mechanism, finding that it is in a government's interest to manipulate the media in a way that will polarize social preferences over policy. I qualify prior claims about the benefits of divide-and-rule, showing that whether a leader benefits from social fractionalization depends on whether the source of rents is the output of the population's labor (tribute) or a fixed resource stock (plunder). The key theoretical distinction between my model and previous theories is that I allow for inefficient internal competition among the social factions. The tradeoff between this competition and productive labor drives my finding that fractionalization reduces how much a tribute-seeking ruler can extract.

My model of conflict within the society draws from a wide literature on the political economy of appropriation and group conflict (Hirshleifer 1991; Skaperdas 1992; Grossman and Kim 1995; Azam 2002; Dal Bó and Dal Bó 2011; Silve 2017). Existing work studies the tradeoff between production and appropriation in societies with imperfect protection of property rights. Following this line of research, particularly Hirshleifer (1991) and Skaperdas (1992), I model internal conflict

as a contest in which the size of the prize shrinks with the amount of effort devoted to the contest, reflecting how appropriative competition draws labor away from socially productive activities. In order to study how this internal competition affects the policies of an predatory ruler (and vice versa), I extend these models of horizontal competition to have a vertical aspect. I introduce a government that seeks to expropriate from the society as a whole, and I allow the political factions within the society to devote labor to resisting this expropriation. All else equal, predatory tax policies decrease the level of internal conflict compared to the baseline environment considered in previous models, as higher taxes increase groups' incentive to partake in collective resistance instead of internal competition. A government that seeks tribute and therefore benefits from internal harmony may also prefer to enact policies, such as legal protection of claims to property, that further reduce the amount of internal conflict. The effect of predation by a plunder-seeking government is more ambiguous, however, as such states may seek to arm competing factions or otherwise increase social conflict, offsetting the direct effects of predatory taxation on internal competition.

In studying how fractionalization affects a predatory state's ability to stifle resistance, I also tap into the growing interest in formally modeling collective action against autocratic regimes. Existing models have focused on informational dynamics, examining how groups' or individuals' beliefs about each other's preferences affect their ability to coordinate their actions (e.g., Bueno de Mesquita 2010; Casper and Tyson 2014; Little 2017; Zhou 2017). I abstract away from the informational issues considered in these models in order to more closely examine the conflicts of interest that may arise among challengers in a weakly institutionalized political economy. Specifically, I model how political factions' incentives to appropriate from each other may detract from their ability to collectively resist autocratic rule.

The relationship between resource dependence and internal conflict is a longstanding concern in the empirical civil war literature (for a review, see Ross 2015). In a close analogue of the distinction I draw between tribute and plunder, Dube and Vargas (2013) find that participation in civil war in Colombia falls as labor-intensive commodities like coffee become more economically important, but rises with the price of capital-intensive commodities like oil. My theory suggests that

there may also be an resource curse for communal violence—as opposed to rebellion against the state—stemming from resource-dependent predatory governments’ incentives to engage in divide-and-rule politics. Insofar as communal rivalries are a stronger cause of separatist conflicts than center-seeking wars (Lacina 2015), recent studies finding that the oil curse is concentrated among such conflicts (Paine 2016; Hunziker and Cederman 2017) provide suggestive evidence in favor of this claim, as does the finding that mineral wealth is associated with acts of violence at a local level (Berman et al. 2017).

Finally, as predation and exploitation of social divisions were both hallmarks of colonialism,² this paper contributes to the recent literature on the long-run consequences of colonial legacies and the mechanisms through which they operate. Engerman and Sokoloff (1997) argue that differences in precolonial factor endowments explain variation in subsequent growth patterns. Acemoglu, Johnson and Robinson (2001) argue, to the contrary, that this variation is best explained by whether colonial political institutions were primarily extractive and therefore left a weak foundation for property rights protection. This paper suggests how the *interaction* of factor endowments and extractive politics may explain long-run variation in governance and growth. In particular, I find that predatory states that draw rents from the population’s labor typically have an incentive to protect subjects’ property rights against threats from each other, while those that profit from fixed natural resource stocks do not. My analysis of how predatory states’ governance strategies vary with internal fractionalization is also related to recent empirical findings that precolonial political institutions and colonial boundaries splitting ethnic groups have long-run effects on conflict and economic performance (Michalopoulos and Papaioannou 2013, 2016).

2 The Model

I begin with the model of a predatory government that seeks to profit from the product of the population’s labor, or tribute. The players are the government, denoted G , and a set of N identical

²For a wide-ranging historical analysis of how empires managed social difference, see Burbank and Cooper (2010).

factions within society, denoted $\mathcal{N} = \{1, \dots, N\}$. Let $i \in \mathcal{N}$ denote a generic faction.

The interaction proceeds in two stages. First, the government chooses a tax rate, $t \in [0, 1]$. Second, after observing the tax rate, each faction simultaneously allocates its labor among activities that affect the level and distribution of economic output. These are production, denoted p_i ; resistance against the government, r_i ; and internal competition, c_i . Each faction has a finite amount of labor, meaning its allocation must satisfy the constraint

$$\frac{p_i}{\pi^p} + \frac{r_i}{\pi^r} + \frac{c_i}{\pi^c} = \frac{L}{N}, \quad (1)$$

where $L > 0$ denotes the total size of the population and each $\pi^p, \pi^r, \pi^c > 0$ denotes the society's productivity in the respective activity.³ For example, the greater π^r is, the less labor is required to produce the same amount of resistance; total resistance cannot exceed $\pi^r L$. After these choices are made, the game ends and each player receives her payoffs.

Each player's goal, including the government's, is to capture as much economic output as possible. Production, the first potential outlet for labor, creates valuable output that the government and the factions compete over. I assume output simply equals the total amount of labor devoted to production,

$$f(p) = \sum_{i=1}^N p_i, \quad (2)$$

where $p = (p_1, \dots, p_N)$ denotes the vector of each faction's production choice. The linear production technology rules out complementarity between different factions' economic production (see Silve 2017). This is a reasonable assumption for the most common forms of labor extraction by predatory states, such as staple crop production and mining of raw materials or precious metals, but would be less applicable to more developed economies. Since the goal of each player is to consume as much as possible, each player's utility is ultimately a fraction of $f(p)$. The other choice variables—the tax rate, resistance, and internal competition—determine what these fractions are.

³I write each faction's labor as a fraction of L so that I can take comparative statics on the number of factions while holding fixed the total size of the society. In the Appendix, I derive equilibrium existence and uniqueness results for the more general case in which factions may differ in their size and productivities.

Resistance, the second way the factions can expend their labor, determines how much the government can actually collect in taxes. Given the nominal tax rate t and the resistance allocations $r = (r_1, \dots, r_N)$, let $\tau(t, r)$ denote the proportion of economic output that goes to the government, or the effective tax rate, and let $\bar{\tau} = 1 - \tau$ denote the share that remains to be divided among the factions. Resistance determines what proportion of the nominal tax rate the government can collect:

$$\tau(t, r) = t \times g\left(\sum_{i=1}^N r_i\right), \quad (3)$$

where $g : [0, \pi^r L] \rightarrow [0, 1]$ is a strictly decreasing function. Given total resistance $R = \sum_{i=1}^N r_i$, the function $g(R)$ may represent either the proportion of t that the government can collect or the probability that it collects t as opposed to nothing. As regularity conditions to ensure the existence of an equilibrium and ease its characterization, I assume g is twice continuously differentiable, convex, and log-concave.⁴ Convexity implies that resistance has diminishing returns, so the factions face a classic collective action problem (Olson 1965): the more each faction expects the others to contribute to resistance, the less it prefers to contribute itself. I also assume the government fully collects the announced tax rate if there is no resistance, so $g(0) = 1$. The linear function $g(R) = 1 - R/\pi^r L$ is one example of the many functions that satisfy these conditions.

Resistance consists of any activity that directly reduces the government's ability to extract the population's economic output. The most obvious example is anti-government violence, such as in the 1791 revolt against French rule in Saint-Domingue or the 1857 Indian mutiny against the British East India Company. But resistance can also take more subtle forms. Scott (2008) describes means of "everyday resistance" employed by peasants against elites, including acts as simple as dragging one's feet. There are also overt but nonviolent forms of tax evasion, like the rampant smuggling of silver out of Spain's American colonies to circumvent the royal monopoly on bullion imports (Scammell 1989, 28).

Internal competition, the third and final outlet for the factions' labor, determines the share each group receives of what is left over after the government takes its cut. I model the internal

⁴Given the first condition, the latter two are equivalent to $0 \leq g''(R) \leq g'(R)^2/g(R)$ for all $R \in [0, \pi^r L]$.

competition over resources as a contest, in which each faction expends costly effort to increase its share of the pie. Following Hirshleifer (1991) and Skaperdas (1992), I assume the cost of participation in the contest is an opportunity cost—the more labor a faction spends increasing its own share of the pie through competition, the less it has to spend increasing the total size of the pie through production or reducing the effective tax rate through resistance. Given the competition allocations $c = (c_1, \dots, c_N)$, a faction's share of post-tax output is given by the contest success function

$$\omega_i(c) = \frac{\phi(c_i)}{\sum_{j=1}^N \phi(c_j)}, \quad (4)$$

where $\phi : [0, \pi^c L/N] \rightarrow \mathbb{R}_+$ is strictly increasing. Factions that devote more effort to internal competition end up with larger shares of the output. If every faction spends the same amount, or they all spend nothing, they all end up with equal shares, $\omega_i(c) = 1/N$.⁵ Again, as regularity conditions to ensure equilibrium existence, I assume ϕ is twice continuously differentiable and log-concave. Both of the most popular contest success functions satisfy these criteria: the ratio form, with $\phi(c_i) = c_i$, and the difference form, with $\phi(c_i) = e^{c_i}$ (Hirshleifer 1989).

The contest determines the division of all of the post-tax output among the factions. In this sense the model features an environment with weak protection of property rights, in which possession is determined through appropriation (Skaperdas 1992).⁶ By separating resistance and internal competition into separate choices, the model assumes that effort spent resisting government predation does not help a faction in resource competition with other groups, and vice versa. In reality, some activities, such as building fortifications, may serve both purposes. However, it is analytically useful to focus on the case in which there is a stark separation between the two activities. Most importantly, the negative effect of fractionalization on incentives for collective action is strongest in an environment without spillovers between collective resistance and internal competition. The less complementarity there is between efforts toward internal competition and collective resistance, the easier it should be for the divide-and-rule logic to emerge. Therefore, my assumption of minimal

⁵If $\phi(0) = 0$, in which case (4) is not well-defined at $c = 0$, let each $\omega_i(0) = 1/N$.

⁶For a model of competition with (endogenously) partial property rights, see Grossman and Kim (1995).

complementarity makes it all the more surprising that I identify conditions under which internal division decreases the revenue of the predatory state.

Each faction's utility is simply the amount of economic output it receives. This is a function of how much is produced, how much the government extracts through taxation, and the faction's standing in the internal competition. Together, these yield faction i 's utility function,

$$u_i(t, p, r, c) = \omega_i(c) \times \bar{\tau}(t, r) \times f(p). \quad (5)$$

The multiplicative payoff structure is similar to that of Hirshleifer (1991) and Skaperdas (1992), which I extend to incorporate extraction by a predatory government and collective resistance against that extraction.

The government in the model is predatory insofar as its motivation is to increase revenues for its own consumption (Levi 1989). Its utility is how much of the economic output it receives, accounting for reductions in the effective tax rate due to resistance:

$$u_G(t, p, r, c) = \tau(t, r) \times f(p). \quad (6)$$

The government does not use tax revenues to provide public goods or redistribute wealth within society. In addition, the government does not have any preferences over the distribution of post-tax consumption among the factions—internal competition does not directly enter its utility, though it matters indirectly insofar as it reduces production or resistance. Given the government's predatory incentives and lack of connection to the internal factions, the model is particularly well suited to study extractive policies of empires, colonial powers, and military occupiers.

The model is a multistage game of complete information, so the appropriate solution concept is subgame perfect equilibrium. In the remainder of the analysis, I refer to the subgame in which the factions choose divisions of labor after learning the tax rate as the labor allocation subgame. I find the equilibrium by backward induction, first characterizing the equilibrium of the labor allocation subgame following each $t \in [0, 1]$, then solving for the government's optimal choice of tax rate

given the factions' equilibrium responses.

This model sets up stark strategic tradeoffs for both the factions and the government. Each faction must choose between increasing the total size of the post-tax pie, namely through production or resistance, and securing its own share of that pie through internal competition. They face a collective action problem, insofar as production and resistance are collectively beneficial while one's share of the internal competition only has private benefits. I will consider under what circumstances the government can exploit this tradeoff for its own benefit. For the government, the key tradeoff comes in how it calibrates its extractive demand. Holding fixed the behavior of the factions, the government would always prefer a higher tax rate. But a high rate may in fact be counterproductive if it diverts social effort away from production and into resistance.

In analyzing the model, I focus on two parameters related to the idea of social order. The first is the number of factions, N . Each faction is treated as a unitary actor in the model—i.e., a faction is a political unit that has solved its internal collective action problems well enough to coordinate on a division of labor that maximizes the group's total welfare. Therefore, a greater number of factions corresponds to a more politically fractionalized society. As I show in the analysis below, the equilibrium level of internal conflict increases with the level of fractionalization. A key question for the analysis is whether this internal conflict, and thus fractionalization itself, works to the benefit of the government. When is it more profitable to govern a fractionalized society rife with internal conflict, versus a more unified society that has better overcome its own collective action problems?

The second parameter I focus on is the factions' "productivity" in internal competition, π^c , which gauges how easily the factions can appropriate from each other. I will refer to π^c as competition effectiveness, since it reflects how effectively a faction can translate its labor into strength in the internal competition. The lower the value of π^c , the greater the opportunity cost of appropriating from other factions as opposed to engaging in production or resistance. Unlike the number of factions, which would be relatively difficult to manipulate directly, it is plausible that the government could shape the relative return to appropriation, at least at the margin. The introduction of

new military technology, such as horses or firearms, might increase the ratio of effective strength to labor spent, thereby corresponding to an increase in π^c . On the other hand, stronger protection of the factions' property rights (namely against each other, not against state expropriation) would correspond to a decrease in π^c . In the analysis of the model, I look for conditions under which the government prefers to increase (or decrease) the opportunity cost of internal conflict, and how these relate to the promotion of social order. When the government benefits from encouraging conflict between factions, it is fair to say it pays to divide and rule.

3 Tribute

In the baseline model set up in the previous section, the sole source of economic value is what the population produces. If the population does not produce anything, the government comes away with nothing. In this sense it is a model of tribute—expropriation of the output of the population's labor. The analysis in this section therefore concerns the interplay of fractionalization and social order in a political economy of tribute. In the next section, I modify the model, replacing endogenous production with an exogenously fixed resource stock, in order to examine what changes when the government's objective is plunder.

I focus on how taxation, fractionalization, and the relative labor cost of internal competition affect social order in equilibrium, and how this in turn affects how much the government can extract. In order to focus on the most substantively relevant applications of the model, I impose the following assumption throughout this section:

Assumption 1 (State of Nature). $(N - 1)/N > \phi(0)/\phi'(0)\pi^c L$.

This assumption holds if and only if the baseline level of internal conflict—what would occur in equilibrium if the government imposed no taxes—is nonzero.⁷ If the State of Nature assumption does not hold, then for every tax rate $t \in [0, 1]$, the equilibrium of the subsequent labor allocation

⁷Assumption 1 holds only if $N > 1$, as the right-hand side of the condition is non-negative.

subgame entails each faction spending nothing on internal competition. These cases, in which internal conflict is effectively impossible regardless of the tax rate, are of relatively little substantive interest for the analysis of how the predatory state influences social order.

3.1 Division of Labor

I begin by solving for each faction's equilibrium division of labor in the labor allocation subgame, after the government chooses a tax rate $t \in [0, 1]$. Remember that each faction divides its labor between production, p_i ; resistance, r_i ; and internal competition, c_i .

In an equilibrium of the labor allocation subgame, each faction's division of labor maximizes its own payoff, taking as fixed the other factions' actions. In an equilibrium labor allocation, each faction must devote its labor to the activity (or activities) with the greatest marginal benefit per unit of labor. If it expends labor on two or more activities, the marginal benefits from the two must be equal. For example, if there is an equilibrium (p, r, c) in which faction i expends labor on both production and internal competition, the following condition must hold:

$$\pi^p \frac{\partial u_i(t, p, r, c)}{\partial p_i} = \pi^c \frac{\partial u_i(t, p, r, c)}{\partial c_i}.$$

If the marginal benefit of production were greater than that of internal competition, then the faction could strictly increase its consumption by shifting a bit of labor out of internal competition and into production, violating the condition of equilibrium.

