DIVIDING THE CONQUERED

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Work in progress. Comments welcome.

ABSTRACT. When an outside power gains control of a piece of territory, such as in a colonial endeavor or a military occupation, it inherits the burden of managing internal conflict among the groups already residing there. Using a formal model, this paper develops a new theory of how the policies of an extractive outside force affect the occurrence of internal social conflict, and vice versa. When an outside ruler confronts a highly fractionalized society, the most profitable policy is one that maximizes internal conflict and minimizes resistance against external rule. However, the broader analysis does not lend support to the conventional wisdom of "divide and rule." All else equal, a ruler would rather govern a unified society than a divided one, as internal conflict is economically destructive. Similarly, I find that an outside force has no incentive to create artificial inequality between identical social groups. The results of the model also suggest that harsh extractive policies and violent resistance may be more likely to occur when insurgents are concentrated in a few groups than when they have a broad base. In extensions to the model, I show that natural resource wealth is less valuable than it might appear, and that productive investment can reduce commitment problems that otherwise lead to inefficiently high levels of appropriation.

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1. Introduction

Violent political conflict often ends—or begins—with a piece of territory coming under the rule of an outside power. One of the most challenging and contentious tasks for these external rulers is the management of internal conflict among the territory's population. History is rife with examples of civil conflict either aiding or hindering outside forces' attempts to gain control and appropriate resources in the territory they govern. From Cortés in 16th-century Mexico to De Gaulle in 20th-century Algeria, European powers faced colonial populations that engaged in internal struggles while simultaneously resisting conquest. This combination of internal strife and resistance against external rule is also common in military occupations, such as the French occupation of Switzerland during the Revolutionary Wars, the German occupations in Eastern Europe in World War II, and the recent U.S. invasion of Iraq. Even some separatist conflicts, namely those in ethnically heterogeneous regions, occur at this nexus of external and internal conflict. One example is the Xinjiang region of China, in which there is both separatist resistance against the central government's rule and localized ethnic conflict between Han Chinese and Uyghur groups.

This general phenomenon, the external rule of an internally fractionalized society, can be thought of as a two-layered conflict. At the top layer, there is an outside force that seeks to control the society as a whole and expropriate its valuable goods. At the bottom layer, within the territory that has fallen under external rule, various groups and factions seek to appropriate from one another as well. To understand historical and contemporary instances of governance of a territory by an outside party, including such widespread phenomena as colonialism and military occupations, we must understand the workings of this type of two-layered conflict. However, even simple questions about this topic lack intuitive answers. For example, all else equal, does the presence of internal conflict help or hinder an outside force's efforts to extract valuable resources from a piece of territory? A plausible case can be made for either answer. If civil strife prevents the factions within society from presenting a united front against their common enemy, then it would appear to benefit the external ruler. On the other hand, if internal conflict results in the destruction of the resources that are valuable to the ruler, then it would appear detrimental. In order to answer questions like this clearly and consistently, a systematic theoretical approach is necessary.

This paper develops a unified theory of the relationship between extractive outside rule and internal social conflict. In order to develop a systematic account of such conflicts, which involve numerous actors with complex, cross-cutting incentives, I situate the analysis in a formal model. The analysis addresses questions like the following: How does material appropriation by a third party shape the occurrence and severity of violent civil conflict? Under what circumstances, if any, does social fractionalization allow an outside force to extract more rents than if society were unified? How does natural resource wealth affect the "value" of governing a piece of territory, given its effects on internal strife?

The main premise of the theory is that a social or political group faces a difficult set of trade-offs when confronted with appropriation from both an outside force and other groups within society. The formal model developed in this paper incorporates these trade-offs, yielding clearly grounded answers to important questions about the relationship between internal and external conflict. Specifically, I model an interaction between an outside ruler that has gained control of a piece of territory and the set of groups comprising the native population of that territory. At the outset of the interaction, the outside ruler places an extractive demand on the occupied society, laying claim to a percentage of whatever economic output it produces. Each social group within the territory then must allocate its limited supply of labor among three potential activities. The first is economic production, the creation of intrinsically valuable goods that each actor—both the outside ruler and the internal groups—wish to consume. The groups' other choices determine who receives what share of this productive output. The second activity in which a group may partake is resistance against the outside ruler, which decreases the ruler's ability to carry out its extractive demand. Resistance represents any activity that hinders an external force's extractive efforts, encompassing anything from violent insurgency to forms of "everyday resistance" like workers dragging their feet or concealing productive output (Scott 1987). But groups within a fractionalized society do not just have to worry about external theft of their resources; they must also protect their share from each other. Accordingly, the third and final activity to which a group may devote labor is appropriation from the other groups. Instead of producing valuable goods or contributing to the joint effort against outside rule, a group may simply decide to steal from the others. In this sense, the theoretical model includes two layers of conflict—social groups against external rule, and against each other.

This simple theoretical framework encompasses the important trade-offs that the actors at the nexus of internal and external conflict must face. Because a group's labor and other resources are limited, any effort it takes against the invading force may leave its goods more vulnerable to extraction by the other groups, and vice versa. Meanwhile, any energy devoted to either arena of conflict detracts from the only intrinsically valuable activity—namely, economic production, the fruits of which are the object of the fighting in the first place. Of course, how each social group decides to resolve these trade-offs will depend on the scope of the outside actor's attempts to appropriate the society's goods. All else equal, harsher policies will be met with more resistance, which in turn means less internal conflict, less economic production, or both. Therefore, if the outside force is forward-looking, meaning it bases its policy decisions on the likely response, it also faces a trade-off: massive extractive efforts are self-defeating if they engender violent resistance and discourage the population from engaging in valuable productive activity. The model accounts for each of these trade-offs and allows me to analyze how rational, forward-looking actors would resolve them.

I obtain the main results from the baseline form of the model, in which I assume symmetry in size and productivity among the groups in society. Each group's incentive to engage in appropriate activity depends on the total number of groups—the more there are, the greater the temptation to obtain goods by taking from the others instead of by direct production. At the same time, greater extractive demands by the outside power increase the groups' incentives to contribute to collective resistance instead of internal competition. In fact, regardless of the number of groups or the level of internal conflict, society as a whole can credibly deter the outsider from making high demands with the threat of commensurately high resistance. The optimal policy for the external force is to demand just enough not to provoke resistance.

In the analysis of the baseline model, I find an apparent paradox regarding the philosophy of *divide et impera*—the notion that an outside ruler is best served by provoking the factions in society to point their guns at one another instead of at their common enemy, the external ruler. When there are enough group divisions that some level of internal conflict is unavoidable, the level of demand that minimizes resistance (and thereby is best for the ruler) also results in the maximal level of civil strife. Thus it would appear that social conflict works to an outside ruler's benefit, confirming the wisdom of *divide et impera*. But this addresses how an outside force can best rule a hopelessly fractionalized society, not whether it would rather be ruling a unified one. In fact, examining the relationship between the number of groups and the external ruler's payoff, I find that the amount of economic surplus an outsider can extract is inversely related to the number of groups. In other words, the ruler would want to minimize group divisions if it could. Although internal conflict reduces resistance, it also siphons effort away from economic production, at an even greater cost to an extractive ruler.

Moving away from the baseline model, I find that quite different behavior is possible when the groups in society are not identical in terms of their size or productivity. I examine a special case of asymmetry in which there are two types of groups: workers, who specialize in economic production, and insurgents, who specialize in resistance against external appropriation. Under this form of specialization, greater extractive demands by the ruler may actually lead to higher economic production. Consequently, unlike in the baseline symmetric case, in equilibrium the outside power may choose a sizable extractive demand that engenders violent resistance. This result suggests that harsh policies and violent resistance against outside rule may be more prevalent when insurgents are concentrated in a few groups than when they have a broad base.

One of the most attractive features of the theoretical model developed in this paper is that it can easily be extended to address more specific questions about factors that affect outside rule of a divided population. I illustrate three such extensions in the paper. In the first extension, I allow for natural resource wealth in addition to the economic output produced by the social groups. I show that natural resources are less valuable to a ruler than one might suspect, as they increase the incentives for both internal conflict and resistance against outside appropriation. The second extension introduces a "commitment problem" whereby the outside force cannot credibly promise not to implement the harshest feasible policy, leaving all actors worse off than in the original game. I then illustrate how the shadow of the future can alleviate the problem, specifically if society can use its share of economic output to make productive investments whose value grows over time. In the third extension, I allow the size of the ruler's demand to vary by group, introducing the possibility of making policies that create endogenous inequality. Surprisingly, I find that the equilibrium policy is the same as in the original game if the groups are symmetric; the outside force cannot benefit from creating arbitrary differences between otherwise identical groups.

Previous studies of "divide and rule" have examined how social fractionalization affects a government's ability to extract rents from society (Acemoglu, Verdier and Robinson 2004; Debs 2007). In these models, social divisions aid autocrats' efforts to remain in power insofar as they allow a ruler to impede coordination or offer particularized benefits to a concentrated base of support. Unlike the theory presented in this paper, these models only allow social groups to differ in their policy preferences, not to appropriate resources from each other. As such, the model here is better suited to address questions about outside attempts to extract resources that are also the object of internal conflict.

My conceptualization of internal conflict is adapted from the political economy literature on civil conflict and ethnic conflict, specifically in its use of a contest model to represent competition over resources. A contest is a game in which each player's share of some disputed good is determined by the amount of costly effort it spends in competition with the others (Hirshleifer 1989, 1991; Grossman 1991). A player's share in a contest can be thought of as its probability of success in a military engagement against the others, where the contest success function describes the relationship between resources and military outcomes. Because contest models can easily accommodate conflict among a large set of groups, they are common in the literature on ethnic conflict and other forms of social conflict (e.g., Esteban and Ray 1999; Robinson 2001; Dal Bó and Dal Bó 2011). The model employed in the present paper is most closely related to that of Skaperdas (1992), in which the players must divide their labor between production, which determines the total size of the prize, and the contest, which determines their share of the prize. I extend Skaperdas's framework by including a third activity to which the players can devote effort—resistance, which reduces the amount of the prize that the outside actor can extract.