The marginal benefit of one activity depends on the values of the others. For example, labor spent on internal competition, c_i , increases a faction's share of the output that is left over after taxation. Production and resistance increase the amount of post-tax output and, consequently, raise the marginal benefit of internal competition. Because of these interdependencies, the equilibrium labor allocation involves a mixture of activities. There cannot be an equilibrium with high production and no internal competition, because the return to internal competition would be too great for the factions to refrain. But there also cannot be an equilibrium with high internal competition and

no production, because then the competition would have no benefit.

The benefits of resistance against government taxation depend not only on the level of production and a faction's share in the internal competition, but also on the tax rate itself. Naturally, the more the government demands, the more there is to be gained from resistance. In the extreme case of no taxes, $t = 0$, resistance has no effect on the outcome. Each faction's labor would be better spent either on increasing total output through production or increasing its own share of output through internal competition. Therefore, the equilibrium division of labor following $t = 0$ will entail no resistance. More generally, when taxes are low enough, the marginal benefit of resistance remains too low to justify diverting effort from production or internal conflict, and there is no resistance in equilibrium.

Without resistance, the equilibrium division of labor when taxes are low entails a mixture of production and internal competition. Because the factions are identical in terms of size and productivity, in equilibrium each devotes the same amount of labor to internal competition. As a result, each ends up with an equal share of the pie, $\omega_i(c) = 1/N$. The following proposition states the form of the equilibrium in this low-tax case.⁸ I call this the baseline equilibrium, since it represents what would occur if the government imposed no taxes at all.

Proposition 1 (Baseline Equilibrium). *There is a tax rate $\hat{t}_0 \in (0, 1)$ such that $\sum_i r_i = 0$ in every equilibrium of the labor allocation subgame if and only if $t \leq \hat{t}_0$. Every subgame with $t \leq \hat{t}_0$ has the same unique equilibrium, in which $\sum_i p_i = \bar{P}_0 > 0$ and each $c_i = \bar{c}_0 > 0$.*

Even when the tax rate is zero, the equilibrium outcome is Pareto inefficient for the factions. Every faction would receive the same share, $1/N$, of a larger pie if they devoted all their labor to production. A kind of prisoner's dilemma logic explains why this Pareto efficient allocation of labor is not sustainable: if no faction planned to spend on the internal competition, then any single faction could obtain a large share by spending relatively little. Under Assumption 1, the temptation is large enough that every faction has an incentive to deviate from a strategy profile

⁸All proofs are in the Appendix.

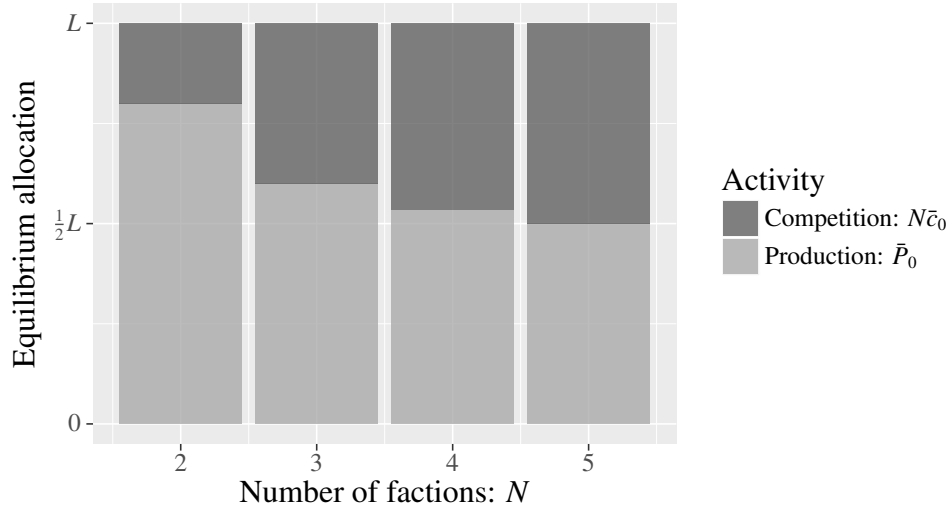


Figure 1. Division of labor in the baseline equilibrium (Proposition 1) as a function of the number of factions.

with no competition.

Holding the tax rate fixed below the baseline equilibrium cutoff, $t \leq \hat{t}_0$, the government benefits from greater social order—i.e., less internal conflict. To be clear, the government does not intrinsically care about internal conflict. However, when there is no resistance, any internal competition must come at the expense of production, to the government’s detriment. For $t \leq \hat{t}_0$, the government’s payoff is $t\bar{P}_0$. Therefore, to analyze how fractionalization and the opportunity costs of internal competition affect the government’s profits, I take comparative statics on baseline equilibrium production with respect to these parameters.

The comparative statics on the number of factions, N , which I interpret as fractionalization, are straightforward—more factions means less production. The following remark states this result, which Figure 1 illustrates.⁹

Remark 1. *Total production in the baseline equilibrium, \bar{P}_0 , is strictly decreasing in the number of factions, N .*

This result follows from the same logic as the classic finding that public good provision de-

⁹All figures use the parameters $\pi^p = \pi^r = \pi^c = 1$ and $L = 2.5$, and the functional forms $g(R) = 1 - R/\pi^r L$ and $\phi(c_i) = e^{c_i}$.

creases with group size (Olson 1965). In the baseline equilibrium, because of the internal competition among factions, each faction only recoups $1/N$ of what it produces. Because of appropriation by the other groups, a faction only partially internalizes the benefits of its own production, less so as the number of factions increases. But each faction always fully internalizes the benefits of the labor it devotes to internal competition, regardless of the number of other factions. Therefore, as N increases, the relative return to internal competition increases, so the equilibrium division of labor entails less production. For any fixed tax rate that results in the baseline equilibrium, the government is better off when the society is less fractionalized.

The effects of competition effectiveness, π^c , on the baseline equilibrium are more complicated. Specifically, a marginal increase in π^c —which represents a decrease in the opportunity cost of expending labor on internal competition—has two cross-cutting effects. The first, which I call the *incentive effect*, is to raise the marginal benefit per unit of labor of internal competition. Since the marginal benefits of competition and production must be equal in the baseline equilibrium, on its own this would lead to greater competition and less production. However, the second effect of increasing π^c , which I call the *labor-saving effect*, works the other way. If a faction's competition effectiveness increases, it can achieve the same amount of effective strength, $\phi(c_i)$, with a smaller labor force. It may use some of the freed-up labor to even further push its advantage in the internal competition, but it may also devote some to production.

The overall effect of π^c on economic output, and thus the government's payoff in the baseline equilibrium, depend on whether the incentive effect or the labor-saving effect dominates. When the incentive effect is stronger, a marginal increase in π^c (e.g., arming the population or weakening protection of property rights) would lead to more internal conflict and lower total production, reducing government revenues. The opposite is true when the labor-saving effect is stronger. The following condition characterizes the relative strength of the two effects. For any $C > 0$, I say the incentive effect outweighs the labor-saving effect at C if

$$\frac{d \log \phi(C)}{dC} + C \frac{d^2 \log \phi(C)}{dC^2} > 0,$$

and the labor-saving effect outweighs the incentive effect if the opposite inequality holds.¹⁰

Remark 2. *Total production in the baseline equilibrium, \bar{P}_0 , strictly decreases with a marginal increase in competition effectiveness, π^c , if and only if the incentive effect outweighs the labor-saving effect at \bar{c}_0 .*

The motivating example of the Dutch East India Company in the Sulawesi region of Indonesia illustrates how a change in competition effectiveness alters division of labor choices (Henley 2004). In the absence of a “stranger king” to mediate disputes, effort devoted to appropriation was relatively effective and therefore common. However, once the Company began enforcing claims to property against raiding by rival neighbors, the return to labor spent raiding plunged, corresponding to a decrease in π^c . Consequently, Sulawesi groups spent relatively more effort on economic production, increasing Company profits.

These comparative statics results show that, for any fixed tax rate below the threshold, a tribute-seeking predatory state benefits from social order. Revenues always decrease with fractionalization, and they decrease with competition effectiveness unless the labor-saving effect is so strong that internal conflict decreases with π^c . However, these results are conditional on taxes being so low that the government does not face any resistance. When internal competition can only take place at the expense of economic output, of course the government prefers less internal competition. Before inferring from these results that the government benefits from social order, I must show that the equilibrium tax rate results in no resistance, and thus the baseline division of labor.

In fact, the location of the cutpoint tax rate, \hat{t}_0 , itself depends on fractionalization and competition effectiveness. This cutpoint—the most the government can demand in taxes without engendering resistance—is the point at which the factions are at the margin indifferent about diverting some labor away from their baseline equilibrium allocations and into resistance. Since there are diminishing marginal returns to production, this indifference occurs at a relatively low tax rate when

¹⁰In Lemma 12 in the Appendix, I verify that the incentive effect always outweighs the labor-saving effect for the “difference” contest success function ($\phi(C) = \theta e^{\lambda C}$), and the two effects are exactly offsetting in case of a “ratio” contest success function ($\phi(C) = \theta C^\lambda$).

\bar{P}_0 is relatively high. Therefore, factors that decrease baseline production (like fractionalization) increase the cutpoint tax rate.

Remark 3. *The cutpoint tax rate,*

$$\hat{t}_0 = \frac{\pi^p}{\pi^p - \pi^c \bar{P}_0 g'(0)},$$

is strictly increasing in fractionalization, N . It strictly increases with a marginal increase in competition effectiveness, π^c , if and only if the incentive effect outweighs the labor-saving effect at \bar{c}_0 .

This finding complicates the picture of how social order affects revenues. On one hand, for any fixed tax rate below the threshold, a marginal increase in fractionalization or competition effectiveness (if it increases internal conflict at the expense of production) reduces the amount the government extracts. However, these same effects push the threshold upward. When the incentive for social strife increases, the economic pie shrinks, but the government can extract a greater portion of it without engendering resistance. I return to the interplay of these cross-cutting effects below when I analyze the government's equilibrium choice of tax rate.

For higher tax rates, the baseline equilibrium is no longer sustainable. Once the tax rate crosses the threshold, $t \geq \hat{t}_0$, the marginal return to resistance is too high for the factions to prefer spending nothing on resistance. Resistance increases gradually with taxes and may reach a point where it crowds out internal conflict entirely. However, there is always positive production. Resistance pushes the effective tax rate down, so there remains some incentive to produce even at the maximal nominal tax rate, $t = 1$, unlike in the usual Laffer curve. The following proposition states the form of the equilibrium above the baseline; Figure 2 illustrates equilibrium labor allocations as a function of the tax rate and the number of factions.

Proposition 2 (Resistance Equilibrium). *There is a tax rate $\hat{t}_1 > \hat{t}_0$ such that in every equilibrium of the labor allocation subgame with tax rate t :*

- *If $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = \tilde{P}_1(t) > 0$ (weakly decreasing in t), $\sum_i r_i = \tilde{R}_1(t) > 0$ (strictly*

increasing), and each $c_i = \tilde{c}_1(t) > 0$ (strictly decreasing).

- If $t \geq \hat{t}_1$, then $\sum_i p_i = \tilde{P}_2(t) > 0$ (strictly decreasing in t), $\sum_i r_i = \tilde{R}_2(t) > 0$ (strictly increasing), and each $c_i = 0$.

A high tax rate in effect unifies the population, giving the factions an incentive to act in concert to reduce government expropriation. For example, we see excessive extraction by a colonial empire leading to unity both among the American colonies during the Stamp Act crisis of 1765 and between creoles and Indians during a contemporaneous tax revolt in Quito (Elliott 2007, 310–314).

3.2 Optimal Tax Rate

I now solve for a tribute-seeking government's choice of tax rate. Given the equilibrium responses to each potential choice, as characterized above, the government faces a tradeoff. A higher tax rate has an obvious benefit—the government gets a greater share of the output, all else equal. But all else is not equal. Greater tax rates are met with greater resistance, reducing the government's effective share of output. Moreover, total output itself changes, shrinking as the factions devote more labor to resistance and thereby less to production when taxes are greater.

These tradeoffs create a Laffer curve, such that the highest tax rate does not maximize the government's revenue. At low tax rates, namely those that result in the baseline equilibrium characterized by Proposition 1, the tax rate has no marginal effect on the factions' behavior. Therefore, the government's payoff strictly increases with the tax rate in this low range, as it receives a larger share of the same pie. Beyond that, however, greater taxes are self-defeating. As taxes increase above \hat{t}_0 , the increase in resistance and the concomitant decrease in production are large enough to offset the gain the government might get from demanding a greater proportional share. As the following proposition states, the equilibrium tax rate is \hat{t}_0 , the highest at which there is no resistance.

Proposition 3 (Optimal Tax Rate). *There is an equilibrium in which the government chooses the greatest tax rate that engenders no resistance, $t = \hat{t}_0$. If g or ϕ is strictly log-concave, this is the unique equilibrium tax rate.*

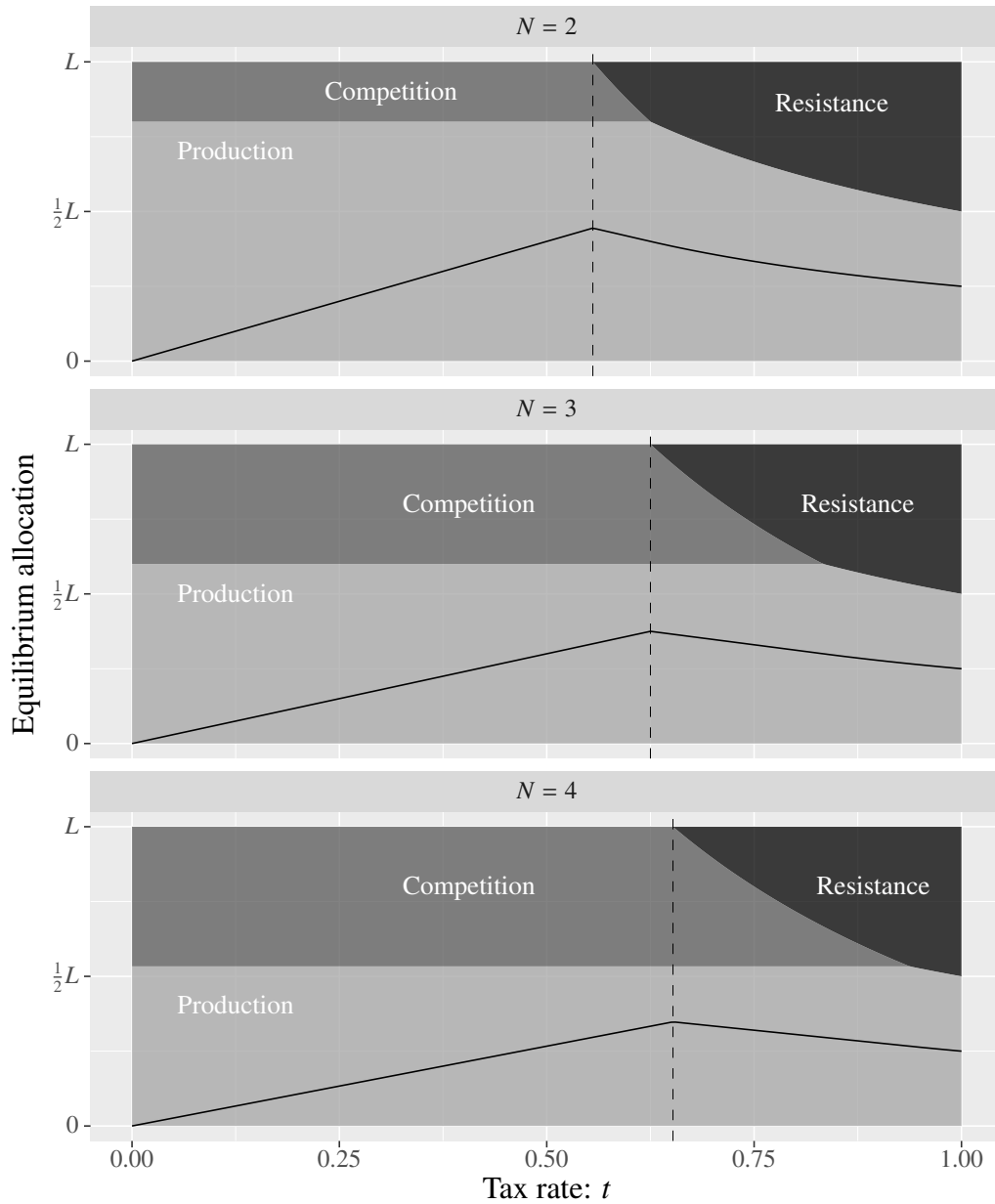


Figure 2. Equilibrium labor allocations, as described in Propositions 1 and 2, as a function of the tax rate and the number of factions. The solid curve is the government's payoff as a function of the tax rate, and the dashed line is the equilibrium tax rate.

To see why it is optimal for the government to avoid engendering resistance, consider a tax rate $t' > \hat{t}_0$ that does lead to resistance. This will result in an effective tax rate of $\tau(t', r) < t$, with some of the increase in resistance coming at the expense of production. It would be better for the government to make the announced tax rate equal the effective one, $t'' = \tau(t', r)$, and reap the gains of the additional production. Of course, if $t'' > \hat{t}_0$, there will still be positive resistance at t'' and the government will not recoup the full share. Nonetheless, as I show in the proof of this proposition in the Appendix, the increase in production by moving to a lower rate is great enough to be profitable for the government.

At a glance, this result might appear to imply that the government benefits from internal disorder. It is evident from Figure 2 that the equilibrium tax rate corresponds to a high point for internal competition. But this correlation is not exactly causal. Where internal competition is at its maximum, production is also at its maximum, and resistance is at its minimum. To analyze whether a tribute-seeking government benefits from social order or disorder, it would be more apt to imagine an exogenous shock to fractionalization or the opportunity cost of internal conflict.

To better understand how social order affects the government's ability to extract economic surplus, I return to the main structural determinants of internal competition—fractionalization (N) and competition effectiveness (π^c). I showed above that these have cross-cutting effects on the baseline equilibrium. Since the equilibrium tax rate is the cutpoint \hat{t}_0 , per Proposition 3, the government's equilibrium payoff is this fraction of baseline production, $\hat{t}_0 \bar{P}_0$. Fractionalization and competition effectiveness (when the incentive effect outweighs the labor-saving effect) increase the cutpoint tax rate, per Remark 3, but decrease baseline production, per Remarks 1 and 2. In other words, as the structural incentives for internal violence increase, the government ends up with a large share of a smaller pie in equilibrium. The critical question is which of these effects dominates. On the whole, if the objective is to secure tribute from the population, would a predatory government prefer to rule a society that is more or less prone to internal conflict? As the following proposition states, the answer is the latter—as the structural determinants of internal conflict increase, the government's equilibrium payoff decreases.

Proposition 4. *The government's equilibrium payoff is strictly decreasing in the number of factions, N . It strictly decreases with a marginal increase in competition effectiveness, π^c , if and only if the incentive effect outweighs the labor-saving effect at \bar{c}_0 .*

The most striking result here is that additional fractionalization always makes the government worse off. Why does the decrease in production always more than offset the increase in the equilibrium tax rate? Once again, the answer lies in the relative marginal benefits of production, resistance, and internal competition for the factions. At the equilibrium tax rate \hat{t}_0 , each faction's payoff is $(1 - \hat{t}_0)\bar{P}_0/N$ and the government's payoff is $\hat{t}_0\bar{P}_0$. The marginal benefit of production at the equilibrium point is therefore

$$\pi^p \frac{\partial u_i}{\partial p_i} = \frac{\pi^p}{N}(1 - \hat{t}_0),$$

which decreases with the tax rate, \hat{t}_0 . The marginal benefit of resistance in equilibrium is

$$\pi^r \frac{\partial u_i}{\partial r_i} = \frac{-\pi^r g'(0)}{N} \hat{t}_0 \bar{P}_0,$$

where $g'(0) < 0$ represents how quickly the effective tax rate shrinks with a marginal increase in resistance. Notice that the marginal benefit to resistance increases with the government's payoff, $\hat{t}_0\bar{P}_0$. Since the equilibrium tax rate pushes the factions just to the point where resistance would become profitable, in equilibrium the marginal benefits of production and resistance must be equal even though no resistance takes place. Equality of the above expressions is equivalent to

$$\pi^p(1 - \hat{t}_0) = -\pi^r g'(0)\hat{t}_0\bar{P}_0.$$

Remark 3 shows that fractionalization increases \hat{t}_0 and thereby decreases the marginal return to production (the left-hand side of the above expression). Therefore, in order to maintain the equality of marginal benefits, production \bar{P}_0 must decrease enough with fractionalization that the government's payoff, $\hat{t}_0\bar{P}_0$, also decreases. Fractionalization directly reduces the benefits of resistance, but it also reduces the benefits of production. Therefore, even as fractionalization increases, the

threat that the factions will divert labor from production into resistance remains strong enough to prevent the government from profiting.

The upshot of Proposition 4 is that social order promotes the government's ability to extract tribute. As long as the government's objective is to profit from the fruits of the population's labor, it is better off governing a society where structural conditions are favorable to social order. If we think of these conditions as at least partially endogenous, this implies that it is in the government's interest to promote social order if the costs of doing so are low enough. For example, the government might seek to divert labor away from looting and into productive economic activity by enforcing subjects' claims to property against appropriation by other factions. We have already seen this with the Dutch in Sulawesi (Henley 2004). Similarly, on the frontiers of Latin Christendom prior to Frankish conquest in the late Middle Ages, "direct predation . . . was not the occasional excess of the lawless but the prime activity of the free adult male population" (Bartlett 1993, 303). By establishing free villages and improving the protection of property rights, the immigrant Frankish nobility was able to profit by directing labor into productive activity rather than banditry. Later in Europe, the Ottoman empire maintained an advantage in trade and revenue in part by maintaining peace among the diverse religious and cultural groups that constituted its subjects (Burbank and Cooper 2010, 132–133). In yet another example, the Mughal empire that preceded British rule on the Indian subcontinent "defined their task as to keep an ordered balance between the different forces which constituted Indian society" (Wilson 2016, 17).

In a tribute economy, in which the government expropriates the product of the population's labor, the logic of divide and rule appears inoperative. Internal conflict may distract the population from resistance against taxation, but it also reduces the incentives for economic production. On the whole, it is more profitable to control a society that is more unified, and therefore more productive. The benefits of social order might be difficult to detect from casual observation, however. The optimal policy choice for the government involves some level of internal competition among the factions—not because this competition is beneficial in itself, but because that policy happens also to be the one that maximizes production and minimizes resistance.

4 Plunder

The analysis so far has considered the case in which the government's objective is to expropriate the product of the population's labor; if the population does not work, then there is no profit to be made. I have shown that the government has an incentive to promote social order under these conditions. In this section, I consider an alternative political economy, in which the main source of value—what the government and the factions want to appropriate—is a fixed stock whose value does not depend on labor inputs from the subject population. In contrast with the political economy of tribute analyzed above, I call this kind of expropriation plunder. I find that whereas a tribute-seeking government benefits from social order, a plunder-seeking government benefits instead from chaos.

The clearest example of the kind of fixed stock of value that could be plundered is natural resources, particularly oil. Oil exploitation is capital-intensive, often takes place offshore, and can be conducted by workers imported from abroad (Le Billon 2013, 28–30). In the absence of mass resistance, a government can extract value from oil deposits regardless of local labor contributions. Land itself may also play the role of a fixed resource that is valued in its own right. Whereas Spanish settlers in South America acquired property to be worked by native labor, English settlers in North America sought sparsely populated territories and fought to expel Indians where they settled (Elliott 2007, 36–38). In the terms I use here, the Spanish system of exploiting American Indian labor for mining and agriculture was tribute, whereas the English drive to dispossess American Indians of their land for English settlement was plunder.

When the government's objective is to expropriate from a fixed stock whose value does not depend on the population's labor, its strategic tradeoffs are significantly different than in the tribute case considered above. For a tribute-seeking government, internal conflict has cross-cutting effects: it reduces resistance, allowing the government to impose a higher effective tax rate, but it also reduces production. The situation is simpler for a plunder-seeking government. Since the value of the resources available to expropriate does not depend on the society's labor inputs, there is no longer a tradeoff between internal competition and the size of the pie. Internal conflict merely

helps prevent the factions from cooperating to resist government expropriation. Consequently, a plunder-seeking government benefits from internal divisions and will profit from policies that heighten social conflict.

To model the political economy of plunder, I make a simple change to the baseline model. In the original model, the total size of the pie is $f(p)$, the endogenous result of production by the factions. In the *plunder model*, the value of the economic product available for expropriation is fixed at the exogenous value $X > 0$.¹¹ The government's choice is still the tax rate $t \in [0, 1]$. With production out of the picture,¹² the factions now only choose to allocate their labor between resistance, r_i , and internal competition, c_i . The budget constraint for each faction is still given by (1), with p_i fixed to 0. Utility functions in the plunder model are

$$u_i(t, r, c) = \omega_i(c) \times \bar{\tau}(t, r) \times X,$$

$$u_G(t, r, c) = \tau(t, r) \times X,$$

so all players' payoffs sum to the total value X . This model of internal appropriation with a fixed resource stock is similar to that of Hodler (2006), extended to include an predatory government and potential resistance to it.

The relationship between the tax rate and the equilibrium division of labor in a plunder economy mirrors that of the tribute economy studied above, except with production taken out. As the tax rate increases, so too does resistance, at the expense of internal competition. The following proposition summarizes the equilibrium, and Figure 3 illustrates.

Proposition 5. *In the plunder model, every labor allocation subgame has a unique equilibrium.*

There exists a tax rate $\hat{t}_0^X \in (0, 1)$ such that each $r_i = 0$ in equilibrium if and only if $t \leq \hat{t}_0^X$.

¹¹Different actors might value the same resources differently. For example, control of oil fields might be more valuable in an absolute sense to the government than to rebel groups (Le Billon 2013, 29–30). The results of the analysis would not change if each actor valued the pie at a potentially different level $X_i > 0$, since this would simply entail the multiplication of each actor's utility function by a positive constant.

¹²The results would be similar substantively but more cumbersome to derive if economic output were the combination of exogenous resources and endogenous production, such as $f(p) = X + \sum_i p_i$.

There exists $\hat{t}_1^X > \hat{t}_0^X$ such that each $c_i = 0$ in equilibrium if and only if $t \geq \hat{t}_1^X$. For $t \in (\hat{t}_1^X, \hat{t}_0^X)$, in equilibrium each $r_i = \tilde{R}_X(t)/N > 0$ (strictly increasing in t) and each $c_i = \tilde{c}_X(t) > 0$ (strictly decreasing).

Unlike in the tribute model, it is not necessarily true that there will be no resistance on the equilibrium path.¹³ Whereas a tribute-seeking government always chooses the tax rate that results in maximal internal conflict (given equilibrium responses by the factions), a plunder-seeking government may not do so. But this is not because the incentives for social order are stronger for a plunder-seeking government. Instead, it simply reflects how the government's strategic tradeoffs change when it no longer values production by the population. Higher tax rates now have only one drawback (greater resistance), compared to two (that and less production) in a tribute economy.

In fact, looking at the structural parameters that drive internal competition, it is clear that plunder-seeking governments prefer social conflict over social order. Their incentives are therefore opposite those of tribute-seeking governments. For example, consider fractionalization, as represented by the number of factions, N . In an economy based on tribute, fractionalization leads to lower production (Remark 1) and thereby reduces how much the government can extract (Proposition 4), even though it also reduces the incentives for collective resistance. But when the source of value for the government is exogenously fixed, the negative effect of fractionalization on the population's economic activity is irrelevant, while its negative effect on resistance remains. Therefore, for a plunder-seeking government, additional fractionalization is a net benefit. The story is similar for competition effectiveness, π^c : as long as the incentive effect dominates the labor-saving effect, meaning a marginal increase in competition effectiveness results in greater social conflict in equilibrium, the plunder-seeking predatory state will benefit from such an increase. The following proposition summarizes how a plunder-seeking government benefits from structural conditions that favor internal conflict.

Proposition 6. *In the plunder model, the government's equilibrium payoff is increasing in the*

¹³In Figure 3 the equilibrium tax rate is \hat{t}_0^X , as in the original model, but this is an artifact of the particular functional forms used to make the figures.

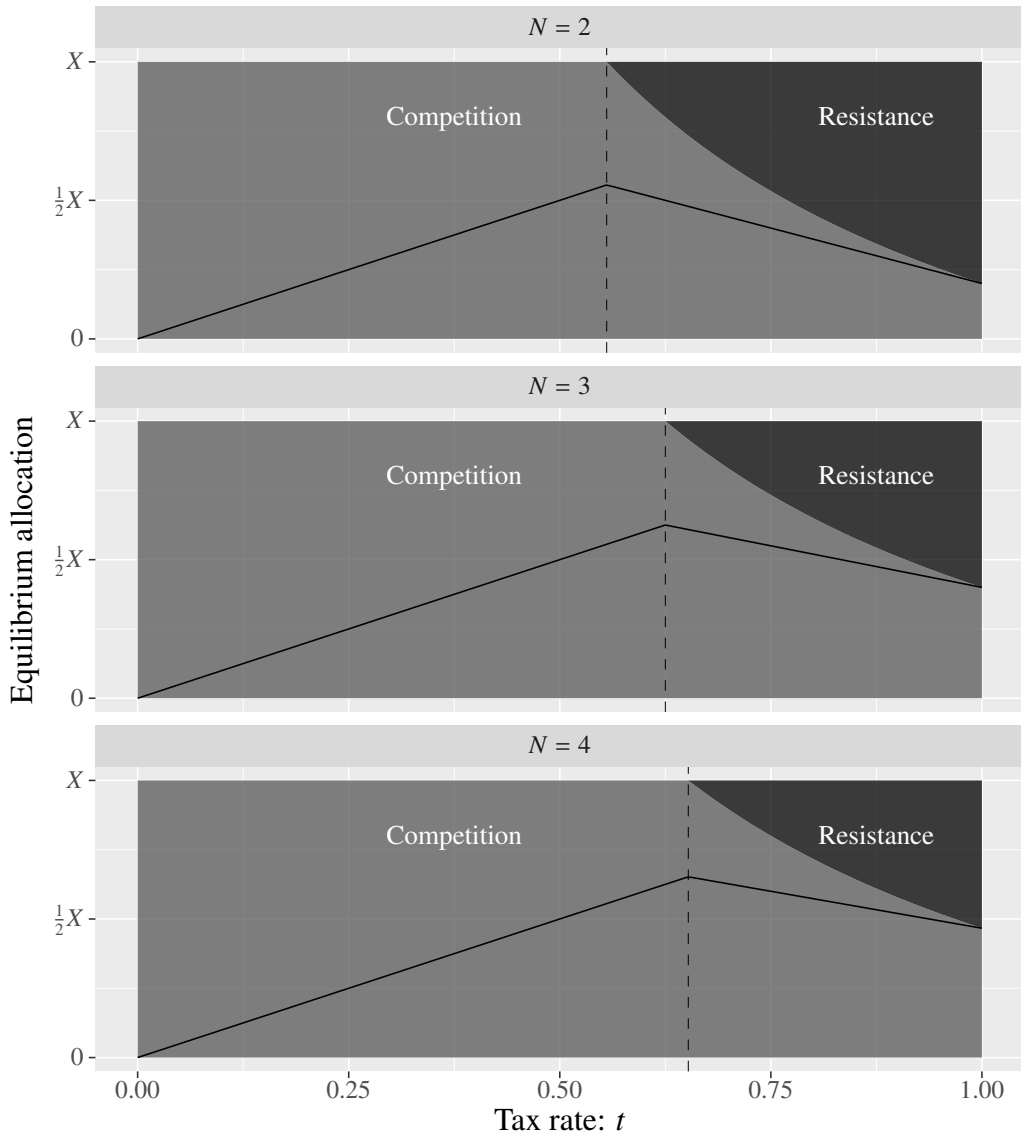


Figure 3. Equilibrium labor allocations in the plunder model, as described in Proposition 5, as a function of the tax rate and the number of factions. The solid curve is the government's payoff as a function of the tax rate, and the dashed line is the equilibrium tax rate. Parameters and functional forms are the same as in the previous figures, with $X = L = 2.5$.

number of factions, N . If there is a unique equilibrium tax rate t^ , the government's equilibrium payoff is locally increasing in competition effectiveness, π^c , if and only if the incentive effect outweighs the labor-saving effect at the corresponding equilibrium level of internal competition.*

Proposition 6, in combination with the results of the previous section, shows that the profitability of divide-and-rule politics depends critically on the nature of the predatory state's revenue base. If the main source of value is the product of the population's labor, as in the tribute model, then fractionalization and internal disorder decrease the incentive to produce, ultimately reducing the profits of expropriation. But if the source of contention between the government and the population is some fixed pool of goods or resources, as in the plunder model, the opposite logic prevails. In this case, internal conflict does not reduce the value of the pie, but it does keep the population distracted from resistance against the government.

The major empirical implication of these results is that the relationship between social order and the policies and profitability of extractive governance should be conditional on the nature of what is being extracted. All else equal, when a predatory state seeks tribute—i.e., to extract from the output of its subjects' labor—we should expect it to impose policies that reduce conflict and promote internal order, at least at the margins. By reducing appropriation among various ethnic groups or political factions, these policies raise the overall productivity of the governed population, which is profitable for the government. Moreover, we should expect extractive governance to be more profitable and stable in polities where structural conditions favor internal order. For example, imperial regimes funded by tribute should be less willing to expend resources to gain or maintain control over internally divided societies.