With the focus on appropriation and material conflict, the theory here is most applicable to disputes over the distribution of economic production or wealth, as opposed to non-economic matters such as the structure of political institutions or fear of future repression. To be clear, only in a limited sense is this emphasis on material conflict related to the empirical question of whether "greed" or "grievance" (or neither) is the primary cause of civil war (Collier and Hoeffler 2004; Humphreys 2005). In the first place, I address only those internal conflicts that also involve an outside force, the causes of which may systematically differ from those of civil conflicts with no external involvement. Moreover, although economic appropriation may seem most naturally connected to "greedy" motivations, it is also connected to the "grievance" of wishing to rectify economic inequality. Finally, inasmuch as some internal conflicts with external interference have economic motivations and others do not, the theory here applies to the cases in the former category, regardless of whether such cases represent the majority.

The remainder of the paper proceeds as follows. The next section contains the formal statement of the theoretical model. Section 3 contains some basic results including equilibrium existence and discusses the general principles used to solve for the groups' choices. The equilibrium of the baseline model with symmetric groups is derived in Section 4. Section 5 presents results under asymmetry. Extensions to the model appear in Section 6. I give concluding remarks and discuss directions for future research in Section 7.

2. The Model

I model an interaction in which an outside force attempts to extract material surplus from a divided society. The model is designed to capture the features and trade-offs inherent in this process, as described in the introduction. I refer to the outside force as the conqueror, denoted C.[1] The population of the conquered territory consists of n groups, where i denotes a generic group and $N = \{1, \ldots, n\}$ is the set of all groups. Both the conqueror and the groups have the same general objective: to obtain as much material surplus as possible. Their choices determine how much economic output is created and how it is distributed.

The timing of the game is as follows. At the outset of the game, the conqueror announces a policy of tribute, denoted by $\tau \in [0, 1]$, which represents a demand for a certain proportion of society's economic output. After the conqueror has set the tribute level, each group i simultaneously chooses how to allocate its labor supply among three activities: economic production, denoted p_i; resistance against the conqueror, r_i; and conflict or competition with the other groups, c_i. The vector of all production allocations is written $p = (p_1, \ldots, p_n)$, with $c = (c_1, \ldots, c_n)$ and $r = (r_1, \ldots, r_n)$ similarly defined. Each group has a fixed labor supply of $L_i > 0$ units, which may also be interpreted as the size of the group. A group's choices must satisfy the budget constraint

$$\frac{c_i}{\alpha_i^c} + \frac{p_i}{\alpha_i^p} + r_i = L_i, \tag{2.1}$$

where $\alpha_i^c > 0$ and $\alpha_i^p > 0$ represent how many units of c_i and p_i, respectively, the group produces per unit of labor.[2] A group's economic productivity is represented by α_i^p. If $\alpha_i^p > \alpha_j^p$, then group i is more productive than group j: it needs less labor to produce the same amount. The term α_i^c represents a group's effectiveness or productivity in conflict with the other groups. For a more concrete interpretation, one might think of α_i^c as being inversely related to a group's vulnerability to extractive efforts by the other groups. In this sense, having superior military technology, living in rugged terrain, and possessing inherent advantages in the political system (e.g., speaking the national language) can all increase a group's value of α_i^c.

Economic production forms the "pie" that the groups and the conqueror are in conflict over. Society's economic output is the total amount contributed to production, $P = \sum_{i \in N} p_i$. I assume that there are

[1] I use the terminology of territorial conquest for expositional convenience; it should not be interpreted as narrowing the model's scope of application. For example, the extractive outside force that I refer to as the "conqueror" may represent a metropole governing a peripheral region or a military temporarily occupying territory in wartime.

[2] Without loss of generality, α_i^r is normalized to 1 for all groups.

no property rights; i.e., a group does not automatically get to keep all or part of what it produces. Instead, the distribution of the good among the groups and the conqueror is determined by contentious political processes. Specifically, the fraction of P that each player receives depends on the other three components of the model—tribute, resistance, and internal conflict.

Tribute and resistance determine how the economic surplus is divided between the conqueror and the groups as a whole. Tribute is the proportion of P that the conqueror demands, while resistance by the groups hinders the conqueror's ability to carry out this demand fully. To represent this process formally, let $R = \sum_{i \in N} r_i$ denote the total amount of labor contributed toward resistance. The proportion of the tribute demand τ that the conqueror can collect—or, equivalently, the probability that it collects τ as opposed to nothing—is a strictly decreasing function of total resistance, $g(R)$. To be as general as possible, I allow this to be any function $g : \Re_+ \to [0,1]$ that meets a mild set of technical conditions: it must be twice continuously differentiable, convex, and log-concave.[3] Given the tribute level τ and total resistance R, the conqueror's share of economic surplus is $\tau g(R)$, leaving a share of $1 - \tau g(R)$ for the groups as a whole.

Intergroup conflict determines how the remaining economic output is apportioned among the groups. I model this process as a contest, in which groups that devote more labor to the competition receive greater shares of the goods. In particular, group i's share is proportional to $f(c_i)$, a strictly increasing function of its allocation toward conflict. The only technical conditions necessary are that f be twice continuously differentiable and log-concave, and that $f(0) > 0$. Group i's share of the output available to the groups (i.e., the amount not taken by the conqueror through tribute) is given by the contest success function

$$s_i(c) = \frac{f(c_i)}{\sum_{j \in N} f(c_j)}. \tag{2.2}$$

Three facts are immediate from this definition. First, each group's share is strictly positive. Second, the shares always sum to one. Third, if all groups devote the same amount of labor to conflict, then every group's share is $s_i(c) = \frac{1}{n}$.

With the technologies of production, resistance, and intergroup conflict now defined, it is possible to derive the utility functions for each actor. Recall that the conqueror's share of the total output P is $\tau g(R)$. Its utility function can thus be written

$$u_C(c, p, r, \tau) = \tau g(R) \times P. \tag{2.3}$$

[3] Log-concavity of g is equivalent to the function $\log g$ being concave. As g must be twice differentiable, a sufficient condition is that $g''(R) \leq g'(R)^2 / g(R)$ for all R.

The conqueror's utility does not depend directly on conflict between groups. However, insofar as such conflict detracts from resistance or production, it may indirectly increase or decrease the conqueror's payoff. The total amount of economic surplus available to the groups after the conqueror's extraction is $(1-\tau g(R)) \times P$, of which each group receives its contest share, $s_i(c)$. The utility function for a group is therefore

$$u_i(c, p, r, \tau) = s_i(c) \times (1 - \tau g(R)) \times P. \tag{2.4}$$

The groups do not pay a direct cost for their choices of production, resistance, and conflict. Instead, costs enter indirectly via the budget constraint. For example, the more labor a group devotes to production, the less it has available to protect its share of the output against appropriation by the conqueror or the other groups. On the other hand, if a group focuses solely on resisting the conqueror or competing with other groups to the exclusion of economically productive activity, it may find itself with a large slice of a meager pie. In this way, the model incorporates the significant trade-offs inherent to situations with both internal and external conflict.

As this is a multistage game of complete information, the natural solution concept is subgame perfect equilibrium. For every potential tribute level $\tau \in [0,1]$, there is a corresponding subgame, denoted $\Gamma(\tau)$, in which the groups choose how to allocate their labor given the conqueror's choice. An equilibrium is a set of functions $(c^*(\tau), p^*(\tau), r^*(\tau))$ and a value τ^* that meet the following conditions:

(1) For every $\tau \in [0,1]$, the group strategy profile $(c^*(\tau), p^*(\tau), r^*(\tau))$ is a Nash equilibrium of the subgame $\Gamma(\tau)$.

(2) The conqueror's choice τ^* maximizes the induced utility function

$$u_C(\tau) = u_C(c^*(\tau), p^*(\tau), r^*(\tau), \tau).$$

As usual, the game is solved by backward induction—i.e., by first finding the equilibria of every subgame, then using these to determine the level of tribute that is optimal for the conqueror.

3. Basic Results

In this section, I briefly establish some basic properties of the model that inform the equilibrium analysis in the remainder of the paper. I begin by showing that an equilibrium exists and identifying its uniqueness properties. I then describe and explain the set of conditions used to characterize the groups' equilibrium choices.

3.1. **Existence and uniqueness.** The most important preliminary step of the analysis is to verify that an equilibrium of the game exists. For the present model, this requires two steps—first, to prove that an equilibrium exists in every subgame, and second, to show that there is an optimal tribute level for the conqueror given some set of equilibrium responses by the groups. For the first step, I apply a standard result for continuous games, the Debreu–Glicksberg–Fan Theorem (Fudenberg and Tirole 1991, p. 34). The key to the proof is to show that each group's utility function u_i is quasiconcave in its own strategy, which follows from the regularity conditions imposed on the functions f and g. The result is formally stated in the following proposition. All proofs, as well as some intermediate results not stated in the text, appear in the Appendix.

Proposition 1. *In every group subgame $\Gamma(\tau)$, an equilibrium exists.*

Before considering the conqueror's choice of tribute, I show that every subgame has a unique equilibrium *outcome*. This means that even if multiple equilibria exist in a particular subgame, all of them are identical in terms of their payoff-relevant features: total production P, total production R, and the individual conflict allocations $c = (c_1, \dots, c_n)$.[4] All that may differ across equilibria of a subgame is how total production and resistance are apportioned among the groups. This allows us to write the equilibrium values of conflict, total production, and total resistance as functions of the tribute τ, as stated in the proposition.

Proposition 2. *All equilibria of $\Gamma(\tau)$ have the same outcome, $(c^*(\tau), P^*(\tau), R^*(\tau))$.*

The main import of this result is that it simplifies the equilibrium analysis. Because all payoffs depend only on $c^*(\tau)$, $P^*(\tau)$, and $R^*(\tau)$, it is sufficient to characterize these three functions, as opposed to solving for a particular equilibrium in each subgame. Under the assumption that the groups' strategies form an equilibrium in every subgame, the conqueror's induced utility function may be written as

$$u_C(\tau) = \tau g(R^*(\tau)) \times P^*(\tau). \tag{3.1}$$

The final requirement to prove that an equilibrium of the full game exists is to show that there exists a maximizer of $u_C(\tau)$ on $[0, 1]$. The main task of the proof is to show that the functions P^* and R^* are continuous, after which it is immediate that u_C attains a maximum on $[0, 1]$, giving the following result.

[4] To be clear, this result applies only to the equilibria of a particular subgame, not across subgames. For example, the equilibrium outcome of the subgame in which the conqueror demands no tribute, $\tau = 0$, will typically differ from that of the subgame in which $\tau = 1$.

Proposition 3. *An equilibrium of the game exists.*

3.2. **Group choices.** I now describe some basic properties of the groups' behavior in equilibrium, following the conqueror's choice of tribute. One basic result, stated formally in the following lemma, is that there must be some production in equilibrium, and not all of it may be extracted by the conqueror.

Lemma 1. *For all* $\tau \in [0,1]$, $P^*(\tau) > 0$ *and* $\tau g(R^*(\tau)) < 1$.

The proof is as follows. If either of these conditions fails to hold, then every group's payoff is 0. However, any group can assure itself a strictly positive payoff by choosing a strategy with $p_i > 0$ and $r_i > 0$. Therefore, no strategy profile with $P = 0$ or $\tau g(R) = 1$ can be an equilibrium.

Throughout the rest of the paper, I use a simple set of criteria to solve for the groups' equilibrium responses to each potential tribute level. The basic principle is that a group will devote labor only to the activity (or activities) with the greatest marginal benefit. This principle is expressed formally in the first-order conditions for a solution to the constrained optimization problem each group faces,

$$\begin{aligned} \max_{c_i, p_i, r_i} \quad & u_i(c, p, r, \tau) \\ \text{s.t.} \quad & \frac{c_i}{\alpha_i^c} + \frac{p_i}{\alpha_i^p} + r_i = L_i. \end{aligned}$$

The first-order necessary conditions for a solution to this problem, derived by the usual Karush–Kuhn–Tucker method, can be stated as follows. A group may partake only in those actions where the marginal effect of its labor—defined as productivity (the associated α_i) times marginal utility (the associated partial derivative of u_i)—is greatest. For example, if some group $i \in N$ partakes in conflict ($c_i > 0$), the following condition must hold:

$$\alpha_i^c \frac{\partial u_i(c, p, r, \tau)}{\partial c_i} \geq \max \left\{ \alpha_i^p \frac{\partial u_i(c, p, r, \tau)}{\partial p_i}, \frac{\partial u_i(c, p, r, \tau)}{\partial r_i} \right\}.$$

The necessary conditions for $p_i > 0$ and $r_i > 0$ have similar expressions. These conditions fail to hold only if some group could raise its payoff by shifting a small amount of labor from one activity to another, violating the requirements of equilibrium.

I use these conditions in the rest of the analysis to rule out particular types of equilibrium or to pin down the form the equilibrium must take. To illustrate, consider the subgame in which the conqueror demands no tribute, $\tau = 0$. Since nothing is extracted by the conqueror, each group's payoff is given by $s_i(c)P$. The marginal utility of resistance in this case is 0 for every group. Meanwhile, each group's marginal utility of production is its share in the intergroup contest, $s_i(c) > 0$. A strategy profile in which $R > 0$ thus would violate the first-order optimality conditions, as it entails a group partaking in resistance even though production has a strictly greater marginal benefit. Therefore, there cannot be resistance in the subgame in which $\tau = 0$; i.e., $R^*(0) = 0$.

4. Symmetric Groups and 'Divide et Impera'

The baseline form of the model is the symmetric case, in which the groups are identical to one another. The symmetric model is defined by all groups $i \in N$ having the same economic productivity, $\alpha_i^p = \alpha^p$; the same effectiveness in intergroup conflict, $\alpha_i^c = \alpha^c$; and the same labor endowment, $L_i = L/n$. In this setting, it is possible to examine how the number of groups affects equilibrium choices and the payoffs to extraction while holding fixed all other aspects of the model, including society's total labor endowment L.

The main findings in the baseline symmetric model are as follows. First, higher extractive demands by the conqueror lead to greater resistance among the groups, causing production and internal conflict to decrease. In fact, beyond a certain threshold, the increase in resistance and decline in production are always enough to offset the conqueror's gains from higher tribute levels. This leads to the second main result: in equilibrium, the conqueror demands little enough to avoid provoking resistance. The threat of unified resistance is effective in taming the conqueror's extractive ambitions. The last set of results concerns the relationship between the number of groups, the amount of internal conflict, and how much the conqueror can extract from the society. I find that conflict among groups increases with the number of groups, to the detriment of both production and resistance; this leaves the conqueror with a larger share of a smaller pie. Contrary to the apparent logic of *divide et impera*, there is no incentive for an extractive outside force to encourage or create artificial divisions in an otherwise symmetric society.

4.1. **The groups' choices.**

Proceeding via backward induction, I begin the analysis of the symmetric game by solving for the groups' responses to each potential tribute level. The basic form of the equilibrium is intuitive: at higher levels of τ, representing greater extractive efforts by the conqueror, the groups devote more labor to resistance and less to production. The amount of conflict between groups increases with the number of groups, n, and their level of "conflict productivity," α^c. The value of α^c is directly related to how much one group can take from the others, given a fixed amount of labor devoted to conflict. In this sense, the groups in society are mutually secure if α^c is low, and mutually vulnerable if it is high.

The assumption of symmetry means the groups are *a priori* identical. No group is more economically productive than the others, nor does any group have an inherent advantage in the competition over resources. This equality of characteristics leads to equality of outcomes. In equilibrium, all groups devote the same amount of labor to conflict, meaning they all receive the same share of output—and thus the same payoff.

Lemma 2. *If the groups are symmetric,* $s_i(c^*(\tau)) = \frac{1}{n}$ *for all* $i \in N$ *and* $\tau \in [0,1]$.

Each group's share of the output not extracted by the conqueror is $\frac{1}{n}$, which is the same as if there were no conflict. Therefore, under any equilibrium with internal conflict, it would be a strict Pareto improvement for each group to divert all of the labor it spends on conflict to production instead. The problem is that this collective improvement would not be individually rational. In order for peace among groups to be sustainable in equilibrium, there must be no group that could benefit by devoting a bit of labor to conflict when the others spend nothing—i.e., the marginal return to conflict at $c = (0,\dots,0)$ must be sufficiently low. Three factors determine whether this condition holds. First, the "conflict productivity" α^c determines how much a group can increase its share with a fixed amount of labor, so it sets the baseline for the attractiveness of conflict relative to production and resistance. Second, as the value of the groups' total intake $(1 - \tau g(R))P$ increases, so does the benefit of using conflict to increase one's share of it. Third, the greater the number of groups, and thus the smaller each group's initial share, the greater the incentive for each group to raise its payoff by taking from the others instead of contributing directly to total output.

The relationship between the number of groups and the incentive to partake in conflict can be used to characterize the conditions under which peace prevails in equilibrium. Specifically, I show that there is a cutpoint, which I call the *conflict threshold*, determining whether conflict occurs. If the number of groups is at or below the threshold, then every equilibrium of every subgame is peaceful, with each group choosing $c_i = 0$. On the other hand, if the number of groups exceeds the threshold, then there is internal conflict in every subgame with sufficiently low tribute. The threshold is formally derived in the following lemma.

Lemma 3. *Let the groups be symmetric and let* $\eta^* = \frac{f(0)}{\alpha^c L f'(0)}$. *There is no internal conflict in any equilibrium of any subgame, i.e.,* $c_i^*(\tau) = 0$ *for all* $i \in N$ *and* $\tau \in [0,1]$, *if and only if* $\frac{n-1}{n} \leq \eta^*$.

With these results in hand, it is now possible to derive the groups' equilibrium responses to each potential tribute level. Naturally, the form the equilibrium takes depends on whether the number of groups exceeds the conflict threshold. I begin with the case below the threshold. As just shown, no group partakes in conflict in this scenario, so the remaining task is to determine the levels of production and resistance at each value of τ. If the conqueror demands nothing, or very little, the groups have no incentive to resist. So at low values of τ, the equilibrium response is for every group to spend its full labor complement on production, putting total output at its highest feasible value, $P = \alpha^p L$. At a certain point, however, the conqueror's demands become great enough that full production is no longer in the groups' interests. In this case, they divert some of their effort into resistance in order to keep down the conqueror's share of the output. The exact levels of production and resistance are determined by the optimality conditions for the groups' choices, as discussed in Section 3.2. Specifically, the marginal returns to resistance and production must be equal, a condition that is stated formally in the following equation:

$$\zeta^{pr}(P,R;\tau) = \alpha^p(1-\tau g(R)) + \tau P g'(R) = 0$$
$$\iff \alpha^p \frac{\partial u_i(c,p,r,\tau)}{\partial p_i} = \frac{\partial u_i(c,p,r,\tau)}{\partial r_i} \text{ for all } i \in N \quad (4.1)$$

One of the implications of this condition is that as the tribute level increases, so must the amount of resistance. The following proposition, which is illustrated in Figure 1, contains the formal statement of equilibrium outcomes below the conflict threshold.