When the object of government extraction is a fixed source, such as a natural resource, we should expect these relationships to go the other direction. A plunder-seeking government will, at the margin, prefer policies that increase internal conflict and thereby reduce anti-government resistance, such as lax enforcement of competing groups' property rights. Colonies or occupations whose main objective is to secure control of some existing resource will be more successful when the population is more divided, or local conditions otherwise favor internal chaos. The answer to

“Does it pay to divide and rule?” is “It depends”—specifically, on what the ruler wants to get out of the society.

5 Conquest

The preceding analysis takes the identity of the ruler as fixed. I have shown that the profitability of divide-and-rule politics depends on the type of economic product from which the ruler’s rents derive. I now briefly consider the process of taking control, prior to the selection of the tax rate and subsequent division of society’s labor. When an outside force seeks to usurp authority, is it more likely to succeed when the population is more divided?

Whereas internal fractionalization is only *conditionally* beneficial for predatory governance, namely when the government’s objective is plunder, it *unconditionally* increases the prospects of an outsider seeking to gain control in the first place. This follows for much the same reason that in the post-conquest stage modeled above, fractionalization increases the tax rate the government can impose without engendering resistance (Remark 3). Resistance against an outsider’s attempt to take control is effectively a public good. As the number of factions increases, the incentive to provide this public good rather than to fend for oneself decreases (Olson 1965). Therefore, an outsider can more easily take control of a divided society than a unified one.

In the *conquest model*, a set of N factions compete with each other and with an outsider, denoted O , for the chance to be the government in the future. The incremental value of being the government is $v(N) > 0$, which may increase with N (as in a plunder economy) or decrease (as in a tribute economy). Each faction has L/N units of labor, which it may divide between two activities: $s_i \geq 0$, to prevent the outsider from taking over; and $d_i \geq 0$, to influence its own chance of becoming the government if the outsider fails. Each faction’s budget constraint is¹⁴

$$s_i + d_i = \frac{L}{N}. \quad (7)$$

¹⁴The assumption of unit productivity for each activity is without loss of generality. The model here with functional forms $\chi(S) = \tilde{\chi}(\pi^s S)$ and $\psi(D) = \tilde{\psi}(\pi^d D)$ is isomorphic to a model with the common budget constraint $s_i/\pi^s + d_i/\pi^d = L/N$ and functional forms $\tilde{\chi}$ and $\tilde{\psi}$.

The success of the attempted takeover depends on how much the factions spend to combat the outsider. I assume the outsider's military strength is a fixed value, $\bar{s}_O > 0$, so the outsider is not a strategic player here. The assumption that the outsider's strength is exogenous is of course a simplification, but it is plausible in situations where the outsider marshals its forces before fully understanding the internal political situation—such as in Cortés's incursion into the Mexican mainland, and other early maritime colonial ventures.¹⁵ The probability that the outsider becomes the government is

$$\frac{\bar{s}_O}{\bar{s}_O + \chi(\sum_{i=1}^N s_i)}, \quad (8)$$

where $\chi : [0, L] \rightarrow \mathbb{R}_+$ represents the translation of society's labor into its strength against the outsider. In case the outsider fails, the probability that faction i becomes the government is

$$\frac{\psi(d_i)}{\sum_{j=1}^N \psi(d_j)}, \quad (9)$$

where $\psi : [0, L/N] \rightarrow \mathbb{R}_+$ represents the translation of an individual faction's labor into its proportional chance of success against other factions. As with the function ϕ in the original model, I assume χ and ψ are strictly increasing and log-concave.

The factions simultaneously choose how to allocate their labor, subject to the budget constraint (7). A faction's utility function is

$$u_i(s, d) = \frac{\psi(d_i)}{\sum_{j=1}^N \psi(d_j)} \times \frac{\chi(\sum_{j=1}^N s_j)}{\bar{s}_O + \chi(\sum_{j=1}^N s_j)} \times v(N), \quad (10)$$

where $s = (s_1, \dots, s_N)$ and $d = (d_1, \dots, d_N)$.

The strategic tradeoff for the factions here is analogous to the tradeoff between resistance and internal competition in the tribute and plunder models. Critically, the relative marginal benefit of fighting the outsider declines as the number of factions increases. When the number of factions is large, any individual faction's chance of becoming the government if the outsider loses is small,

¹⁵With some additional restrictions on the parameters of the model, the results of the conquest game would be essentially the same if the outsider's strength were chosen endogenously.

which in turn reduces its incentive to contribute to the collective effort against the government. Consequently, as the following result states, the outsider is more likely to win the more divided the society is.

Proposition 7. *In the conquest model, the probability that the outsider wins is increasing in the number of factions, N .*

To be clear, unlike some of the earlier results, Proposition 7 does not address how fractionalization affects the outsider's overall welfare in equilibrium. In particular, if the outsider's ultimate objective is tribute, there is no guarantee that the increase in the chance of winning due to greater fractionalization would offset the decrease in $v(N)$. Proposition 7, in combination with the earlier results, implies that a society that is easy to conquer may nevertheless be difficult to govern. Specifically, fractionalization benefits a tribute-seeking government in the conquest stage, but not in the governance stage. Only for a plunder-seeking government, which benefits from internal chaos even while governing, does fractionalization have the same effect on ease of conquest and the profitability of rule.

The conquest model captures how Cortés benefited from internal divisions in Aztec society. He was able to conquer with significantly less military support than would have been necessary otherwise, because the incumbent regime also had to contend with its internal enemies (Elliott 2007; Burkholder and Johnson 2015). The Dutch East India Company similarly exploited internal divisions when initially establishing its foothold in present-day Indonesia (Scammell 1989, 20). In pure military competition, the political economy issues that arise in tribute extraction—namely, the tradeoff between internal conflict and economic productivity—are sidelined. Only after establishing control does internal fractionalization become a potential problem for the predatory state.

6 Conclusion

I have characterized the political economy of predatory rule in a divided society. The main result is that the profitability of divide-and-rule politics depends on the nature of the economic product the ruler wishes to extract. When it is the output of the population's labor, or tribute, the ruler is better off when society is less deeply divided. The opposite is true for a government that seeks to expropriate from an exogenously fixed source of value, or plunder. Additionally, regardless of the ultimate extractive aims, internal fractionalization increases the likelihood of gaining control in the first place.

Besides its contributions to the literatures on social conflict and the political economy of colonialism, the model here may also have applications in the study of state formation. Early states are typically thought of as predatory institutions (Tilly 1985; Olson 1993; Sánchez de la Sierra 2017). My results suggest that it sometimes may be profitable for such a state to refrain from establishing total sovereignty—to allow some banditry to take place within its sphere of ostensible authority. If the proto-state's main revenue source is the output of its subjects, then eliminating internal appropriation and establishing full sovereignty would indeed be optimal if possible. But if it benefits mainly from control of some fixed resource, such as control of an economically or strategically important waterway, the early state may be better off tolerating some infighting so as to minimize the chance of a serious competitor emerging from within.

My results also speak to the literature on international conflict. The most influential theory of conflict posits that war is the result of bargaining failure (Fearon 1995). Although international relations theorists have made tremendous progress identifying how and why bargaining might break down, the theoretical literature has relatively little to say about the issues that states bargain over. The model here provides a novel explanation of why some territory is more valuable than others—an important question, in light of the centrality of territorial disputes as a cause of war (Goertz and Diehl 1992). In particular, there is an important interaction between natural resource wealth and social fractionalization. Internal divisions should increase the value of resource-rich territory, but decrease the value of territory with relatively few natural resources. Consequently, in the empirical

record, we should expect fractionalization to have differential effects on the probability of a crisis breaking out. Similarly, the theory provides a political economy foundation for the regularity that new interstate borders tend to follow previous administrative frontiers (Carter and Goemans 2011): boundary changes that split ethnic groups may threaten a predatory state's revenues even if its territorial holdings increase overall, namely by increasing communal violence (see also Michalopoulos and Papaioannou 2016).

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A Appendix to “Social Conflict and the Predatory State”

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A.1 Additional Notation

Throughout the appendix, let $\Phi(c) = \sum_i \phi(c_i)$. We have $\log \omega_i(c) = \log \phi(c_i) - \log \Phi(c)$ and thus

$$\begin{aligned} \frac{\partial \log \omega_i(c)}{\partial c_i} &= \frac{\phi'(c_i)}{\phi(c_i)} - \frac{\phi'(c_i)}{\Phi(c)} \\ &= \frac{\phi'(c_i)}{\phi(c_i)} \left(1 - \frac{\phi(c_i)}{\Phi(c)}\right) \\ &= \hat{\phi}'(c_i)(1 - \omega_i(c)), \end{aligned}$$

where $\hat{\phi} = \log \phi$. Because ϕ is strictly increasing and log-concave, $\hat{\phi}' > 0$ and $\hat{\phi}'' \leq 0$.

A.2 Equilibrium Existence and Uniqueness

For the existence and uniqueness results, I consider a more general version of the model presented in the text. I allow groups to be asymmetric in their size and productivities, which entails generalizing each faction i 's budget constraint (1) to

$$\frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} = L_i, \quad (11)$$

where $\pi_i^p, \pi_i^r, \pi_i^c, L_i > 0$. In addition, the results here do not depend on Assumption 1.

Let $\Gamma(t)$ denote the subgame that follows the government's selection of t , in which the factions simultaneously decide how to allocate their labor. Let $\sigma_i = (p_i, r_i, c_i)$ be a strategy for faction i in the subgame, and let

$$\Sigma_i = \left\{ (p_i, r_i, c_i) \left| \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} = L_i \right. \right\}$$

denote the strategy space. Let $\sigma = (\sigma_1, \dots, \sigma_N)$ and $\Sigma = \times_{i=1}^N \Sigma_i$.

I begin by proving that a Nash equilibrium exists in each subgame. The task is complicated by the potential discontinuity of the factions' payoffs, namely at $c = 0$ when $\phi(0) = 0$. I rely

on Reny's (1999) conditions for the existence of pure strategy equilibria in a discontinuous game. The key condition is *better-reply security*—informally, that at least one player can assure a strict benefit by deviating from any non-equilibrium strategy profile, even if the other players make slight deviations.

Lemma 1. $\Gamma(t)$ is better-reply secure.

Proof. Let $U^t : \Sigma \rightarrow \mathbb{R}_+^N$ be the vector payoff function for the factions in $\Gamma(t)$, so that $U^t(\sigma) = (u_1(t, \sigma), \dots, u_N(t, \sigma))$. Take any convergent sequence in the graph of U^t , call it $(\sigma^k, U^t(\sigma^k)) \rightarrow (\sigma^*, U^*)$, such that σ^* is not an equilibrium of $\Gamma(t)$. Because production and the effective tax rate are continuous in (p, r) , we have

$$U_i^* = w_i^* \times \bar{\tau}(t, r^*) \times f(p^*)$$

for each i , where $w_i^* \geq 0$ and $\sum_{i=1}^N w_i^* = 1$. I must show there is a player i who can secure a payoff $\bar{U}_i > U_i^*$ at σ^* ; i.e., there exists $\bar{\sigma}_i \in \Sigma_i$ such that $u_i(t, \bar{\sigma}_i, \sigma'_{-i}) \geq \bar{U}_i$ for all σ'_{-i} in a neighborhood of σ^*_{-i} (Reny 1999, 1032).

If $N = 1$ or $\Phi(c^*) > 0$, then U^t is continuous in a neighborhood of σ^* , so the conclusion is immediate. If $\bar{\tau}(t, r^*) \times f(p^*) = 0$, then each $U_i^* = 0$ and each faction can assure a strictly greater payoff by deviating to a strategy with positive production, resistance, and competition. For the remaining cases, suppose $N > 1$, $\bar{\tau}(t, r^*) \times f(p^*) > 0$, and $\Phi(c^*) = 0$, the latter of which implies $c^* = 0$ and $\phi(0) = 0$. Since $N > 1$, there is a faction i such that $w_i^* < 1$. Take any $\epsilon \in (0, (1 - w_i^*)/2)$ and any $\delta_1 > 0$ such that

$$\bar{\tau}(t, r') \times f(p') \geq (w_i^* + 2\epsilon) \times \bar{\tau}(t, r^*) \times f(p^*)$$

for all σ' in a δ_1 -neighborhood of σ^* . Since $w_i^* + 2\epsilon < 1$ and $\bar{\tau}(t, r) \times f(p)$ is continuous in (p, r) , such a δ_1 exists. Then let $\bar{\sigma}_i = (\bar{p}_i, \bar{r}_i, \bar{c}_i)$ be any strategy in a δ_1 -neighborhood of σ_i^* such that $\bar{c}_i > 0$. Because $c^*_{-i} = 0$ and ϕ is continuous, there exists $\delta_2 > 0$ such that

$$\omega_i(\bar{c}_i, c'_{-i}) = \frac{\phi(\bar{c}_i)}{\phi(\bar{c}_i) + \sum_{j \in N \setminus \{i\}} \phi(c'_j)} \geq \frac{w_i^* + \epsilon}{w_i^* + 2\epsilon}$$

for all σ'_{-i} in a δ_2 -neighborhood of σ^*_{-i} . Therefore, for all σ'_{-i} in a $\min\{\delta_1, \delta_2\}$ -neighborhood of σ^*_{-i} , we have

$$u_i(t, \bar{\sigma}_i, \sigma'_{-i}) \geq (w_i^* + \epsilon) \times \bar{\tau}(t, r^*) \times f(p^*) > U_i^*,$$

establishing the claim. □

The other main condition for equilibrium existence is that each faction's utility function be quasiconcave in its own actions. I prove this by showing that the logarithm of a faction's utility function is concave in its actions.

Lemma 2. $\Gamma(t)$ is log-concave.

Proof. Take any (p, r, c) such that $u_i(t, p, r, c) > 0$, and let $P = \sum_j p_j$ and $R = \sum_j r_j$. First, assume

$\sum_{j \neq i} \phi(c_j) > 0$, so that u_i is continuously differentiable in (p_i, r_i, c_i) . We have

$$\begin{aligned}\frac{\partial \log u_i(t, p, r, c)}{\partial p_i} &= \frac{1}{P}, \\ \frac{\partial \log u_i(t, p, r, c)}{\partial r_i} &= \frac{-tg'(R)}{1 - tg(R)}, \\ \frac{\partial \log u_i(t, p, r, c)}{\partial c_i} &= \hat{\phi}'(c_i)(1 - \omega_i(c)),\end{aligned}$$

and therefore

$$\begin{aligned}\frac{\partial^2 \log u_i(t, p, r, c)}{\partial p_i^2} &= \frac{-1}{P^2} < 0, \\ \frac{\partial^2 \log u_i(t, p, r, c)}{\partial r_i^2} &= \frac{-tg''(R)(1 - tg(R)) - (tg'(R))^2}{(1 - tg(R))^2} \leq 0, \\ \frac{\partial^2 \log u_i(t, p, r, c)}{\partial c_i^2} &= \hat{\phi}''(c_i)(1 - \omega_i(c)) - \hat{\phi}'(c_i) \frac{\partial \omega_i(c)}{\partial c_i} \leq 0, \\ \frac{\partial^2 \log u_i(t, p, r, c)}{\partial p_i \partial r_i} &= \frac{\partial^2 \log u_i(t, p, r, c)}{\partial p_i \partial c_i} = \frac{\partial^2 \log u_i(t, p, r, c)}{\partial r_i \partial c_i} = 0,\end{aligned}$$

so $\log u_i$ is concave in (p_i, r_i, c_i) . By the same token, $\bar{\tau}(t, r) \times f(p)$ is log-concave in (p, r) regardless of whether $\sum_{j \neq i} \phi(c_j) > 0$.

Now assume $\sum_{j \neq i} \phi(c_j) = 0$. Take any (p', r', c') such that $u_i(t, p', r', c') > 0$, where $(p', r', c') = ((p'_i, p_{-i}), (r'_i, r_{-i}), (c'_i, c_{-i}))$. Take any $\alpha \in [0, 1]$, and let $(p^\alpha, r^\alpha, c^\alpha) = \alpha(p, r, c) + (1 - \alpha)(p', r', c')$. If $c_i = c'_i = 0$, then $\omega_i(c^\alpha) = \omega_i(c) = \omega_i(c') = 1/N$ and thus

$$\begin{aligned}\log u_i(t, p^\alpha, r^\alpha, c^\alpha) &= \log \frac{1}{N} + \log \bar{\tau}(t, r^\alpha) + \log f(p^\alpha) \\ &\geq \log \frac{1}{N} + \alpha (\log \bar{\tau}(t, r) + \log f(p)) \\ &\quad + (1 - \alpha) (\log \bar{\tau}(t, r') + \log f(p')) \\ &= \alpha \log u_i(t, p, r, c) + (1 - \alpha) \log u_i(t, p', r', c'),\end{aligned}$$

where the inequality follows from the log-concavity of $\bar{\tau}(t, r) \times f(p)$ in (p, r) . If $c_i > 0$ and $c'_i = 0$, then $\omega_i(c^\alpha) = \omega_i(c) = 1$, $\omega_i(c') = 1/N$, and thus

$$\begin{aligned}\log u_i(t, p^\alpha, r^\alpha, c^\alpha) &= \log \bar{\tau}(t, r^\alpha) + \log f(p^\alpha) \\ &\geq \alpha (\log \bar{\tau}(t, r) + \log f(p)) \\ &\quad + (1 - \alpha) \left(\log \frac{1}{N} + \log \bar{\tau}(t, r') + \log f(p') \right) \\ &= \alpha \log u_i(t, p, r, c) + (1 - \alpha) \log u_i(t, p', r', c').\end{aligned}$$

The same argument holds in case $c_i = 0$ and $c'_i > 0$. It is easy to see that the same conclusion holds if $c_i > 0$ and $c'_i > 0$, in which case $\omega_i(c^\alpha) = \omega_i(c) = \omega_i(c') = 1$. Therefore, $\log u_i$ is concave in

(p_i, r_i, c_i) . □

Equilibrium existence follows immediately from the two preceding lemmas.

Proposition 8. $\Gamma(t)$ has a pure strategy equilibrium.

Proof. The strategy space Σ is compact, each payoff function u_i is bounded on Σ , and $\Gamma(t)$ is better-reply secure (Lemma 1) and quasiconcave (Lemma 2). Therefore, a pure strategy equilibrium exists (Reny 1999, Theorem 3.1). □

I now turn to the question of uniqueness. I show that although $\Gamma(t)$ may have multiple equilibria, these equilibria are identical in terms of three essential characteristics: total production, $\sum_i p_i$; total resistance, $\sum_i r_i$; and the vector of individual expenditures on internal competition, c .

To prove essential uniqueness, I must characterize the equilibrium more fully than I have up to this point. The following result rules out equilibria in which (1) a faction's share in the internal competition is zero or (2) a faction could raise its share to one by an infinitesimal change in strategy.

Lemma 3. If $N > 1$, then each $\phi(c_i) > 0$ in any equilibrium of $\Gamma(t)$.

Proof. Assume $N > 1$, and let (p, r, c) be a strategy profile of $\Gamma(t)$ in which $c_i = 0$ for some $i \in \mathcal{N}$. The claim holds trivially if $\phi(0) > 0$, so assume $\phi(0) = 0$. If $\Phi(c) > 0$ or $\bar{\tau}(t, r) \times f(p) = 0$, then $u_i(t, p, r, c) = 0$. But i could ensure a strictly positive payoff with any strategy that allocated nonzero labor to production, resistance, and competition, so (p, r, c) is not an equilibrium. Conversely, suppose $\Phi(c) = 0$, which implies $c_j = 0$ for all $j \in \mathcal{N}$, and $\bar{\tau}(t, r) \times f(p) > 0$. Then $u_i(t, p, r, c) = (\bar{\tau}(t, r) \times f(p))/N$. But i could obtain a payoff arbitrarily close to $\bar{\tau}(t, r) \times f(p)$ by diverting an infinitesimal amount of labor away from production or resistance and into internal competition, so (p, r, c) is not an equilibrium. □

This result is important because it implies the game is continuously differentiable in the neighborhood of any equilibrium. Equilibria can therefore be characterized in terms of first-order conditions.

Lemma 4. (p', r', c') is an equilibrium of $\Gamma(t)$ if and only if, for each $i \in \mathcal{N}$,

$$p'_i \left(\pi_i^p \frac{\partial \log f(p')}{\partial p_i} - \mu_i \right) = 0, \quad (12)$$

$$r'_i \left(\pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} - \mu_i \right) = 0, \quad (13)$$

$$c'_i \left(\pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} - \mu_i \right) = 0, \quad (14)$$

$$\frac{p'_i}{\pi_i^p} + \frac{r'_i}{\pi_i^r} + \frac{c'_i}{\pi_i^c} - L_i = 0, \quad (15)$$

where

$$\mu_i = \max \left\{ \pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i}, \pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} \right\}.$$

Proof. In equilibrium, each faction's strategy must solve the constrained maximization problem

$$\begin{aligned} \max_{p_i, r_i, c_i} \quad & \log u_i(t, p, r, c) \\ \text{s.t.} \quad & \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} - L_i = 0, \\ & p_i \geq 0, r_i \geq 0, c_i \geq 0. \end{aligned}$$

It follows from Lemma 3 that each u_i is C^1 in (p_i, r_i, c_i) in a neighborhood of any equilibrium. This allows use of the Karush–Kuhn–Tucker conditions to characterize solutions of the above problem. The “only if” direction holds because (12)–(15) are the first-order conditions for the problem and the linearity constraint qualification holds. The “if” direction holds because $\log u_i$ is concave in (p_i, r_i, c_i) , per Lemma 2. \square

A weak welfare optimality result follows almost immediately from this equilibrium characterization. If (p', r', c') is an equilibrium of $\Gamma(t)$, then there is no other equilibrium (p'', r'', c'') such that $c'' = c'$ and $\bar{\tau}(t, r'') \times f(p'') > \bar{\tau}(t, r') \times f(p')$. In other words, taking as fixed the factions' allocations toward internal competition, there is no inefficient misallocation of labor between production and resistance.

Corollary 1. *If (p', r', c') is an equilibrium of $\Gamma(t)$, then (p', r') solves*

$$\begin{aligned} \max_{p, r} \quad & \log \bar{\tau}(t, r) + \log f(p) \\ \text{s.t.} \quad & \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} = L_i - \frac{c'_i}{\pi_i^c}, \quad i = 1, \dots, N, \\ & p_i \geq 0, r_i \geq 0, \quad i = 1, \dots, N. \end{aligned}$$

Proof. This is a C^1 concave maximization problem with linear constraints, so the Karush–Kuhn–Tucker first-order conditions are necessary and sufficient for a solution. The result then follows from Lemma 4. \square

I next prove that if post-tax output is weakly greater in one equilibrium of $\Gamma(t)$ than another, then each of the two individual components (production and the factions' total share) is weakly greater. The proof relies on the fact that if $c'_i \leq c''_i$ and $\omega_i(c') \leq \omega_i(c'')$, then

$$\frac{\partial \log \omega_i(c')}{\partial c_i} = \hat{\phi}'(c'_i)(1 - \omega_i(c')) \geq \hat{\phi}'(c''_i)(1 - \omega_i(c'')) = \frac{\partial \log \omega_i(c'')}{\partial c_i}.$$

If in addition $\omega_i(c') < \omega_i(c'')$, the inequality is strict.

Lemma 5. *If (p', r', c') and (p'', r'', c'') are equilibria of $\Gamma(t)$ such that $\bar{\tau}(t, r') \times f(p') \geq \bar{\tau}(t, r'') \times f(p'')$, then $\bar{\tau}(t, r') \geq \bar{\tau}(t, r'')$ and $f(p') \geq f(p'')$.*

Proof. Suppose the claim of the lemma does not hold, so there exist equilibria such that $\bar{\tau}(t, r') \times f(p') \geq \bar{\tau}(t, r'') \times f(p'')$ but $\bar{\tau}(t, r') < \bar{\tau}(t, r'')$. Together, these inequalities imply $f(p') > f(p'')$. (The proof in case $\bar{\tau}(t, r') > \bar{\tau}(t, r'')$ and $f(p') < f(p'')$ is analogous.)

I will first establish that $p'_i > 0$ implies $r''_i = 0$. Per Lemma 4 and the log-concavity of f and $\bar{\tau}$, $p'_i > 0$ implies

$$\pi_i^p \frac{\partial \log f(p'')}{\partial p_i} > \pi_i^p \frac{\partial \log f(p')}{\partial p_i} \geq \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} > \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i}.$$

Therefore, again by Lemma 4, $r''_i = 0$.

Next, I establish that $\Phi(c'') > \Phi(c')$. Since $f(p') > f(p'')$, there is a faction $i \in \mathcal{N}$ such that $p'_i > p''_i$. As this implies $r''_i = 0$, the budget constraint gives $c''_i > c'_i$. If $\Phi(c'') \leq \Phi(c')$, then $\omega_i(c'') > \omega_i(c')$ and thus by Lemma 4

$$\pi_i^c \frac{\partial \log \omega_i(c'')}{\partial c_i} > \pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} \geq \pi_i^p \frac{\partial \log f(p'')}{\partial p_i} > \pi_i^p \frac{\partial \log f(p')}{\partial p_i}.$$

But this implies $p'_i = 0$, a contradiction. Therefore, $\Phi(c'') > \Phi(c')$.

Using these intermediate results, I can now establish the main claim by contradiction. Since $\bar{\tau}(t, r'') > \bar{\tau}(t, r')$, there is a faction $j \in \mathcal{N}$ such that $r''_j > r'_j$. This implies $p'_j = 0$, so the budget constraint gives $c''_j < c'_j$. Since $\Phi(c'') > \Phi(c')$, this in turn gives $\omega_j(c'') < \omega_j(c')$ and thus

$$\pi_j^c \frac{\partial \log \omega_j(c'')}{\partial c_j} > \pi_j^c \frac{\partial \log \omega_j(c')}{\partial c_j} \geq \pi_j^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j} > \pi_j^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j}.$$

But this implies $r''_j = 0$, a contradiction. □

I can now state and prove the essential uniqueness of the equilibrium of each labor allocation subgame.

Proposition 9. *If (p', r', c') and (p'', r'', c'') are equilibria of $\Gamma(t)$, then $f(p') = f(p'')$, $\bar{\tau}(t, r') = \bar{\tau}(t, r'')$, and $c' = c''$.*

Proof. First I prove that $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$. Suppose not, so that, without loss of generality, $\bar{\tau}(t, r') \times f(p') > \bar{\tau}(t, r'') \times f(p'')$. Then Lemma 5 implies $\bar{\tau}(t, r') \geq \bar{\tau}(t, r'')$ and $f(p') \geq f(p'')$, at least one strictly so, and thus

$$\max \left\{ \pi_i^p \frac{\partial \log f(p'')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i} \right\} \geq \max \left\{ \pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} \right\}$$

for all $i \in \mathcal{N}$, strictly so for some $j \in \mathcal{N}$. Since $\bar{\tau}(t, r'') \times f(p'') < \bar{\tau}(t, r') \times f(p')$, it follows from Corollary 1 that the set $\mathcal{N}^+ = \{i \in \mathcal{N} \mid c''_i > c'_i\}$ is nonempty. For any $i \in \mathcal{N}^+$ such that $\omega_i(c'') > \omega_i(c')$,

$$\begin{aligned} \pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} &> \pi_i^c \frac{\partial \log \omega_i(c'')}{\partial c_i} \\ &\geq \max \left\{ \pi_i^p \frac{\partial \log f(p'')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i} \right\} \\ &\geq \max \left\{ \pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} \right\}. \end{aligned}$$

But this implies $p'_i = r'_i = 0$, contradicting $c''_i > c'_i$. So $\omega_i(c'') \leq \omega_i(c')$ for all $i \in \mathcal{N}^+$. Since \mathcal{N}^+ is nonempty and the competition shares are increasing in c_i and sum to one, this can hold only if $\mathcal{N}^+ = \mathcal{N}$ and $\omega_i(c'') = \omega_i(c')$ for all $i \in \mathcal{N}$. This implies

$$\begin{aligned} \pi_j^c \frac{\partial \log \omega_j(c')}{\partial c_j} &\geq \pi_j^c \frac{\partial \log \omega_j(c'')}{\partial c_j} \\ &\geq \max \left\{ \pi_j^p \frac{\partial \log f(p'')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j} \right\} \\ &> \max \left\{ \pi_j^p \frac{\partial \log f(p')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j} \right\}, \end{aligned}$$

which in turn implies $p'_j = r'_j = 0$, contradicting $c''_j > c'_j$. I conclude that $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$ and thus, by Lemma 5, $\bar{\tau}(t, r') = \bar{\tau}(t, r'')$ and $f(p') = f(p'')$.

Next, I prove that $c' = c''$. Suppose not, so $c' \neq c''$. Without loss of generality, suppose $\Phi(c') \geq \Phi(c'')$. Since $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$ yet $c' \neq c''$, by Corollary 1 there exists $i \in \mathcal{N}$ such that $c'_i > c''_i$ and $j \in \mathcal{N}$ such that $c'_j < c''_j$. It follows from $\Phi(c') \geq \Phi(c'')$ that $\omega_j(c') < \omega_j(c'')$ and therefore

$$\begin{aligned} \pi_j^c \frac{\partial \log \omega_j(c')}{\partial c_j} &> \pi_j^c \frac{\partial \log \omega_j(c'')}{\partial c_j} \\ &\geq \max \left\{ \pi_j^p \frac{\partial \log f(p'')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j} \right\} \\ &= \max \left\{ \pi_j^p \frac{\partial \log f(p')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j} \right\}. \end{aligned}$$

But this implies $p'_j = r'_j = 0$, contradicting $c''_j > c'_j$. □

Proposition 9 allows me to write the equilibrium values of total production, total resistance, and individual conflict allocations as functions of the tax rate. For each $t \in [0, 1]$, let $P^*(t) = P$ if and only if there is an equilibrium (p, r, c) of $\Gamma(t)$ such that $\sum_i p_i = P$. Let the functions $R^*(t)$ and $c^*(t)$, the latter of which is vector-valued, be defined analogously.

The only remaining step to prove the existence of an equilibrium in the full game is to show that an optimal tax rate exists. An important consequence of Proposition 9 is that the optimal tax rate (if one exists) does not depend on the equilibrium that is selected in each labor allocation subgame, since the government's payoff depends only on total production and resistance. The main step toward proving the existence of an optimal tax rate is to show that total production and resistance are continuous in t .

Lemma 6. P^* , R^* , and c^* are continuous.

Proof. Define the equilibrium correspondence $E : [0, 1] \rightrightarrows \Sigma$ by

$$E(t) = \{(p, r, c) \mid (p, r, c) \text{ is an equilibrium of } \Gamma(t)\}.$$

Standard arguments (e.g., Fudenberg and Tirole 1991, 30–32) imply that E has a closed graph.¹⁶ This in turn implies E is upper hemicontinuous, as its codomain, Σ , is compact. Let $F : \Sigma \rightarrow \mathbb{R}_+^{N+2}$ be the function defined by $F(p, r, c) = (\sum_i p_i, \sum_i r_i, c)$. Since F is continuous as a function, it is upper hemicontinuous as a correspondence. Then we can write the functions in the lemma as the composition of F and E :

$$(P^*(t), R^*(t), c^*(t)) = \{F(p, r, c) \mid (p, r, c) \in E(t)\} = (F \circ E)(t).$$

As the composition of upper hemicontinuous correspondences, (P^*, R^*, c^*) is upper hemicontinuous (Aliprantis and Border 2006, Theorem 17.23). Then, as an upper hemicontinuous correspondence that is single-valued (per Proposition 9), (P^*, R^*, c^*) is continuous as a function. \square

Continuity of total production and resistance in the tax rate imply that the government's payoff is continuous in the tax rate, so an equilibrium exists.

Proposition 10. *There is a pure strategy equilibrium.*

Proof. For each labor allocation subgame $\Gamma(t)$, let $\sigma^*(t)$ be a pure strategy equilibrium of $\Gamma(t)$. Proposition 8 guarantees the existence of these equilibria. By Proposition 9, the government's payoff from any $t \in [0, 1]$ is

$$u_G(t, \sigma^*(t)) = t \times g(R^*(t)) \times P^*(t).$$

This expression is continuous in t , per Lemma 6, and therefore attains its maximum on the compact interval $[0, 1]$. A maximizer t^* exists, and the pure strategy profile $(t^*, (\sigma^*(t))_{t \in [0, 1]})$ is an equilibrium. \square

A.3 Proof of Propositions 1 and 2

In these and all remaining proofs, I consider the special symmetric case of the model discussed in the text, in which each $\pi_i^p = \pi^p$, $\pi_i^r = \pi^r$, $\pi_i^c = \pi^c$, and $L_i = L/N$. An important initial result for the symmetric case is that in every equilibrium of every labor allocation subgame, every faction spends the same amount on internal competition.