Proposition 4. *If the groups are symmetric and* $\frac{n-1}{n} \leq \eta^*$*,*

(a) $R^*(\tau) = 0$ *for all* $\tau \leq (g(0) - Lg'(0))^{-1} \equiv \bar{\tau}_0$*,*

(b) $R^*(\tau)$ *solves* $\zeta^{pr}(\alpha^p(L-R),R;\tau) = 0$ *for all* $\tau > \bar{\tau}_0$*,*

(c) $P^*(\tau) = \alpha^p(L - R^*(\tau))$ *for all* τ*.*

The outcomes given by this proposition are optimal from the standpoint of the groups' overall welfare. If there were a benevolent social planner whose objective were to maximize the sum of the groups' payoffs given the conqueror's choice of tribute, she would implement the outcome prescribed by Proposition 4. The key to this result is the absence of conflict between groups. From the hypothetical social planner's standpoint, any such conflict is purely destructive. Conflict merely shifts the distribution of spoils across groups, having no direct impact on total welfare, while indirectly reducing the groups'

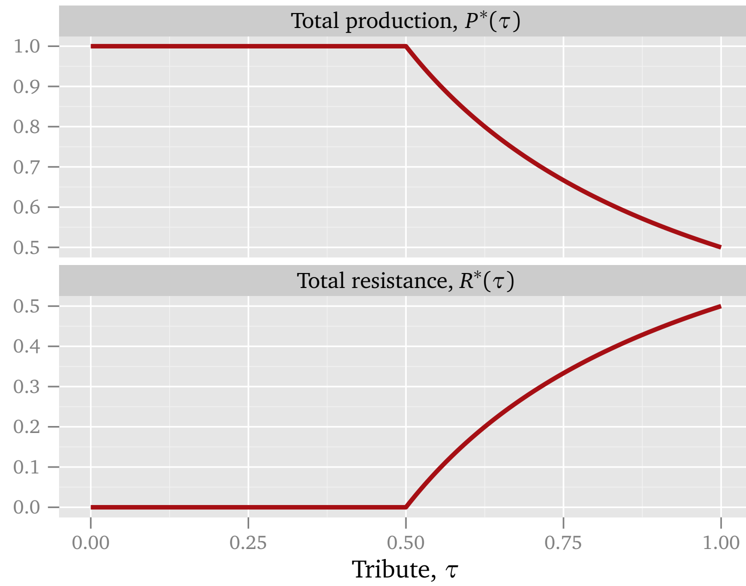

FIGURE 1. The groups' equilibrium choices as a function of tribute when the number of groups is below the conflict threshold, as given by Proposition 4.[5]

total surplus by siphoning labor away from production or resistance. Conversely, in the absence of such conflict, the groups' equilibrium choices result in a socially optimal outcome.

Proposition 5. *If the groups are symmetric, the social welfare–maximizing outcome of each subgame is the corresponding outcome given in Proposition 4.*

I now turn to the form of the equilibrium when the number of groups exceeds the conflict threshold. Above the threshold, there is no equilibrium in which the groups spend all of their labor on production; there must be internal conflict, resistance, or both. As in the prior case, the form of the equilibrium is defined by cutpoints on the tribute level. When τ is below the first cutpoint, meaning the conqueror's demands are minimal, the groups divide all their effort between production and internal conflict. There is then a middle range, when τ is between the two cutpoints, in which production, resistance, and conflict among groups all take place in equilibrium. In this range, resistance increases with tribute, while the other two activities decrease. When τ is above the second cutpoint, representing very high demands by the conqueror, internal conflict ceases altogether. In this sense, greater extractive demands unify the groups, giving each an incentive to contribute toward the public good of resistance instead of appropriating from the others. Only in this high range are the equilibrium outcomes equivalent to

[5]The functional forms and parameter values used in this graph, as well as in Figures 2 and 3, are as follows: $f(c_i) = \exp(c_i)$, $g(R) = 1 - \frac{R}{L}$, $\alpha^c = 2$, $\alpha^p = 1$, and $L = 1$. Note that $\eta^* = \frac{1}{2}$, so the conflict threshold is exceeded if and only if $n > 2$.

those given in Proposition 4. Otherwise, in the cases where tribute is low enough to allow for internal conflict, the equilibrium outcomes are suboptimal in terms of the groups' total welfare.

As with resistance in the previous case, if internal conflict occurs in equilibrium, its marginal return to the groups must equal that of production. This condition, which is stated formally in the following equation, can be used to derive the equilibrium level of c_i in each subgame:

$$\zeta^{pc}(P,R;n) = \frac{\alpha^p}{P} - \alpha^c \frac{n-1}{n} \frac{f'(\frac{\alpha^c}{n}(L - \frac{P}{\alpha^p} - R))}{f(\frac{\alpha^c}{n}(L - \frac{P}{\alpha^p} - R))} = 0$$

$$\Leftrightarrow \alpha^p \frac{\partial u_i(c,p,r,\tau)}{\partial p_i} = \alpha^c \frac{\partial u_i(c,p,r,\tau)}{\partial c_i} \text{ for all } i \in N. \quad (4.2)$$

With this expression defined, I can now state the equilibrium outcome at each tribute level when the number of groups exceeds the conflict threshold.

Proposition 6. *If the groups are symmetric and* $\frac{n-1}{n} > \eta^*$,

(a) $P^*(\tau)$ *solves* $\zeta^{pc}(P,0;n) = 0$ *and* $R^*(\tau) = 0$ *for all* $\tau \le \bar{\tau}_1(n)$,

(b) $P^*(\tau)$ *and* $R^*(\tau)$ *solve* $\zeta^{pc}(P,R;n) = \zeta^{pr}(P,R;\tau) = 0$ *for all* $\tau \in (\bar{\tau}_1(n), \bar{\tau}_2(n))$,

(c) $R^*(\tau)$ *solves* $\zeta^{pr}(\alpha^p(L-R),R;\tau) = 0$ *and* $P^*(\tau) = \alpha^p(L - R^*(\tau))$ *for all* $\tau \ge \bar{\tau}_2(n)$

(d) $c_i^*(\tau) = \frac{\alpha^c}{n}(L - \frac{P^*(\tau)}{\alpha^p} - R^*(\tau))$ *for all* $i \in N$ *and all* τ, *with* $c_i^*(\tau) > 0$ *if and only if* $\tau < \bar{\tau}_2(n)$.

The cutpoints are $\bar{\tau}_1(n) \equiv \left(g(0) - \frac{\tilde{P}_1(n)}{\alpha^p} g'(0)\right)^{-1}$ *and* $\bar{\tau}_2(n) \equiv \left((g(L - \frac{\tilde{P}_2(n)}{\alpha^p}) - \frac{\tilde{P}_2(n)}{\alpha^p} g'(L - \frac{\tilde{P}_2(n)}{\alpha^p})\right)^{-1}$, *where* $\tilde{P}_1(n)$ *and* $\tilde{P}_2(n)$ *solve* $\zeta^{pc}(P,0;n) = 0$ *and* $\zeta^{pc}(P, L - \frac{P}{\alpha^p}; n) = 0$ *respectively.*

These results are illustrated in Figure 2. In this case, unlike the form of the equilibrium below the conflict threshold, the tribute cutpoints and the levels of each activity depend on the number of groups, n. This is driven by the fact that the return to conflict is greater when there are more groups. One specific consequence is that the baseline amount of production at low levels of tribute, $\tilde{P}_1(n)$, decreases with n. Another is that the highest level of tribute at which there is no resistance, the cutpoint $\bar{\tau}_1(n)$, increases with the number of groups. Notice the cross-cutting effects on the conqueror's utility: as the number of groups increases, it can extract proportionally more without engendering resistance, but overall output is lower. I return to the question of how the number of groups affects the conqueror's welfare in Section 4.2. Before that, I move up the game tree to solve for the equilibrium level of tribute.

4.2. **Optimal tribute: a policy of accommodation.** The conqueror faces a complex set of trade-offs in choosing τ, the proportion of economic output to demand from society. If production and resistance

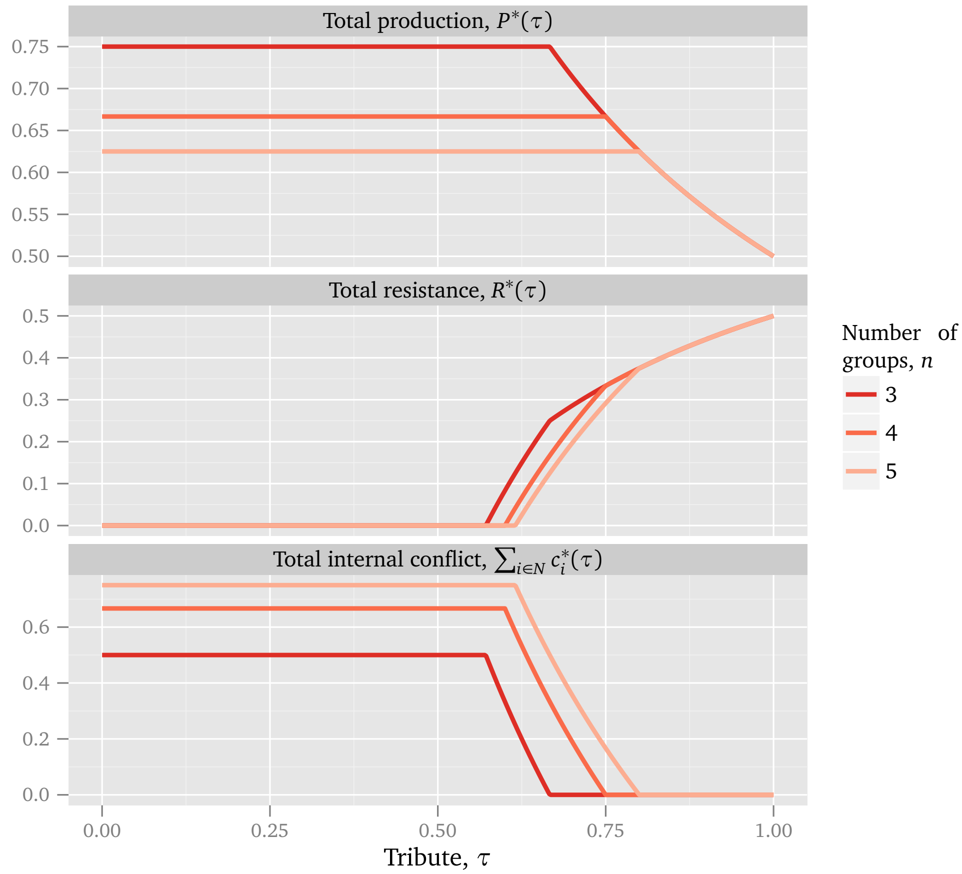

FIGURE 2. The groups' equilibrium choices as a function of tribute when the number of groups exceeds the conflict threshold, as given by Proposition 6.

were fixed, its choice would simply be to demand as much as possible, $\tau = 1$. However, the conqueror must also consider how its choice affects the groups' equilibrium responses. In particular, greater tribute leads to less production and more resistance, both of which reduce the conqueror's payoff. In making its choice, the conqueror must weigh the direct effect of demanding more tribute against these indirect effects on the total amount available.