Lemma 7. *If the game is symmetric and (p, r, c) is an equilibrium of $\Gamma(t)$, then $c_i = c_j$ for all $i, j \in \mathcal{N}$.*

Proof. Consider an equilibrium in which $c_i > c_j$. This implies $\omega_i(c) > \omega_j(c)$ and therefore

$$\begin{aligned} \pi^c \frac{\partial \log \omega_j(c)}{\partial c_j} &> \pi^c \frac{\partial \log \omega_i(c)}{\partial c_i} \\ &\geq \max \left\{ \pi^p \frac{\partial \log f(p)}{\partial p_i}, \pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} \right\} \end{aligned}$$

¹⁶The only complication in applying the usual argument is that the model is discontinuous at $c = 0$ in case $\phi(0) = 0$. However, by the same arguments as in the proof of Lemma 1, if $\phi(0) = 0$ there cannot be a sequence $(t^k, (p^k, r^k, c^k))$ in the graph of E such that $c^k \rightarrow 0$.

$$= \max \left\{ \pi^p \frac{\partial \log f(p)}{\partial p_j}, \pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_j} \right\}.$$

Lemma 4 then gives $p_j = r_j = 0$. But since i and j have the same budget constraint, this contradicts $c_i > c_j$. \square

In the equilibrium of each labor allocation subgame, total production, total resistance, and the amount each faction spends on conflict (the same for all factions by Lemma 7) solve some subset of the following system of equations. These equations give the conditions for equal marginal benefits per unit of labor across activities, as well as the budget constraint (1). I write them as functions of t as well as the exogenous parameters $\pi = (\pi^p, \pi^r, \pi^c, L, N)$ to allow for comparative statics via implicit differentiation:

$$Q^{pr}(P, R, C; t, \pi) = \pi^p(1 - tg(R)) + \pi^r t P g'(R) = 0, \quad (16)$$

$$Q^{pc}(P, R, C; t, \pi) = \frac{\pi^p}{P} - \frac{N-1}{N} \pi^c \hat{\phi}'(C) = 0, \quad (17)$$

$$Q^{rc}(P, R, C; t, \pi) = \frac{\pi^r t g'(R)}{1 - tg(R)} + \frac{N-1}{N} \pi^c \hat{\phi}'(C) = 0, \quad (18)$$

$$Q^b(P, R, C; t, \pi) = L - \frac{P}{\pi^p} - \frac{R}{\pi^r} - \frac{NC}{\pi^c} = 0. \quad (19)$$

The condition (18) is redundant when (16) and (17) both hold, but I use it later in the plunder model.

The quantities defined in Propositions 1 and 2 are as follows. (\bar{P}_0, \bar{c}_0) is the solution to the system

$$Q^{pc}(\bar{P}_0, 0, \bar{c}_0; t, \pi) = \frac{\pi^p}{\bar{P}_0} - \frac{N-1}{N} \pi^c \hat{\phi}'(\bar{c}_0) = 0, \quad (20)$$

$$Q^b(\bar{P}_0, 0, \bar{c}_0; t, \pi) = L - \frac{\bar{P}_0}{\pi^p} - \frac{N\bar{c}_0}{\pi^c} = 0. \quad (21)$$

$(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t))$ is the solution to the system

$$Q^{pr}(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = \pi^p(1 - tg(\tilde{R}_1(t))) + \pi^r t \tilde{P}_1(t) g'(\tilde{R}_1(t)) = 0, \quad (22)$$

$$Q^{pc}(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = \frac{\pi^p}{\tilde{P}_1(t)} - \frac{N-1}{N} \pi^c \hat{\phi}'(\tilde{c}_1(t)) = 0, \quad (23)$$

$$Q^b(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = L - \frac{\tilde{P}_1(t)}{\pi^p} - \frac{\tilde{R}_1(t)}{\pi^r} - \frac{N\tilde{c}_1(t)}{\pi^c} = 0. \quad (24)$$

$(\tilde{P}_2(t), \tilde{R}_2(t))$ is the solution to the system

$$Q^{pr}(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi) = \pi^p(1 - tg(\tilde{R}_2(t))) + \pi^r t \tilde{P}_2(t) g'(\tilde{R}_2(t)) = 0, \quad (25)$$

$$Q^b(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi) = L - \frac{\tilde{P}_2(t)}{\pi^p} - \frac{\tilde{R}_2(t)}{\pi^r} = 0. \quad (26)$$

The first cutpoint tax rate is

$$\hat{t}_0 = \frac{\pi^p}{\pi^p - \pi^r \bar{P}_0 g'(0)}. \quad (27)$$

Lemma 8 below shows that $\bar{P}_0 > 0$ and therefore, since $g'(0) < 0$, that $\hat{t}_0 < 1$. The second cutpoint tax rate is

$$\hat{t}_1 = \frac{\pi^p}{\pi^p g(\bar{R}_1) - \pi^r \bar{P}_1 g'(\bar{R}_1)}, \quad (28)$$

where

$$\bar{P}_1 = \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(0)}, \quad (29)$$

$$\bar{R}_1 = \pi^r \left(L - \frac{\bar{P}_1}{\pi^p} \right). \quad (30)$$

The next three lemmas give conditions on the tax rate under which there is positive production, resistance, and internal competition in the equilibrium of the labor allocation subgame. Jointly, these lemmas constitute the bulk of the proof of Propositions 1 and 2. The proofs rely on the following equalities:

$$\begin{aligned} \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, 0)}{\partial r_i} &= -\pi^r \frac{\hat{t}_0 g'(0)}{1 - \hat{t}_0} = \frac{\pi^p}{\bar{P}_0}, \\ \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, (\bar{R}_1/N) \mathbf{1}_N)}{\partial r_i} &= -\pi^r \frac{\hat{t}_1 g'(\bar{R}_1)}{1 - \hat{t}_1 g(\bar{R}_1)} = \frac{\pi^p}{\bar{P}_1} \end{aligned}$$

for all $i \in \mathcal{N}$, where $\mathbf{1}_N$ is the N -vector each of whose elements equals one.

Lemma 8. *If the game is symmetric, Assumption 1 holds, and (p, r, c) is an equilibrium of $\Gamma(t)$, then $0 < \sum_i p_i \leq \bar{P}_0 < \pi^p L$.*

Proof. Assumption 1 implies

$$Q^{pc}(\pi^p L, 0, 0; 0, \pi) = \frac{1}{L} - \frac{N-1}{N} \pi^c \hat{\phi}'(0) < 0.$$

Since Q^{pc} is decreasing in P and weakly increasing in C , this gives $\bar{P}_0 < \pi^p L$.

Let $P = \sum_i p_i$, and suppose $P > \bar{P}_0$. The budget constraint and Lemma 7 then give $c_i = C < \bar{c}_0$ for each $i \in \mathcal{N}$. But then we have

$$\pi^c \frac{\partial \log \omega_i(c)}{\partial c_i} \geq \frac{N-1}{N} \pi^c \hat{\phi}'(\bar{c}_0) = \frac{\pi^p}{\bar{P}_0} > \pi^p \frac{\partial \log f(p)}{\partial p_i}$$

for each $i \in \mathcal{N}$. By Lemma 4, this implies each $p_i = 0$, a contradiction. Therefore, $P \leq \bar{P}_0$.

Finally, since $P = 0$ implies each $u_i(t, p, r, c) = 0$, but any faction can assure itself a positive payoff with any $(p_i, r_i, c_i) \gg 0$, in equilibrium $P > 0$. \square

Lemma 9. *If the game is symmetric, Assumption 1 holds, and (p, r, c) is an equilibrium of $\Gamma(t)$,*

then $\sum_i r_i > 0$ if and only if $t > \hat{t}_0$.

Proof. Let $P = \sum_i p_i$ and $R = \sum_i r_i$. To prove the “if” direction, suppose $t > \hat{t}_0$ and $R = 0$. Since $P \leq \bar{P}_0$, this implies each $c_i = C > 0$; the first-order conditions of Lemma 4 then give $P = \bar{P}_0$ and $C = \bar{c}_0$. It follows that

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, r)}{\partial r_i} = \frac{\pi^p}{\bar{P}_0} = \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

This implies each $p_i = 0$, a contradiction.

To prove the “only if” direction, suppose $t \leq \hat{t}_0$ and $R > 0$. For each $i \in \mathcal{N}$,

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, 0)}{\partial r_i} = \frac{\pi^p}{\bar{P}_0} \leq \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

This implies each $r_i = 0$, a contradiction. □

Lemma 10. *If the game is symmetric and Assumption 1 holds, then $\hat{t}_1 > \hat{t}_0$. If, in addition, (p, r, c) is an equilibrium of $\Gamma(t)$, then each $c_i > 0$ if and only if $t < \hat{t}_1$.*

Proof. To prove that $\hat{t}_1 > \hat{t}_0$, note that

$$\bar{P}_1 = \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(0)} \leq \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(\bar{c}_0)} = \bar{P}_0$$

by log-concavity of ϕ . This implies $\bar{R}_1 > 0$, so $g(\bar{R}_1) < g(0) = 1$ and $g'(0) \leq g'(\bar{R}_1) < 0$. Therefore,

$$\pi^p - \pi^r \bar{P}_0 g'(0) > \pi^p g(\bar{R}_1) - \pi^r \bar{P}_1 g'(\bar{R}_1) > 0,$$

which implies $\hat{t}_1 > \hat{t}_0$.

Let $P = \sum_i p_i$ and $R = \sum_i r_i$. To prove the “if” direction of the second statement, suppose $t \geq \hat{t}_1$ and some $c_i > 0$. By Lemma 7, $c_j = c_i = C > 0$ for each $j \in \mathcal{N}$. Since $P > 0$ by Lemma 8, the first-order conditions give

$$P = \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(C)} \geq \bar{P}_1.$$

The budget constraint then gives $R < \bar{R}_1$ and thus

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, (\bar{R}_1/N)\mathbf{1}_N)}{\partial r_i} = \frac{\pi^p}{\bar{P}_1} \geq \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

But this implies each $p_i = 0$, a contradiction.

To prove the “only if” direction, suppose $t < \hat{t}_1$ and each $c_i = 0$. The first-order conditions then give $P \leq \bar{P}_1$, so $R \geq \bar{R}_1 > 0$ by the budget constraint. This in turn gives

$$\pi^p \frac{\partial \log f(p)}{\partial p_i} = \frac{\pi^p}{P} \geq \frac{\pi^p}{\bar{P}_1} \geq \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i}$$

for each $i \in \mathcal{N}$. But this implies each $r_i = 0$, a contradiction. \square

The last thing we need to prove the propositions is how the labor allocations change with the tax rate when $t > \hat{t}_0$.

Lemma 11. *Let the game be symmetric and Assumption 1 hold. For all $t \in (\hat{t}_0, \hat{t}_1)$,*

$$\frac{d\tilde{P}_1(t)}{dt} = \frac{-(N-1)\pi^p\pi^c\hat{\phi}''(\tilde{c}_1(t))}{N\pi^r t\Delta_1(t)} \leq 0, \quad (31)$$

$$\frac{d\tilde{R}_1(t)}{dt} = \frac{-\pi^p \left(N\pi^p/\pi^c \tilde{P}_1(t)^2 - (N-1)\pi^c \hat{\phi}''(\tilde{c}_1(t))/N\pi^p \right)}{t\Delta_1(t)} > 0, \quad (32)$$

$$\frac{d\tilde{c}_1(t)}{dt} = \frac{(\pi^p)^2}{\pi^r t \tilde{P}_1(t)^2 \Delta_1(t)} < 0, \quad (33)$$

where

$$\begin{aligned} \Delta_1(t) &= \left(\pi^p t g'(\tilde{R}_1(t)) - \pi^r t \tilde{P}_1(t) g''(\tilde{R}_1(t)) \right) \left(\frac{N\pi^p}{\pi^c \tilde{P}_1(t)^2} - \frac{N-1}{N} \frac{\pi^c}{\pi^p} \hat{\phi}''(\tilde{c}_1(t)) \right) \\ &\quad - \frac{N-1}{N} \pi^c t g'(\tilde{R}_1(t)) \hat{\phi}''(\tilde{c}_1(t)) \\ &< 0. \end{aligned} \quad (34)$$

For all $t > \hat{t}_1$,

$$\frac{d\tilde{P}_2(t)}{dt} = \frac{-\pi^p}{\pi^r t \Delta_2(t)} < 0, \quad (35)$$

$$\frac{d\tilde{R}_2(t)}{dt} = \frac{1}{t \Delta_2(t)} > 0, \quad (36)$$

where

$$\Delta_2(t) = \frac{\pi^r}{\pi^p} t \tilde{P}_2(t) g''(\tilde{R}_2(t)) - 2t g'(\tilde{R}_2(t)) > 0. \quad (37)$$

Proof. Throughout the proof, let $\eta = (N-1)/N$.

First consider $t \in (\hat{t}_0, \hat{t}_1)$. To reduce clutter in what follows, I omit the evaluation point $(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi)$ from all partial derivative expressions. The Jacobian of the system of equations that defines $(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t))$ is

$$\begin{aligned} \mathbf{J}_1(t) &= \begin{bmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R & \partial Q^{pr}/\partial C \\ \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial R & \partial Q^{pc}/\partial C \\ \partial Q^b/\partial P & \partial Q^b/\partial R & \partial Q^b/\partial C \end{bmatrix} \\ &= \begin{bmatrix} \pi^r t g'(\tilde{R}_1(t)) & \pi^r t \tilde{P}_1(t) g''(\tilde{R}_1(t)) - \pi^p t g'(\tilde{R}_1(t)) & 0 \\ -\pi^p/\tilde{P}_1(t)^2 & 0 & -\eta \pi^c \hat{\phi}''(\tilde{c}_1(t)) \\ -1/\pi^p & -1/\pi^r & -N/\pi^c \end{bmatrix}. \end{aligned}$$

It is easy to verify that $|\mathbf{J}_1(t)| = \Delta_1(t) < 0$. Notice that

$$\begin{aligned}\frac{\partial Q^{pr}}{\partial t} &= \pi^r \tilde{P}_1(t) g'(\tilde{R}_1(t)) - \pi^p g(\tilde{R}_1(t)) \\ &= \pi^r \left(-\frac{\pi^p (1 - t g(\tilde{R}_1(t)))}{\pi^r t g'(\tilde{R}_1(t))} \right) g'(\tilde{R}_1(t)) - \pi^p g(\tilde{R}_1(t)) \\ &= -\frac{\pi^p}{t}.\end{aligned}$$

Then, by the implicit function theorem and Cramer's rule,

$$\begin{aligned}\frac{d\tilde{P}_1(t)}{dt} &= \frac{\begin{vmatrix} -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial R & \partial Q^{pr}/\partial C \\ -\partial Q^{pc}/\partial t & \partial Q^{pc}/\partial R & \partial Q^{pc}/\partial C \\ -\partial Q^b/\partial t & \partial Q^b/\partial R & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_1(t)|} \\ &= \frac{-\eta \pi^p \pi^c \hat{\phi}''(\tilde{c}_1(t))}{\pi^r t \Delta_1(t)} \\ &\leq 0, \\ \frac{d\tilde{R}_1(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial C \\ \partial Q^{pc}/\partial P & -\partial Q^{pc}/\partial t & \partial Q^{pc}/\partial C \\ \partial Q^b/\partial P & -\partial Q^b/\partial t & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_1(t)|} \\ &= \frac{-\pi^p (N\pi^p/\pi^c \tilde{P}_1(t)^2 - \eta \pi^c \hat{\phi}''(\tilde{c}_1(t))/\pi^p)}{t \Delta_1(t)} \\ &> 0, \\ \frac{d\tilde{c}_1(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R & -\partial Q^{pr}/\partial t \\ \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial R & -\partial Q^{pc}/\partial t \\ \partial Q^b/\partial P & \partial Q^b/\partial R & -\partial Q^b/\partial t \end{vmatrix}}{|\mathbf{J}_1(t)|} \\ &= \frac{(\pi^p)^2}{\pi^r t \tilde{P}_1(t)^2 \Delta_1(t)} \\ &< 0,\end{aligned}$$

as claimed.

Now consider $t > \hat{t}_1$. Again to reduce clutter in what follows, I omit the evaluation point $(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi)$ from all partial derivative expressions. The Jacobian of the system of equations that defines $(\tilde{P}_2(t), \tilde{R}_2(t))$ is

$$\begin{aligned}\mathbf{J}_2(t) &= \begin{bmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R \\ \partial Q^b/\partial P & \partial Q^b/\partial R \end{bmatrix} \\ &= \begin{bmatrix} \pi^r t g'(\tilde{R}_2(t)) & \pi^r t \tilde{P}_2(t) g''(\tilde{R}_2(t)) - \pi^p t g'(\tilde{R}_2(t)) \\ -1/\pi^p & -1/\pi^r \end{bmatrix}.\end{aligned}$$

It is easy to verify that $|\mathbf{J}_2(t)| = \Delta_2(t) > 0$. As before, $\partial Q^{pr}/\partial t = -\pi^p/t$. So by the implicit function theorem and Cramer's rule,

$$\begin{aligned}\frac{d\tilde{P}_2(t)}{dt} &= \frac{\begin{vmatrix} -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial R \\ -\partial Q^b/\partial t & \partial Q^b/\partial R \end{vmatrix}}{|\mathbf{J}_2(t)|} \\ &= \frac{-\pi^p}{\pi^r t \Delta_2(t)} \\ &< 0, \\ \frac{d\tilde{R}_2(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & -\partial Q^{pr}/\partial t \\ \partial Q^b/\partial P & -\partial Q^b/\partial t \end{vmatrix}}{|\mathbf{J}_2(t)|} \\ &= \frac{1}{t \Delta_2(t)} \\ &> 0,\end{aligned}$$

as claimed. □

The proofs of Propositions 1 and 2 follow almost immediately from these lemmas. I prove them jointly.

Proposition 1 (Baseline Equilibrium). *There is a tax rate $\hat{t}_0 \in (0, 1)$ such that $\sum_i r_i = 0$ in every equilibrium of the labor allocation subgame if and only if $t \leq \hat{t}_0$. Every subgame with $t \leq \hat{t}_0$ has the same unique equilibrium, in which $\sum_i p_i = \bar{P}_0 > 0$ and each $c_i = \bar{c}_0 > 0$.*

Proposition 2 (Resistance Equilibrium). *There is a tax rate $\hat{t}_1 > \hat{t}_0$ such that in every equilibrium of the labor allocation subgame with tax rate t :*

- *If $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = \tilde{P}_1(t) > 0$ (weakly decreasing in t), $\sum_i r_i = \tilde{R}_1(t) > 0$ (strictly increasing), and each $c_i = \tilde{c}_1(t) > 0$ (strictly decreasing).*
- *If $t \geq \hat{t}_1$, then $\sum_i p_i = \tilde{P}_2(t) > 0$ (strictly decreasing in t), $\sum_i r_i = \tilde{R}_2(t) > 0$ (strictly increasing), and each $c_i = 0$.*

Proof. For fixed t , every equilibrium of $\Gamma(t)$ has the same total production, total resistance, and individual competition allocations, per Proposition 9. Consider any $t \in [0, 1]$ and let (p, r, c) be an equilibrium of $\Gamma(t)$.

If $t \leq \hat{t}_0$, then $\sum_i p_i = P > 0$, $\sum_i r_i = 0$, and each $c_i = C > 0$ by Lemmas 8–10. The first-order conditions (Lemma 4) imply that P and C solve $Q^{pc}(P, 0, C; t, \pi) = Q^b(P, 0, C; t, \pi) = 0$; therefore, $P = \bar{P}_0$ and $C = \bar{c}_0$. Since each $r_i = 0$, each $p_i = \pi^p(L/N - \bar{c}_0/\pi^c) = \bar{P}_0/N$, so the equilibrium is unique.

Similarly, if $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = P > 0$, $\sum_i r_i = R > 0$, and each $c_i = C > 0$ by Lemmas 8–10. The first-order conditions then imply that these solve the system (16)–(19); therefore, $P = \tilde{P}_1(t)$, $R = \tilde{R}_1(t)$, and $C = \tilde{c}_1(t)$. The comparative statics on \tilde{P}_1 , \tilde{R}_1 , and \tilde{c}_1 follow from Lemma 11.

Finally, if $t \geq \hat{t}_1$, then $\sum_i p_i = P > 0$, $\sum_i r_i = R > 0$, and each $c_i = 0$ by Lemmas 8–10. The first-order conditions then imply that P and R solve $Q^{pr}(P, R, 0; t, \pi) = Q^b(P, R, 0; t, \pi) = 0$; therefore, $P = \tilde{P}_2(t)$ and $R = \tilde{R}_2(t)$. The comparative statics on \tilde{P}_2 and \tilde{R}_2 follow from Lemma 11. \square

A.4 Proof of Proposition 3

Proposition 3 (Optimal Tax Rate). *There is an equilibrium in which the government chooses the greatest tax rate that engenders no resistance, $t = \hat{t}_0$. If g or ϕ is strictly log-concave, this is the unique equilibrium tax rate.*

Proof. As in the proof of Lemma 11, let $\eta = (N - 1)/N$.

For each $t \in [0, 1]$, fix an equilibrium $(p(t), r(t), c(t))$ of $\Gamma(t)$. By Propositions 1, 2, and 9, the government's induced utility function is

$$u_G^*(t) = u_G(t, p(t), r(t), c(t)) = \begin{cases} t\tilde{P}_0 & t \leq \hat{t}_0, \\ tg(\tilde{R}_1(t))\tilde{P}_1(t) & \hat{t}_0 < t < \hat{t}_1, \\ tg(\tilde{R}_2(t))\tilde{P}_2(t) & t \geq \hat{t}_1. \end{cases}$$

It is immediate from the above expression that $u_G^*(t) < u_G^*(\hat{t}_0)$ for all $t < \hat{t}_0$.

Now consider $t \in (\hat{t}_0, \hat{t}_1)$. By Lemma 11,

$$\begin{aligned} \frac{du_G^*(t)}{dt} &= g(\tilde{R}_1(t))\tilde{P}_1(t) + tg'(\tilde{R}_1(t))\frac{d\tilde{R}_1(t)}{dt}\tilde{P}_1(t) + tg(\tilde{R}_1(t))\frac{d\tilde{P}_1(t)}{dt} \\ &= g(\tilde{R}_1(t))\tilde{P}_1(t) - \frac{\pi^p g'(\tilde{R}_1(t))\tilde{P}_1(t) (N\pi^p/\pi^c \tilde{P}_1(t)^2 - \eta\pi^c \hat{\phi}''(\tilde{c}_1(t))/\pi^p)}{\Delta_1(t)} \\ &\quad - \frac{\eta\pi^p \pi^c g(\tilde{R}_1(t))\hat{\phi}''(\tilde{c}_1(t))}{\pi^r \Delta_1(t)}, \end{aligned}$$

where $\Delta_1(t)$ is defined by (34). To reduce clutter in what follows, let $\tilde{P} = \tilde{P}_1(t)$, $\tilde{R} = \tilde{R}_1(t)$, and $\tilde{c} = \tilde{c}_1(t)$. Since $\Delta_1(t) < 0$, the sign of the above expression is the same as that of

$$\begin{aligned} &g'(\tilde{R})\tilde{P} \left(\frac{N(\pi^p)^2}{\pi^c \tilde{P}^2} - \eta\pi^c \hat{\phi}''(\tilde{c}) \right) + \frac{\eta\pi^p \pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})}{\pi^r} - g(\tilde{R})\tilde{P}\Delta_1(t) \\ &= \tilde{P}g'(\tilde{R}) \left(\frac{N(\pi^p)^2}{\pi^c \tilde{P}^2} - \eta\pi^c \hat{\phi}''(\tilde{c}) \right) + \frac{\eta\pi^p \pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})}{\pi^r} \\ &\quad - \tilde{P}g(\tilde{R}) \left(\pi^p tg'(\tilde{R}) - \pi^r t\tilde{P}g''(\tilde{R}) \right) \left(\frac{N\pi^p}{\pi^c \tilde{P}^2} - \eta\frac{\pi^c}{\pi^p} \hat{\phi}''(\tilde{c}) \right) \\ &\quad + \eta\pi^c t\tilde{P}g(\tilde{R})g'(\tilde{R})\hat{\phi}''(\tilde{c}) \\ &= \tilde{P} \left(g'(\tilde{R}) - \frac{g(\tilde{R}) \left(\pi^p tg'(\tilde{R}) - \pi^r t\tilde{P}g''(\tilde{R}) \right)}{\pi^p} \right) \left(\frac{N(\pi^p)^2}{\pi^c \tilde{P}^2} - \eta\pi^c \hat{\phi}''(\tilde{c}) \right) \\ &\quad + \eta\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c}) \left(\frac{\pi^p}{\pi^r} + t\tilde{P}g'(\tilde{R}) \right) \end{aligned}$$

$$\begin{aligned}
&= \frac{\tilde{P}}{g'(\tilde{R})} (1 - tg(\tilde{R})) (g'(\tilde{R})^2 - g(\tilde{R})g''(\tilde{R})) \left(\frac{N(\pi^p)^2}{\pi^c \tilde{P}^2} - \eta\pi^c \hat{\phi}''(\tilde{c}) \right) \\
&\quad + \eta\pi^c g(\tilde{R}) \hat{\phi}''(\tilde{c}) \left(\frac{\pi^p}{\pi^r} tg(\tilde{R}) \right).
\end{aligned}$$

The first term is weakly negative, strictly so if g is strictly log-concave. The second term is weakly negative, strictly so if ϕ is strictly log-concave. Therefore, $du_G^*(t)/dt \leq 0$ for all $t \in (\hat{t}_0, \hat{t}_1)$, strictly so if g or ϕ is strictly log-concave. This implies $u_G^*(\hat{t}_0) \geq u_G^*(t)$ for all $t \in (\hat{t}_0, \hat{t}_1)$, strictly so if g or ϕ is strictly log-concave.

Finally, consider $t > \hat{t}_1$. Again by Lemma 11,

$$\begin{aligned}
\frac{du_G^*(t)}{dt} &= g(\tilde{R}_2(t))\tilde{P}_2(t) + tg'(\tilde{R}_2(t))\frac{d\tilde{R}_2(t)}{dt}\tilde{P}_2(t) + tg(\tilde{R}_2(t))\frac{d\tilde{P}_2(t)}{dt} \\
&= g(\tilde{R}_2(t))\tilde{P}_2(t) + \frac{g'(\tilde{R}_2(t))\tilde{P}_2(t)}{\Delta_2(t)} - \frac{\pi^p g(\tilde{R}_2(t))}{\pi^r \Delta_2(t)},
\end{aligned}$$

where $\Delta_2(t)$ is defined by (37). To reduce clutter in what follows, let $\tilde{P} = \tilde{P}_2(t)$ and $\tilde{R} = \tilde{R}_2(t)$. Since $\Delta_2(t) > 0$, the sign of the above expression is the same as that of

$$\begin{aligned}
&\tilde{P}g(\tilde{R})\Delta_2(t) + \tilde{P}g'(\tilde{R}) - \frac{\pi^p g(\tilde{R})}{\pi^r} \\
&= \tilde{P}g(\tilde{R}) \left(\frac{\pi^r t \tilde{P} g''(\tilde{R})}{\pi^p} - 2tg'(\tilde{R}) \right) + \tilde{P}g'(\tilde{R}) - \frac{\pi^p g(\tilde{R})}{\pi^r} \\
&= \frac{\pi^p}{\pi^r t g'(\tilde{R})^2} \left((1 - tg(\tilde{R}))^2 (g(\tilde{R})g''(\tilde{R}) - g'(\tilde{R})^2) - (tg(\tilde{R})g'(\tilde{R}))^2 \right) \\
&< \frac{\pi^p (1 - tg(\tilde{R}))^2 (g(\tilde{R})g''(\tilde{R}) - g'(\tilde{R})^2)}{\pi^r t g'(\tilde{R})^2} \\
&\leq 0.
\end{aligned}$$

Therefore, $u_G^*(\hat{t}_0) \geq u_G^*(\hat{t}_1) > u_G^*(t)$ for all $t > \hat{t}_1$.

Combining these findings, $u_G^*(\hat{t}_0) \geq u_G^*(t)$ for all $t \in [0, 1] \setminus \{\hat{t}_0\}$, strictly so if g or ϕ is strictly log-concave. Therefore, there is an equilibrium in which $t = \hat{t}_0$, and every equilibrium has this tax rate if g or ϕ is strictly log-concave. \square

A.5 Proof of Remarks 1 and 2

The comparative statics in both remarks come from the same system of equations, so I prove them jointly.

Remark 1. *Total production in the baseline equilibrium, \bar{P}_0 , is strictly decreasing in the number of factions, N .*

Remark 2. *Total production in the baseline equilibrium, \bar{P}_0 , strictly decreases with a marginal increase in competition effectiveness, π^c , if and only if the incentive effect outweighs the labor-*

saving effect at \bar{c}_0 .

Proof. I will treat N as if it were continuous in order to obtain comparative statics by implicit differentiation. Throughout the proof I write \bar{P}_0 and \bar{c}_0 as functions of (N, π^c) .

Recall that $(\bar{P}_0(N, \pi^c), \bar{c}_0(N, \pi^c))$ is defined as the solution to (20) and (21). To reduce clutter in what follows, I omit the evaluation point $(\bar{P}_0(N, \pi^c), 0, \bar{c}_0(N, \pi^c); t, \pi)$ from all partial derivative expressions. The Jacobian of the system is

$$\mathbf{J}_0 = \begin{bmatrix} \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial C \\ \partial Q^b/\partial P & \partial Q^b/\partial C \end{bmatrix} = \begin{bmatrix} -\pi^p/\bar{P}_0(N, \pi^c)^2 & -(N-1)\pi^c \hat{\phi}''(\bar{c}_0(N, \pi^c))/N \\ -1/\pi^p & -N/\pi^c \end{bmatrix},$$

with determinant

$$|\mathbf{J}_0| = \frac{N\pi^p}{\pi^c \bar{P}_0(N, \pi^c)^2} - \frac{N-1}{N} \frac{\pi^c}{\pi^p} \hat{\phi}''(\bar{c}_0(N, \pi^c)) > 0.$$

By the implicit function theorem and Cramer's rule,

$$\begin{aligned} \frac{\partial \bar{P}_0(N, \pi^c)}{\partial N} &= \frac{\begin{vmatrix} -\partial Q^{pc}/\partial N & \partial Q^{pc}/\partial C \\ -\partial Q^b/\partial N & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_0|} \\ &= \frac{\begin{vmatrix} \pi^c \hat{\phi}'(\bar{c}_0(N, \pi^c))/N^2 & -(N-1)\pi^c \hat{\phi}''(\bar{c}_0(N, \pi^c))/N \\ \bar{c}_0(N, \pi^c)/\pi^c & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_0|} \\ &= \frac{1}{|\mathbf{J}_0|} \left(\frac{N-1}{N} \bar{c}_0(N, \pi^c) \hat{\phi}''(\bar{c}_0(N, \pi^c)) - \frac{\hat{\phi}'(\bar{c}_0(N, \pi^c))}{N} \right) \\ &< 0, \end{aligned}$$

as claimed. Similarly,

$$\begin{aligned} \frac{\partial \bar{P}_0(N, \pi^c)}{\partial \pi^c} &= \frac{\begin{vmatrix} -\partial Q^{pc}/\partial \pi^c & \partial Q^{pc}/\partial C \\ -\partial Q^b/\partial \pi^c & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_0|} \\ &= \frac{\begin{vmatrix} (N-1)\hat{\phi}'(\bar{c}_0(N, \pi^c))/N & -(N-1)\pi^c \hat{\phi}''(\bar{c}_0(N, \pi^c))/N \\ -N\bar{c}_0(N, \pi^c)/(\pi^c)^2 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_0|} \\ &= -\frac{N-1}{\pi^c |\mathbf{J}_0|} \left(\hat{\phi}'(\bar{c}_0(N, \pi^c)) + \bar{c}_0(N, \pi^c) \hat{\phi}''(\bar{c}_0(N, \pi^c)) \right), \end{aligned}$$

which is negative if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0(N, \pi^c)$. \square

I also prove the claim in footnote 10.