I find that the optimal tribute level for the conqueror is the highest level at which there is no resistance, which I refer to as a *policy of accommodation* and denote by τ^*. This level represents a Goldilocks solution for the conqueror—neither too high nor too low. At any tribute level less than τ^*, the groups' equilibrium choices are the same as if the conqueror were not part of the game. Within this range, greater demands unambiguously raise the conqueror's payoff, as they have no effect on production and resistance. On the other hand, as the conqueror raises tribute beyond τ^*, lower production and higher resistance offset the positive direct effect of a higher tribute level on the conqueror's utility. Therefore, the best choice for the conqueror is the policy of accommodation, as stated formally in the following proposition.

Proposition 7. *It is an equilibrium for the conqueror to implement the highest tribute level at which there is no resistance,* $\tau^* = \max\{\tau \mid R^*(\tau) = 0\}$.

The key step in the proof is to show that the reduction in production and increase in resistance due to choosing $\tau > \tau^*$ are great enough to hold down the conqueror's overall payoff. The result relies on the fact that the groups' marginal returns to resistance and production must be equal when $R^*(\tau) > 0$. The return to resistance can be expressed as an increasing function of the conqueror's payoff, $\tau g(R)P$. At the same time, the return to production is a function of the groups' share of the total surplus, $1 - \tau g(R)$, so it decreases with tribute. In order for these two values to remain in line when tribute increases beyond τ^*, there must be enough of an increase in resistance and a decrease in production to reduce the conqueror's payoff. In other words, under any strategy profile that gives the conqueror more than it would receive by demanding τ^*, any group with $p_i > 0$ would have an incentive to divert some labor from production to resistance.

The assumption of symmetry between groups is crucial for this result. The logic relies on the fact that the returns to production and resistance must be the same for all groups whenever $P^*(\tau) > 0$ and $R^*(\tau) > 0$. The same condition need not hold if the groups differ in their economic productivity, α_i^p. In particular, the logic used in the proof would not apply to an equilibrium in which one set of groups undertook all production, while a different set were responsible for all resistance. I examine this alternative setting in detail in Section 5. For now, the upshot is that inequality between groups is a necessary condition for resistance to be observed on the equilibrium path in this model. If the groups in a polity are alike in terms of size and resources, the threat of broad resistance will deter an outside force from pursuing highly extractive policies.

This result also has a connection to the concept of *indirect rule*, a style of colonial governance, typically practiced by the British, in which the outside ruler leaves indigenous political structures in place while skimming rents off the top (Crowder 1964). The model in this paper is too sparse to capture all of the features of indirect rule, but there are clear similarities. In particular, when the conqueror chooses a policy of accommodation, the groups' choices—i.e., how much they produce and how much they dedicate to internal conflict—are exactly the same as if the conqueror were not present at all.[6] The only difference is that some share of output ends up going to the outside power instead of being shared among the groups. Consequently, insofar as the equilibrium outcome entails internal political conflict and economic production remaining the same as before conquest, it closely resembles a policy of indirect rule.

The result also speaks to the question of "divide and rule" as a philosophy for appropriating from a fractionalized society. When the number of groups is above the conflict threshold, the conqueror effectively maximizes internal conflict by choosing a policy of accommodation. The amount of conflict between groups observed on the equilibrium path is at least as high as in the equilibrium outcome of any other subgame. In one sense, this provides evidence for the notion that an outside power can extract more by encouraging internal conflict. Conditional on facing a divided society, the optimal policy for the conqueror is one that entails high internal conflict and low resistance. Nonetheless, this does not quite answer the broader question of whether the conqueror benefits from the presence of such social divisions. To that end, the relevant counterfactual is how well the conqueror would fare when facing a society with fewer divisions and thus less internal conflict. Even though a conqueror's best strategy in a divided society is the one that maximizes conflict between groups, it may nonetheless be the case that the conqueror would rather govern a more unified polity if given the choice. To address the broader issue of whether the conqueror prefers more social divisions, I next perform comparative statics analysis on n.

4.3. **Does internal conflict benefit the conqueror?** I now address the relationship between the number of groups and various equilibrium quantities of interest, including the conqueror's payoff. The symmetric form of the game is the natural venue to examine the effects of the number of groups while holding all else equal—without symmetry, the answer to "What if there were one less group?" would depend on which group is subtracted. I find that the equilibrium tribute level increases with n, while total production decreases. In other words, as more group divisions are created, the conqueror ends up with a larger slice of a smaller pie. The overall effect on the conqueror's payoff is negative, as illustrated in Figure 3. I state these comparative statics findings formally in the following proposition.

Proposition 8. *Suppose the groups are symmetric and let their number vary, so the equilibrium outcomes $P^*(\tau; n)$ and $R^*(\tau; n)$ depend on both tribute and the number of groups, as does the conqueror's induced*

[6] Formally, the result is that $P^*(\tau^*) = P^*(0)$, $R^*(\tau^*) = R^*(0) = 0$, and $c_i^*(\tau^*) = c_i^*(0)$ for all groups $i \in N$.

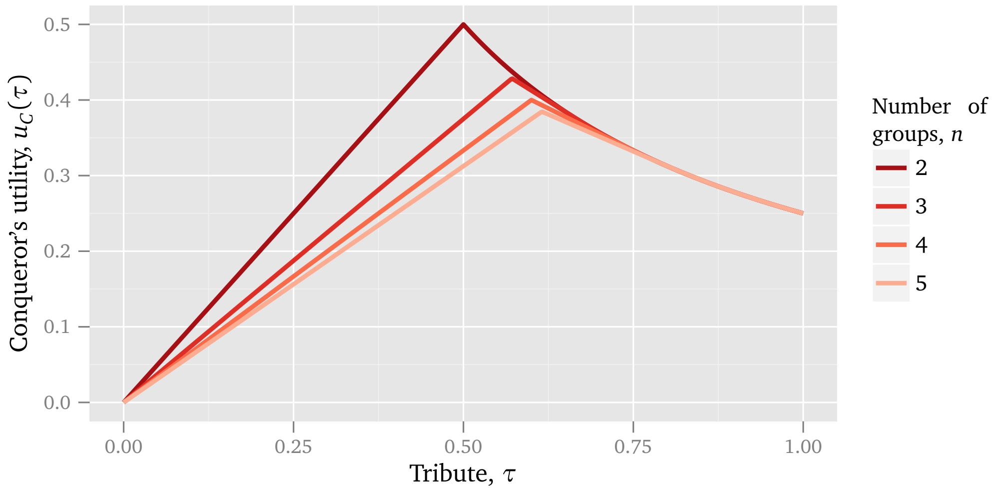

FIGURE 3. The conqueror's payoff as a function of tribute.

utility function $u_C(\tau; n)$*. Let* $\tau^*(n)$ *denote the equilibrium tribute level given by Proposition 7,* $\tau^*(n) = \max\{\tau \mid R^*(\tau; n) = 0\}$*. Take any* $n', n'' \in \mathbb{N}$ *such that* $n'' > n'$*.*

(a) Equilibrium tribute is weakly increasing, $\tau^*(n'') \geq \tau^*(n')$

(b) Equilibrium production is weakly decreasing, $P^*(\tau^*(n''), n'') \leq P^*(\tau^*(n'), n')$*,*

(c) The conqueror's equilibrium utility is weakly decreasing, $u_C(\tau^*(n''); n'') \leq u_C(\tau^*(n'); n')$*,*

(d) Each of these inequalities is strict if $\frac{n''-1}{n''} > \eta^*$*.*

Each of these results can be traced back to the fact that the marginal return to conflict increases with the number of groups. In order for resistance to occur in equilibrium, its marginal return must be at least as great as that of conflict. This condition becomes more stringent as the number of groups (and hence the return to conflict) goes up, allowing the conqueror to extract more without engendering resistance. But by the same token, equilibrium production decreases with the number of groups. Above the conflict threshold, the marginal returns to production and conflict are equal on the equilibrium path. In order to keep these in balance, an increase in n must be met with a decrease in P, as one can verify from (4.2). So as the number of groups increases, the conqueror is left with a larger share of a smaller pie. The remaining question is which of these effects dominates. I show in the proof of the proposition that the overall effect of n on the conqueror's utility is negative, meaning the conqueror would prefer to rule a society with fewer groups.

To summarize, I have found a pair of apparently paradoxical results. On one hand, when the number of groups in society exceeds the conflict threshold, the conqueror chooses a policy that maximizes internal conflict and minimizes resistance. Thus it would appear that the conqueror benefits from exploiting divisions in the population. On the other hand, all else equal, more can be extracted from a relatively united society than from a severely fractionalized one. In this sense, group divisions work against the conqueror's interests. One way to reconcile this apparent contradiction is to think of the equilibrium outcome above the conflict threshold as a second-best solution for the conqueror. The equilibrium tribute in this case maximizes internal conflict not because the conqueror unconditionally benefits from conflict—it does not—but rather because such a policy is the best option in an undesirable situation.

5. Asymmetric Groups and the Consequences of Specialization

I now illustrate how the form of the equilibrium can change if the assumption of group symmetry is relaxed. As it would be intractable to give a full derivation of the equilibrium under general asymmetry between groups, I instead highlight a case whose substantive implications and differences from the baseline model are particularly striking. Specifically, I consider a case in which part of the population specializes in economic production and the other specializes in resistance. In this scenario, unlike in the symmetric case, higher extractive demands by the conqueror may actually lead to greater production, and a policy of accommodation is not necessarily optimal for the conqueror. These results suggest that we may observe less internal conflict and more resistance against external rule when groups have well-defined comparative advantages than when they are essentially similar.