Lemma 12. *Let $\theta, \lambda > 0$. If $\phi(C) = \theta \exp(\lambda C)$, then the incentive effect outweighs the labor-saving effect for all $C \geq 0$. If $\phi(C) = \theta C^\lambda$, then the incentive and labor-saving effects are exactly offsetting for all $C > 0$.*

Proof. First consider the difference contest success function, $\phi(C) = \theta \exp(\lambda C)$. Then $\hat{\phi}(C) = \log \theta + \lambda C$, $\hat{\phi}'(C) = \lambda$, and $\hat{\phi}''(C) = 0$ for all $C \geq 0$. Therefore,

$$\hat{\phi}'(C) + C\hat{\phi}''(C) = \lambda > 0.$$

Now consider the ratio contest success function, $\phi(C) = \theta C^\lambda$. Then $\hat{\phi}(C) = \log \theta + \lambda \log C$, $\hat{\phi}'(C) = \lambda/C$, and $\hat{\phi}''(C) = -\lambda/C^2$. Therefore,

$$\hat{\phi}'(C) + C\hat{\phi}''(C) = -\frac{\lambda}{C^2} + C\frac{\lambda}{C} = 0. \quad \square$$

A.6 Proof of Proposition 4

Proposition 4. *The government's equilibrium payoff is strictly decreasing in the number of factions, N . It strictly decreases with a marginal increase in competition effectiveness, π^c , if and only if the incentive effect outweighs the labor-saving effect at \tilde{c}_0 .*

Proof. By Proposition 3, the government's equilibrium payoff is

$$\hat{t}_0 \bar{P}_0 = \frac{\pi^p \bar{P}_0}{\pi^p - \pi^r \bar{P}_0 g'(0)} = \frac{\pi^p}{(\pi^p / \bar{P}_0) - \pi^r g'(0)}.$$

This expression is strictly increasing in \bar{P}_0 . Since N and π^c only enter through the equilibrium value of \bar{P}_0 , the claim follows from Remark 1 and Remark 2. \square

A.7 Proof of Proposition 5

Let $\Gamma_X(t)$ denote the labor allocation subgame with tax rate t in the plunder model. The quantities defined in Proposition 5 are as follows. $(\tilde{R}_X(t), \tilde{c}_X(t))$ is the solution to the system

$$Q^{pr}(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi) = \frac{\pi^r t g'(\tilde{R}_X(t))}{1 - t g(\tilde{R}_X(t))} + \frac{N-1}{N} \pi^c \hat{\phi}'(\tilde{c}_X(t)) = 0, \quad (38)$$

$$Q^b(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi) = L - \frac{\tilde{R}_X(t)}{\pi^r} - \frac{N \tilde{c}_X(t)}{\pi^c} = 0. \quad (39)$$

The cutpoint tax rates are

$$\hat{t}_0^X = \frac{\eta \pi^c \hat{\phi}'(\pi^c L/N)}{\eta \pi^c \hat{\phi}'(\pi^c L/N) - \pi^r g'(0)}, \quad (40)$$

$$\hat{t}_1^X = \frac{\eta \pi^c \hat{\phi}'(0)}{\eta \pi^c g(\pi^r L) \hat{\phi}'(0) - \pi^r g'(\pi^r L)}, \quad (41)$$

where $\eta = (N-1)/N$. Similar to the cutpoints in the original model,

$$\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} = \eta \pi^c \hat{\phi}'(\pi^c L/N) = \pi^c \frac{\partial \log \omega_i((\pi^c L/N) \mathbf{1}_N)}{\partial c_i},$$

$$\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1^X, (\pi^r L/N)\mathbf{1}_N)}{\partial r_i} = \eta \pi^c \hat{\phi}'(0) = \pi^c \frac{\partial \log \omega_i(0)}{\partial c_i}$$

for each $i \in \mathcal{N}$.

Proposition 5. *In the plunder model, every labor allocation subgame has a unique equilibrium. There exists a tax rate $\hat{t}_0^X \in (0, 1)$ such that each $r_i = 0$ in equilibrium if and only if $t \leq \hat{t}_0^X$. There exists $\hat{t}_1^X > \hat{t}_0^X$ such that each $c_i = 0$ in equilibrium if and only if $t \geq \hat{t}_1^X$. For $t \in (\hat{t}_1^X, \hat{t}_0^X)$, in equilibrium each $r_i = \tilde{R}_X(t)/N > 0$ (strictly increasing in t) and each $c_i = \tilde{c}_X(t) > 0$ (strictly decreasing).*

Proof. The existence and essential uniqueness results from the original game, Proposition 8 and Proposition 9, carry over to the plunder model. So does Lemma 7, showing that all individual allocations toward competition are equal under symmetry. Therefore, $\Gamma_X(t)$ has an equilibrium, and there exists $c_X^*(t)$ such that each $c_i = c_X^*(t)$ in every equilibrium of $\Gamma_X(t)$. The budget constraint then implies each $r_i = \pi^r(L/N - c_X^*(t)/\pi^c)$ in every equilibrium of $\Gamma_X(t)$, so the equilibrium is unique.

Let (r, c) be the equilibrium of $\Gamma_X(t)$. Let $R = \sum_i r_i$ and $C = c_1$, so by Lemma 7 each $c_i = C$. If $t \leq \hat{t}_0^X$ and $R > 0$, then the first-order conditions give

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} = \pi^c \frac{\partial \log \omega_i((\pi^c L/N)\mathbf{1}_N)}{\partial c_i} \leq \pi^c \frac{\partial \log \omega_i(c)}{\partial c_i}$$

for each $i \in \mathcal{N}$. But this implies each $r_i = 0$, contradicting $R > 0$. Therefore, if $t \leq \hat{t}_0^X$, then $R = 0$. Similarly, if $t > \hat{t}_0^X$ and $R = 0$, then each $c_i = \pi^c L/N$ and thus

$$\pi^c \frac{\partial \log \omega_i(c)}{\partial c_i} = \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i}.$$

But this implies each $c_i = 0$, a contradiction. Therefore, if $t > \hat{t}_0^X$, then $R > 0$. The proof that $C > 0$ if and only if $t < \hat{t}_1^X$ is analogous.

For $t \in (\hat{t}_0^X, \hat{t}_1^X)$, the first-order conditions imply that R and C solve $Q^{rc}(0, R, C; t, \pi) = Q^b(0, R, C; t, \pi) = 0$; therefore, $R = \tilde{R}_X(t)$ and $C = \tilde{c}_X(t)$. To reduce clutter in what follows, I omit the evaluation point $(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi)$ from all partial derivative expressions. The Jacobian of the system defining $(\tilde{R}_X(t), \tilde{c}_X(t))$ is

$$\begin{aligned} \mathbf{J}_X &= \begin{bmatrix} \partial Q^{rc} / \partial R & \partial Q^{rc} / \partial C \\ \partial Q^b / \partial R & \partial Q^b / \partial C \end{bmatrix} \\ &= \begin{bmatrix} \pi^r t \frac{g''(\tilde{R}_X(t)) - t g(\tilde{R}_X(t))^2 \hat{g}''(\tilde{R}_X(t))}{(1 - t g(\tilde{R}_X(t)))^2} & \eta \pi^c \hat{\phi}''(\tilde{c}_X(t)) \\ -1/\pi^r & -N/\pi^c \end{bmatrix} \end{aligned}$$

where $\eta = (N - 1)/N$ and $\hat{g} = \log g$. Its determinant is

$$|\mathbf{J}_X| = \frac{\pi^c}{\pi^r} \left(\eta \hat{\phi}''(\tilde{c}_X(t)) - N \pi^r t \frac{g''(\tilde{R}_X(t)) - t g(\tilde{R}_X(t))^2 \hat{g}''(\tilde{R}_X(t))}{(1 - t g(\tilde{R}_X(t)))^2} \right) < 0.$$

By the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{d\tilde{R}_X(t)}{dt} &= \frac{\begin{vmatrix} -\partial Q^{rc}/\partial t & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial t & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X|} \\
&= \frac{\begin{vmatrix} -\pi^r g'(\tilde{R}_X(t))/(1 - tg(\tilde{R}_X(t)))^2 & \eta\pi^c \hat{\phi}''(\tilde{c}_X(t)) \\ 0 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X|} \\
&= \frac{N\pi^r g'(\tilde{R}_X(t))}{\pi^c |\mathbf{J}_X|} \\
&> 0,
\end{aligned}$$

as claimed. The budget constraint then implies $d\tilde{c}_X(t)/dt < 0$, as claimed. \square

A.8 Proof of Proposition 6

Before proving the proposition, I separately derive the comparative statics of \hat{t}_0^X and $\tilde{R}_X(t)$ in N and π^c .

Lemma 13. *In the plunder model, the lower cutpoint \hat{t}_0^X is strictly increasing in the number of factions, N . It is locally decreasing in the effectiveness of competition, π^c , if and only if*

$$\hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N} \hat{\phi}''\left(\frac{\pi^c L}{N}\right) \geq 0.$$

Proof. Recall that

$$\hat{t}_0^X = \frac{((N-1)/N)\pi^c \hat{\phi}'(\pi^c L/N)}{((N-1)/N)\pi^c \hat{\phi}'(\pi^c L/N) - \pi^r g'(0)}.$$

Since $g'(0) < 0$, $(N-1)/N$ is strictly increasing in N , and $\hat{\phi}'(\pi^c L/N)$ is weakly increasing in N , \hat{t}_0^X is strictly increasing in N . Notice that

$$\frac{\partial}{\partial \pi^c} \left[\pi^c \hat{\phi}'\left(\frac{\pi^c L}{N}\right) \right] = \hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N} \hat{\phi}''\left(\frac{\pi^c L}{N}\right),$$

so \hat{t}_0^X is locally increasing in π^c if and only if the above expression is positive. \square

Lemma 14. *In the plunder model, for fixed $t \in (\hat{t}_0^X, \hat{t}_1^X)$, total resistance, $\tilde{R}_X(t)$, is strictly decreasing in the number of factions, N . It is locally decreasing in the effectiveness of competition, π^c , if and only if*

$$\hat{\phi}'(\tilde{c}_X(t)) + \tilde{c}_X(t) \hat{\phi}''(\tilde{c}_X(t)) \geq 0.$$

Proof. As in the proof of Remark 1, I will treat N as if it were continuous in order to obtain comparative statics by implicit differentiation. Throughout the proof I write $\tilde{R}_X(t)$ and $\tilde{c}_X(t)$ as functions of (N, π^c) .

I first consider comparative statics in N . To reduce clutter in what follows, I omit the evaluation point $(0, \tilde{R}_X(t; N, \pi^c), \tilde{c}_X(t; N, \pi^c); t, \pi)$ from all partial derivative expressions. By the implicit function theorem and Cramer's rule,

$$\begin{aligned} \frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial N} &= \frac{\begin{vmatrix} -\partial Q^{rc}/\partial N & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial N & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|} \\ &= \frac{\begin{vmatrix} -\pi^c \hat{\phi}'(\tilde{c}_X(t; N, \pi^c))/N^2 & ((N-1)/N)\pi^c \hat{\phi}''(\tilde{c}_X(t; N, \pi^c)) \\ \tilde{c}_X(t; N, \pi^c)/\pi^c & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|} \\ &= \frac{\hat{\phi}'(\tilde{c}_X(t; N, \pi^c)) - (N-1)\tilde{c}_X(t; N, \pi^c)\hat{\phi}''(\tilde{c}_X(t; N, \pi^c))}{N|\mathbf{J}_X(t; N, \pi^c)|} \\ &< 0, \end{aligned}$$

as claimed, where $|\mathbf{J}_X(t; N, \pi^c)| < 0$ is defined as in the proof of Proposition 5.

I now consider comparative statics in π^c . Again by the implicit function theorem and Cramer's rule,

$$\begin{aligned} \frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial \pi^c} &= \frac{\begin{vmatrix} -\partial Q^{rc}/\partial \pi^c & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial \pi^c & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|} \\ &= \frac{\begin{vmatrix} -((N-1)/N)\hat{\phi}'(\tilde{c}_X(t; N, \pi^c)) & ((N-1)/N)\pi^c \hat{\phi}''(\tilde{c}_X(t; N, \pi^c)) \\ -N\tilde{c}_X(t; N, \pi^c)/(\pi^c)^2 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|} \\ &= \frac{(N-1)\left(\hat{\phi}'(\tilde{c}_X(t; N, \pi^c)) + \tilde{c}_X(t; N, \pi^c)\hat{\phi}''(\tilde{c}_X(t; N, \pi^c))\right)}{\pi^c|\mathbf{J}_X(t; N, \pi^c)|}. \end{aligned}$$

Therefore, $\partial \tilde{R}_X(t; N, \pi^c)/\partial \pi^c \leq 0$ if and only if

$$\hat{\phi}'(\tilde{c}_X(t; N, \pi^c)) + \tilde{c}_X(t; N, \pi^c)\hat{\phi}''(\tilde{c}_X(t; N, \pi^c)) \geq 0,$$

as claimed. □

The proof of Proposition 6 follows mainly from these lemmas.

Proposition 6. *In the plunder model, the government's equilibrium payoff is increasing in the number of factions, N . If there is a unique equilibrium tax rate t^* , the government's equilibrium payoff is locally increasing in competition effectiveness, π^c , if and only if the incentive effect outweighs the labor-saving effect at the corresponding equilibrium level of internal competition.*

Proof. Throughout the proof I write various equilibrium quantities, including the cutpoints \hat{t}_0^X and \hat{t}_1^X , as functions of (N, π^c) . Let the government's equilibrium payoff as a function of these parameters be

$$u_G^*(N, \pi^c) = \max_{t \in [0,1]} t \times g(R^*(t; N, \pi^c)) \times X.$$

I begin with the comparative statics on N . First, suppose $t = \hat{t}_0^X(N', \pi^c)$ is an equilibrium for all N' in a neighborhood of N .¹⁷ Then $u_G^*(N', \pi^c) = \hat{t}_0^X(N', \pi^c) \times X$ in a neighborhood of N' , which by Lemma 13 is strictly increasing in N' . Next, suppose there is an equilibrium with $t \in (\hat{t}_0^X(N', \pi^c), \hat{t}_1^X(N', \pi^c))$ for all N' in a neighborhood of N . Then, by the envelope theorem,

$$\frac{\partial u_G^*(N, \pi^c)}{\partial N} = g'(\tilde{R}_X(t; N, \pi^c)) \frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial N} \times X > 0,$$

where the inequality follows from Lemma 14. Finally, suppose $t = 1$ is an equilibrium for all N' in a neighborhood of N .¹⁸ Then $u_G^*(N, \pi^c) = g(\pi^c L) \times X$ is locally constant in N , and thus weakly increasing.

I now consider the comparative statics on π^c . First, suppose $t = \hat{t}_0^X(N, \pi^{c'})$ is an equilibrium for all $\pi^{c'}$ in a neighborhood of π^c . Then $u_G^*(N, \pi^{c'}) = \hat{t}_0^X(N, \pi^{c'}) \times X$ in a neighborhood of $\pi^{c'}$, which by Lemma 13 is locally increasing at π^c if and only if

$$\hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N} \hat{\phi}''\left(\frac{\pi^c L}{N}\right) \geq 0.$$

Next, suppose there is an equilibrium with $t \in (\hat{t}_0^X(N, \pi^{c'}), \hat{t}_1^X(N, \pi^{c'}))$ for all $\pi^{c'}$ in a neighborhood of π^c . Then, by the envelope theorem,

$$\frac{\partial u_G^*(N, \pi^c)}{\partial \pi^c} = g'(\tilde{R}_X(t; N, \pi^c)) \frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial \pi^c} \times X.$$

This is positive if and only if $\hat{\phi}'(\tilde{c}_X(t)) + \tilde{c}_X(t) \hat{\phi}''(\tilde{c}_X(t)) \geq 0$, per Lemma 14. Finally, suppose $t = 1$ is an equilibrium for all $\pi^{c'}$ in a neighborhood of π^c . Then $u_G^*(N, \pi^c) = g(\pi^c L) \times X$ is locally constant in π^c , and thus weakly increasing. \square

A.9 Proof of Proposition 7

I begin by characterizing the equilibrium of the conquest game. Throughout the proofs, let $\hat{\chi} = \log \chi$ and $\hat{\psi} = \log \psi$. I will characterize equilibria in terms of the criterion function

$$Q^{ds}(S; N) = \frac{N-1}{N} (\psi(S) + \bar{s}_O) \hat{\chi}'\left(\frac{L-S}{N}\right) - \hat{\psi}'(S) \bar{s}_O, \quad (42)$$

which is strictly increasing in both S and N .

Lemma 15. *The conquest game has a unique equilibrium in which each*

$$s_i = \begin{cases} 0 & Q^{ds}(0; N) \geq 0, \\ \tilde{S}(N)/N & Q^{ds}(0; N) < 0, Q^{ds}(L; N) > 0, \\ L/N & Q^{ds}(L; N) \leq 0, \end{cases}$$

¹⁷As in the original game, there cannot be an equilibrium tax rate $t < \hat{t}_0^X$.

¹⁸ $t = 1$ is the only $t \geq \hat{t}_1^X$ that can be an equilibrium, since resistance is constant above \hat{t}_1^X .

and each $d_i = L/N - s_i$, where $\tilde{S}(N)$ is the unique solution to $Q^{ds}(\tilde{S}(N); N) = 0$.

Proof. Like the original game, the conquest game is log-concave, so a pure-strategy equilibrium exists can be characterized by first-order conditions. In addition, the proof of Lemma 7 carries over to the conquest game, so in equilibrium each $d_i = d_j$ for $i, j \in \mathcal{N}$. The claim then follows from the first-order conditions for maximization of each faction's utility. \square

The proof of Proposition 7 follows from this equilibrium characterization.

Proposition 7. *In the conquest model, the probability that the outsider wins is increasing in the number of factions, N .*

Proof. I will prove that the equilibrium value of $\sum_i s_i$ decreases with the number of factions. Let (d, s) and (d', s') be the equilibria at N and N' respectively, where $N' > N$, and let $S = \sum_i s_i$ and $S' = \sum_i s'_i$. If $S = 0$, then $Q^{ds}(0; N) \geq 0$ and thus $Q^{d's'}(0; N') \geq 0$, so $S' = 0$ as well. If $S \in (0, L)$, then $S = \tilde{S}(N)$, which implies $Q^{ds}(L; N) > 0$ and thus $Q^{d's'}(L; N') > 0$. This in turn implies either $S' = \tilde{S}(N') < \tilde{S}(N)$ or $S' = 0 < \tilde{S}(N)$. Finally, if $S = L$, then it is trivial that $S' \leq S$. \square