Suppose there are $n = 2$ groups, one labeled W for workers and the other I for insurgents.[7] Let the workers be more economically productive, so that $\alpha_W^p > \alpha_I^p$. By definition, the workers have a comparative advantage in production, and the insurgents have a comparative advantage in resistance. It is straightforward to verify that there is no equilibrium of any subgame in which each group engages in both production and resistance. If $p_I > 0$, meaning the return to conflict is weakly greater than the return to resistance for the insurgents, then this must hold strictly for the workers, so $r_W = 0$. Equivalently, if $r_W > 0$, then $p_I = 0$. This suggests that the equilibrium in some subgames may involve the workers specializing in production, while the insurgents specialize in resistance. In particular, consider a strategy profile of the following form:

- The workers produce and engage in internal conflict, but do not resist: $c_W, p_W > 0$, $r_W = 0$,
- The insurgents resist and engage in internal conflict, but do not produce: $c_I, r_I > 0$, $p_I = 0$.

[7] All results in this section extend to the case in which there are $n_W \geq 1$ groups each with $(\alpha_i^c, \alpha_i^p, L_i) = (\alpha_W^c, \alpha_W^p, L_W)$ and $n_I \geq 1$ groups each with $(\alpha_i^c, \alpha_i^p, L_i) = (\alpha_I^c, \alpha_I^p, L_I)$.

If an equilibrium of some subgame takes this form, call it a *specialist equilibrium*.

A specialist equilibrium differs from the equilibria of the baseline symmetric game in two substantial ways. The first is particularly counterintuitive: at the margin, greater extractive demands by the conqueror cause production to increase. To see why this holds, recall the optimality conditions that must hold in a specialist equilibrium. The returns to resistance and conflict must be the same for the insurgents, and the returns to production and conflict must be the same for the workers. A marginal increase in tribute increases the incentive for resistance, so the insurgents must respond by slightly decreasing c_I and increasing r_I. The decrease in c_I causes the return to conflict for the workers to decrease, so they in turn must respond with a slight increase in p_I. Just like in the symmetric game, an increase in tribute decreases the incentive to produce rather than resistance. However, this marginal change does not make a difference to W's choice; it still strictly prefers production over resistance. All that matters is the decrease in the workers' incentive to engage in conflict, which is what leads to the increase in production. I state the result formally in the following proposition.

Proposition 9. *Suppose that for all $\tau \in (\tau', \tau'') \subseteq [0,1]$, there is an equilibrium of $\Gamma(\tau)$ that is a specialist equilibrium. Then P^* and R^* are both strictly increasing on (τ', τ'').*

The second major difference from the symmetric form of the game is that resistance may be observed on the equilibrium path. In other words, when some groups specialize in resistance while others specialize in production, it is not necessarily optimal for the conqueror to choose a policy of accommodation. Recall that when the groups are symmetric, an increase in tribute beyond the policy of accommodation has two negative indirect effects on the conqueror's payoff: it increases resistance and decreases production. In the present setting, however, the indirect effects of higher tribute are mixed, one negative (greater resistance) and one positive (greater production). Therefore, unlike in the symmetric case, the overall effect of raising tribute may be to increase the conqueror's payoff, which in turn means we may observe resistance on the equilibrium path.[8]

The logic of the specialist equilibrium can be applied to analyze social and political conflict in Xinjiang, a northwestern province of China. The predominant ethnic group in Xinjiang is the Uyghurs, most of whom are Muslims. Since the mid-20th century, in the course of combating separatist movements

[8] Additionally, the logic of Proposition 7, which established the optimality of a policy of accommodation under symmetry, no longer applies here. Specifically, the proof that resistance grows quickly enough to offset any increase in the conqueror's payoff relies on the fact that, under symmetry, the marginal returns to production and resistance are the same for every group. In a specialist equilibrium, however, the workers' return to production may far exceed its return to resistance, while the opposite may be true for the insurgents.

in Xinjiang led by Uyghurs, the Chinese government has encouraged in-migration by Han Chinese; the Han population share grew from five percent in 1941 to 40 percent in 1970, and has remained at roughly that level (Toops 2004). The ethnic division between Uyghurs and Hans is coincident with an economic division, as Hans tend to receive more education and are more likely to work in the manufacturing or transportation industries, rather than in the less remunerative agriculture sector (Hannum and Xie 1998). The region has a long history of Uyghur separatism, which reached its climax in the short-lived independent East Turkistan Republics of 1933–1934 and 1944–1949 but still continues today. Expressions of Uyghur resistance against Beijing's rule range from "everyday resistance" activities like "grumbling, songs, jokes, satire, and political fantasy" to actual violence against Communist Party authorities (Bovingdon 2002). Related to, but distinct from, such actions against central rule are localized instances of violence between Uyghurs and Hans. The most notable instance of ethnic conflict in recent memory was the July 2009 rioting in Urumqi, Xinjiang's capital, which grew out of protests over earlier murders of Uyghur workers by Hans and resulted in more than 150 deaths (Wong 2009). Although the state media has reported that the Urumqi riots were instigated by foreign terrorists with separatist aims, in truth they—and many other violent episodes designated as "terrorism" by the Chinese government—were rooted in localized ethnic tensions (Rodríguez 2013).

The model illustrates a mechanism by which the central government benefits from Han in-migration to Xinjiang, despite the accompanying exacerbation of internal ethnic strife. Notice that the situation in Xinjiang roughly resembles a specialist equilibrium. The "worker" group W is the productively advantaged Hans, the "insurgent" group I is the Uyghurs responsible for the bulk of resistance, and the "conqueror" is the Chinese government which seeks to control Xinjiang's economic output. This is a vast simplification, of course, but the model does provide some broad insights into the in-migration efforts.[9] Imagine the effects of a marginal increase in L_W, the size of the worker group, holding all else fixed. Group W will divide its additional labor between production p_W and internal conflict c_W. There is no direct effect on the insurgent group, but the increase in c_W will cause group I to respond with a commensurate increase in its own conflict efforts c_I, which in turn necessitates a decrease in resistance r_I. In other words, there are two gains to the outside power: economic production increases, and the increase in ethnic conflict draws the insurgent group's attention away from resistance. This

[9]The clearest difference between the case at hand and the model is that the Uyghurs do participate in economic production; in a specialist equilibrium, by contrast, group I devotes all of its labor to resistance and internal conflict. But because what the "outside power" (i.e., the central government) values most is the manufacturing and transportation industry, which is dominated by the Han group, I see the specialist equilibrium as a reasonable enough approximation for analyzing Chinese policy in Xinjiang.

suggests, perhaps unexpectedly, that the Chinese government's ethnic policies in Xinjiang act to reduce the likelihood of a successful movement for independence. Why does an increase in the population of group W increase, rather than decrease, the incentive for group I to resist? The payoff to group I from successfully overthrowing the conqueror depends on its share from internal conflict, $s_I(c)$, whose value in equilibrium decreases with the size of the other group. To phrase this in terms of the empirical application, the greater the Han population of Xinjiang, the less political power the Uyghurs would have in the new state if it suddenly won independence from China. In line with this logic, Gladney (2004, p. 395) reports that in his conversations with Xinjiang residents, both Hans and Uyghurs recognize that independence would simply lead to more internal conflict, especially because both groups would face barriers to emigration.

6. Extensions

The simple baseline model presented in this paper can easily be extended to answer more specific questions about phenomena related to economic extraction and internal conflict. I illustrate three such applications here. For the sake of simplicity, I maintain the assumption of group symmetry throughout this section. In the first extension, I examine how the introduction of natural resource wealth affects equilibrium behavior in the model. Second, I relax the assumption that the conqueror can commit to a level of extraction in advance of the groups' decisions and show the lack of commitment leads to Pareto-inferior outcomes. In the same context, I also show how repeated interaction and productive investment may partly resolve the problems caused by an inability to make commitments. Finally, in the third extension, I allow the conqueror to set different rates of tribute for different groups. Perhaps surprisingly, I find that the conqueror cannot gain from creating inequality between groups; the equilibrium is the same as in the original symmetric game.

6.1. **Natural resources.** Economists have observed that economic growth has been relatively slow in countries with large natural resource endowments, a phenomenon known as the resource curse (Sachs and Warner 2001). Natural resource wealth has also been identified as a major correlate of violent civil conflict (Collier and Hoeffler 1998; Ross 2004*a*,*b*). Extending the model to allow for natural resources, meaning economic product that is available regardless of the groups' choices, I indeed find that an exogenous increase in resource wealth is associated with a decrease in production and an increase in internal conflict (as well as resistance). The results of this extension are similar to those of Hodler (2006), who finds that natural resource wealth increases internal conflict and detracts from productive activity in a divided society. The main difference between Hodler's model and the one here is that I include an outside actor who is not part of the resource-endowed society but seeks to appropriate from it.

To incorporate the notion of natural resources into the model, I assume that total economic output includes an exogenous resource endowment $X \geq 0$ in addition to the groups' production P. The utility functions are now

$$u_i(c, p, r, \tau; X) = s_i(c) \times (1 - \tau g(R)) \times (P + X), \tag{6.1}$$

$$u_C(c, p, r, \tau; X) = \tau g(R) \times (P + X), \tag{6.2}$$

while all other components of the model (timing, budget constraints, etc.) remain the same as before. In this extension of the model, the existence and uniqueness results of Section 3.1 still hold, as does the fact that every group's contest share is $\frac{1}{n}$ in equilibrium under symmetry (Lemma 2). To facilitate analysis of how the level of natural resources affects the groups' choices, I write the equilibrium outcomes as functions of both tribute and the resource endowment: $P^*(\tau; X)$, $R^*(\tau; X)$, etc.

A substantial difference from the baseline model is that there may not be production in equilibrium. If $X > 0$, lack of production does not imply a payoff of 0 for the groups, so the logic of Lemma 1 no longer holds. In fact, I find that there is a *production threshold* X^* beyond which there is never production in any equilibrium of any subgame.

Lemma 4. *In the symmetric game with natural resources, if* $X \geq X^* \equiv \frac{\alpha^p}{\alpha^c} \frac{f(\alpha^c L/n)}{f'(\alpha^c L/n)} \frac{n}{n-1}$*, then* $P^*(\tau; X) = 0$ *for all* $\tau \in [0, 1]$.

The production threshold is just one consequence of a broader phenomenon: greater natural resources lead to more resistance and conflict at the expense of productive activity. In this sense, as the resource curse literature would suggest, higher resource endowments do not necessarily translate into better outcomes for the conqueror. Consider the effects of an exogenous increase in X. Holding fixed the tribute level and the groups' choices, this shock has no effect on a group's marginal return to production, but it increases the returns to resistance and conflict. Accordingly, the effect of a positive shock to X on the groups' equilibrium responses to a given tribute level is to decrease production, increase resistance, and increase conflict between groups.

Proposition 10. *In the symmetric game with natural resources, if* $X < X'$*, then for all* $\tau \in [0, 1]$*,*

(a) $P^*(\tau; X) \geq P^*(\tau; X') \geq P^*(\tau; X) + X - X'$ *(the first holding strictly if* $0 < P^*(\tau; X) < \alpha^p L$*),*

(b) $R^*(\tau;X) \leq R^*(\tau;X')$ *(with equality if* $P^*(\tau;X) = 0$*),*

(c) $c_i^*(\tau;X) \leq c_i^*(\tau;X')$ *for all* $i \in N$ *(with equality if* $P^*(\tau;X) = 0$*).*

The addition of natural resources to the game also changes the conqueror's decision calculus. In the original symmetric game, the optimal tribute level is a policy of accommodation, the highest level that does not engender resistance. As explained in the discussion of Proposition 7, under any strategy profile giving the conqueror more than it receives from a policy of accommodation, each group would have an incentive to shift labor from production into resistance. This logic does not necessarily hold when natural resources are introduced to the game, since there may be no production in equilibrium. Indeed, the form of the equilibrium shifts if the natural resource endowment is great enough. If X is low, there is an equilibrium similar to that of the original symmetric game, with production and no resistance. But at higher resource levels, in equilibrium the conqueror chooses a tribute level at which there is no production and potentially positive resistance, depending on the other parameters. Simply put, there are two types of societies on the equilibrium path: those with low resources, high production, and low resistance; and those with high resources and low production (resistance indeterminate).

Proposition 11. *In the symmetric game with natural resources, there exists* $X^{**} \in (0, X^*]$ *such that*

(a) If $X < X^{**}$*, it is an equilibrium for the conqueror to select* $\bar{\tau}(X) \equiv \max\{\tau \mid R^*(\tau;X) = 0\}$*,*

(b) If $X > X^{**}$*,* $P^*(\tau^*;X) = 0$ *for every equilibrium tribute level* τ^**.*

The differences in the results of Spanish conquest in the Yucatán peninsula and in other parts of Mexico highlight this relationship between natural resource endowments and the outcome of conquest. Yucatán was notable for its lack of natural resource wealth, specifically precious metals, compared to other conquered regions of Mexico (Farriss 1984, pp. 30–32). The profits to colonialism in Yucatán relied more heavily on the maintenance of indigenous labor than in regions where silver or other precious metals could be extracted at a low labor cost. The necessity of maintaining the native labor force may even partly account for why post-conquest depopulation was less severe in Yucatán than elsewhere in Mexico (Farriss 1984, pp. 81–82). These economic results are in line with the above proposition: reliance on native labor and the tribute economy decreased with the availability of natural resources. Regional differences in the socio-political consequences of conquest also line up with the results of the model. In the low-resource case of the proposition, part (a), the conqueror's choice results in the same level of internal conflict as if it were not there at all; i.e., it is indirect rule, as in the original symmetric game without natural resources. Indeed, the Spanish conquistadors in Yucatán pursued such a policy, maintaining the pre-existing Mayan political structures and operating through its elites (Farriss 1984, pp. 86–103). In central Mexico, by contrast, conquest was highly disruptive to native political institutions, with Spanish colonialists and priests usurping authority at both national and local levels (Hassig 1994, pp. 150–158; Knight 2002, pp. 102–127).

Returning to the model, the presence of natural resources also alters the relationship between group conflict and the amount the conqueror is able to extract. One of the major findings in the original symmetric game is that the conqueror's payoff decreases with the number of groups, because growing intergroup conflict siphons away from production (Proposition 8). If society lives on natural resource wealth instead of producing, as is the case in equilibrium if $X > X^{**}$, this logic no longer holds. When there is no production, the only way for the groups to spend more on internal conflict is to reduce resistance, allowing the conqueror to extract a greater share of X. Internal conflict thus may be a boon to the conqueror in the presence of high natural resource wealth, unlike when the conqueror can only extract society's endogenous economic production. In Example 1 in the Appendix, I highlight some parameter values under which the conqueror would be better off if the number of groups increased.

6.2. **Committing to a level of extraction.** In the baseline model, the tribute level is fixed before the groups make their choices and cannot be changed thereafter. This is akin to assuming that the conqueror can commit to the announced tribute level, refraining from any attempt to extract more than promised once the groups have acted. I now examine how the equilibrium outcomes change when this assumption is relaxed. Specifically, I assume the timing of the game is reversed, so the tribute rate τ is chosen after the groups allocate their labor among production, resistance, and internal conflict. This extension is closely related to the hold-up problem in economics, where the possibility of *ex post* renegotiation in the terms of trade leads to underinvestment (Hart and Moore 1988).

If the conqueror cannot commit to the tribute level in advance, both the groups and the conqueror are worse off than in the original game. To solve the modified model, first consider the conqueror's choice at the end of the game. Since production and resistance are already fixed, rather than being functions of τ as in the original game, the conqueror's payoff is strictly increasing as a function of τ. Therefore, regardless of the groups' choices, the conqueror will always select $\tau = 1$. Now move up the game tree to consider the groups' decisions. Since the tribute will be $\tau = 1$ for certain, their payoffs for any given strategy profile are the same as in the subgame of the original model in which $\tau = 1$. The overall equilibrium outcome will thus be the same as in that subgame, with total production $P^*(1)$ and total resistance $R^*(1)$. If the groups are symmetric, this is Pareto inferior to the equilibrium of the original game in which the conqueror selects a policy of accommodation.[10] The higher tribute level obviously leaves the groups worse off, and I have already shown that the conqueror weakly prefers the outcome under a policy of accommodation to any other (Proposition 7).

Is there any way to avoid maximal extraction in this setting? Various contractual approaches to solving the hold-up problem have been proposed in the economics literature (e.g., Rogerson 1992; Nöldeke and Schmidt 1995), but contracts between subject populations and extractive outside powers typically lack legal force. The U.S. government's treaties with American Indian tribes to extend claims over their land—and the eventual abrogation of many of these treaties in the interest of further annexation—are one illustration of how such contracts are unenforceable (Prucha 1997, ch. 14). Instead, I look at how *repeated interaction* and *productive investment* can give the conqueror an incentive to demand less than maximal tribute. In particular, I examine a setting in which part of the groups' share accrues value and is added to the total output in a later period. Unlike in the game without the possibility of such investment, allowing the groups to keep more in the short term may benefit the conqueror by making the pie larger in the long term.

The extended game takes place in two stages, each of which involves the groups allocating their labor followed by the conqueror setting the tribute level. Part of the groups' share of the first-period surplus is invested; i.e., it gains value and carries over to the second period, rather than being consumed immediately. If the return on investment is large enough, it is in the conqueror's interest not to demand too much in the first period. By leaving the groups with more to invest instead of extracting as much as possible, the conqueror takes a short-term loss for the sake of a long-run gain. The timing of the extended model is as follows:

First period: The groups make their choices, $\sigma^1 = (c^1, p^1, r^1)$. The conqueror then selects the first-period tribute τ^1. The groups immediately consume $\gamma \in (0,1)$ of their share of the first-period output, while the remainder $1-\gamma$ is invested for the next period. Payoffs for the period are as follows:

$$u_i^1(\sigma^1, \tau^1) = s_i(c^1)(1 - \tau^1 g(R^1))\gamma P^1,$$

$$u_C^1(\sigma^1, \tau^1) = \tau^1 g(R^1) P^1.$$

Second period: The groups make their choices, $\sigma^2 = (c^2, p^2, r^2)$. The investment grows to $\frac{\beta}{1-\gamma}$ of its original value ($\beta > 0$) and is added to total production for the period. Finally, the conqueror

[10] Unless $R^*(1) = 0$, in which case $\tau = 1$ *is* a policy of accommodation.

selects the second-period tribute τ^2. The game ends after this stage, so both the groups and the conqueror immediately consume their shares of the second-period output. Payoffs for the period are as follows:

$$u_i^2(\sigma^2, \tau^2; \sigma^1, \tau^1) = s_i(c^2)(1 - \tau^2 g(R^2))(P^2 + \beta(1 - \tau^1 g(R^1))P^1),$$
$$u_C^2(\sigma^2, \tau^2; \sigma^1, \tau^1) = \tau^2 g(R^2)(P^2 + \beta(1 - \tau^1 g(R^1))P^1).$$

Each actor's total payoff is the sum of its first- and second-period payoffs.[11]

As in the one-stage model without commitment, the conqueror always chooses maximal extraction at the end of the game, $\tau^2 = 1$. Because its choice at this point cannot affect any of the groups' decisions, there is no drawback to demanding as much as possible. However, this logic does not apply to the conqueror's choice of τ^1, even though it takes place after the groups have made their first-stage allocations. In particular, τ^1 decreases the value of the investment, which in turn both directly affects total output and indirectly shapes the groups' equilibrium choices in the second stage. When β is small, it may be in the conqueror's interest to reduce the size of the investment. This is because the investment plays a role in the second-stage game similar to the endowment X in the natural resource extension: at low levels, a marginal increase in investment decreases production and increases resistance. But by the same token, if the investment exceeds the analogue of the "production threshold"—as is guaranteed if β is large enough—then raising its value increases total second-stage output and has no effect on resistance. A large value of β also implies that these second-stage gains from a marginal decrease in τ^1 outweigh the immediate loss. Therefore, as stated formally in the following proposition, the possibility of productive investment can hold down the amount of extraction, even in the absence of contracts or commitments.[12]

Proposition 12. *Suppose $g(L) > 0$. There exists $\beta^* > 0$ such that if $\beta > \beta^*$, there is no equilibrium of the symmetric two-stage game in which the conqueror selects $\tau^1 = 1$ on the equilibrium path.*

This result accords with Liberman's (1993) empirical finding that the spoils of conquest are greatest when the conquered society is modernized. Liberman suggests two mechanisms to explain the greater profitability of military occupations in modern times: first, that modern technology enhances the coercive capabilities of the state, and second, that wealthier societies are more likely than poorer ones

[11] None of the substantive conclusions of the extension would change if second-period payoffs were discounted.

[12] The proposition's requirement that $g(L) > 0$ is to ensure that the conqueror's second-stage payoff is non-zero even if the groups devote all their labor to resistance.

to comply with coercive demands because they have more to lose. By contrast, the explanation proposed here is that an investment-driven economy helps solve the commitment problem that leads to inefficiently high extraction in the one-shot model.

A striking example of how the shadow of the future can affect extractive policies is the evolution of German policy during its occupation of Soviet territory in World War II (see Dallin 1981; Mulligan 1988). Pre-war plans, based on the assumption that Operation Barbarossa would be quick and decisive, called for total extraction with no concessions to the Soviet population. The basic principle was to take as much as possible, as quickly as possible, from Soviet territory without regard for long-term economic consequences or the response of the local populace. In agricultural policy, this meant maintaining the Soviet system of collectivized farming, changing only the direction in which the food was sent. For political reasons, the plan also called for the East's industry to be dismantled; however, once the invasion began, this policy was almost immediately jettisoned in favor of maximal exploitation of the few industrial resources left unscathed by the Red Army's scorched-earth tactics (Dallin 1981, p. 376). Harsh economic extraction was paired with punitive political policies, like the infamous Commissar Order mandating the killing of political commissars accompanying captured Red Army units.

In 1942–43, as failures at Stalingrad and elsewhere dashed their hopes of a quick victory in the East, German officials gradually began to support and implement reforms of the original occupation policies. Although the starvation of the Soviet population served the crude racial goals of *Lebensraum*, the Nazi authorities found it counterproductive to the task of supplying the German war effort with output from the East. The goal of the reforms was to maintain the labor supply and increase its productivity over time by making some short-run concessions, the same mechanism proposed in Proposition12. The most prominent reform effort was in agriculture, in which German authorities gradually converted collective farms into "cooperatives" using Russian farming methods from the pre-Soviet era. This entailed relinquishing a degree of control (a minor one, to be sure), but occupation officials reported higher overall yields on the converted farms (Mulligan 1988, p. 97). Meanwhile, German forces also worked to restore some of the industrial capacity destroyed in the Red Army's retreat, and offered incentives like increased rations, private gardening space, and paid leave to factory workers (Mulligan 1988, p. 110). The timing and implementation of these reforms is in line with the theoretical mechanism proposed above. By leaving some goods for the population of the occupied territory to invest—insofar as the preservation of human capital via the reduction of starvation can be considered an investment—the German occupiers increased the amount they could extract over time. Even these minor concessions only became possible when German leaders saw that the occupation would take years rather than months.

6.3. **Creating inequality between groups.** Up to now, I have assumed that the conqueror may choose only what proportion to demand from society as a whole. I now examine an extension to the baseline model in which the amount of tribute demanded may vary across groups. Specifically, I address whether an outside force can extract more if it uses discriminatory policies to create inequality between previously identical groups. I find that this is not the case; there is no equilibrium that leaves the conqueror better off than in the original game with a common tribute level. In other words, there is no way to gain by arbitrarily discriminating or creating inequality where it did not exist before.

To model the possibility of endogenous inequality between groups, I relax the assumption that the tribute τ is common across groups. Let there be $n = 2$ symmetric groups,[13] and suppose the conqueror sets a distinct tribute rate for each one, so that $\tau = (\tau_1, \tau_2)$ is now a vector. I return to the timing of the original model, so the conqueror selects τ before the groups select (c, p, r). Although the conqueror may now demand distinct proportional shares from each group, I still assume that resistance by group i reduces the total amount the conqueror extracts, not just what it receives from group i. This reflects the fact that resistance by any group diverts attention and resources from the conqueror, making it more costly to extract even from more cooperative parts of society. The utility functions in this extension are as follows:

$$u_i(c, p, r, \tau) = s_i(c)(P - (\tau \cdot p)g(R)),$$

$$u_C(c, p, r, \tau) = (\tau \cdot p)g(R),$$

where $\tau \cdot p = \tau_1 p_1 + \tau_2 p_2$. I assume $\tau_1 \geq \tau_2$, which is without loss of generality since the groups are symmetric. Notice that if $\tau_1 = \tau_2$, then this is identical to a subgame of the original game with common tribute.

The conqueror can effectively make one group more productive than the other by selecting $\tau_1 > \tau_2$. At a glance, it is tempting to think of such a policy as making group 1 "disadvantaged" and group 2 "advantaged." That would be the case if the groups had property rights, each consuming only the fruits of its own labor. However, because the groups can appropriate from each other, the situation is actually reversed. Since group 1 has the same labor allocation as group 2 but less incentive to

[13] I assume $n = 2$ for simplicity, but the result holds for any number of groups.

engage in production, in equilibrium it puts at least as much toward conflict, $c_1 \geq c_2$. As a result, the "disadvantaged" group's share of output—and thus its payoff—is at least as great as the other's. The way resistance is apportioned between the two groups when $\tau_1 > \tau_2$ is more in line with what intuition would suggest. Because both groups have the same marginal return to resistance but group 1's marginal return to production is lower, it provides the bulk of resistance in equilibrium, $r_1 \geq r_2$. Conversely, group 2 handles most of the production, $p_1 \leq p_2$.[14]

Imposing a mild additional condition on the function f,[15] I find that the conqueror cannot profit from setting different policies for identical groups.

Proposition 13. *Let there be $n = 2$ groups in the symmetric game with varying tribute, and suppose f'/f is convex. There is no equilibrium that gives the conqueror a greater payoff than its equilibrium payoff in the game with common tribute.*

The proof of this result is almost identical to that of Proposition 7, which established that a policy of accommodation is optimal for the conqueror in the original game with symmetric groups. In the varying-tribute case, in order to extract more than under a policy of accommodation, the conqueror must impose tribute $\tau_i > \tau^*$ on at least one group with positive production, $p_i > 0$. But then the same basic logic holds as in the earlier proof. The equilibrium conditions for group i to produce imply that its return to production must be at least as great as its return to resistance. Just as in the common-tribute case, in order for this to hold with $\tau_i > \tau^*$, the conqueror's overall payoff must be no greater than it is under a policy of accommodation. Therefore, there is no incentive for the conqueror to use discriminatory policies to create inequality between groups that are identical *a priori*.

7. Conclusion

This paper has laid out a new theoretical framework for analyzing external rule, internal conflict, and how these two phenomena affect each other. The basic assumption of the theory is that a social group faces two important trade-offs when its material goods are under threat from both an outside ruler and other groups within society. First, the more labor it dedicates to productive activity, the less it has available to secure its own share of economic output from either the external ruler or other groups. Second, a group must balance its contribution to the joint effort against outside rule with its individual

[14] These three characterization results are proved in Lemma A.8 in the Appendix.

[15] The condition is that f'/f be convex. Since the earlier regularity conditions imply that f'/f is positive and weakly decreasing, the convexity assumption is natural, though it does come at some loss of generality.

interest in appropriating from other groups (or protecting its own holdings against such appropriation). I use a formal model to show precisely which conclusions about the relationship between internal and external conflict follow—or fail to follow—from this relatively intuitive set of premises.

One of the most striking findings is that the logic of "divide and rule" does not hold in this setting. The analysis even shows why this principle might seem well justified: in the baseline form of the model, in any given interaction, the outside force always chooses a policy that maximizes internal conflict. On an initial reading, this result appears to support the notion of *divide et impera*. However, using the model to dig deeper into the logic, I show that it is flawed. What an outside force chooses in the presence of social divisions is only weakly related to whether it would prefer that such divisions did not exist. Indeed, the comparative statics analysis shows that a conqueror is better off dealing with a unified society, in which there is no possibility of internal conflict to begin with, than a highly fractionalized one. In this sense, an outside ruler would prefer that the conquered not be divided.

One natural venue for future work stemming from this paper is systematic empirical analysis. The formal results given here can be used to derive numerous hypotheses about social conflict in the presence of external appropriation. Empirical testing of such hypotheses can show how widely applicable the theoretical framework here is, and suggest how the theoretical premises might be extended or modified to better explain observed variation in outcomes. The analysis here suggests that such empirical inquiries should be undertaken cautiously. Some of the important components of the model, such as the functions that describe how labor allocations are translated into the probability of resistance or a group's share of goods, vary across time and place in ways that might be quite hard to operationalize. Nonetheless, all of the comparative statics statements in the paper depend on these and other parameters being held equal, meaning they must be controlled for in a typical regression analysis with time-series cross-section data. As such, an analysis with micro-level data from a single country or conflict might be better suited for testing the implications of the model than a large-scale cross-sectional study.

There is still much theoretical work to be done on the relationship between outside rule and internal conflict. The model presented here can be extended or embedded in richer models of conflict to address further questions in this area. For example, should we expect an outside force to govern differently if there are other forces trying to gain control of the same territory? In many situations, particularly colonial conflicts, more than one outside actor may be competing for the ability to control a piece of land and extract its resources. It is possible to imagine that this kind of competition leads to better outcomes for the indigenous population, as each of the two forces competes for loyalty by promising to steal less than its rival would. Conversely, the presence of competing colonialists could be harmful to social welfare if the each outside force recruited separate local groups to form a proxy army, resulting in greater internal conflict. These questions could be addressed theoretically by extending the model presented in this paper to a multiple-conqueror setting. Another fruitful area of inquiry is the effects of political boundaries on economic extraction and internal conflict. How should colonial or provincial borders be drawn if the objective is to increase the income that can be taxed or extracted? Previous theoretical examinations of this question (e.g., Alesina and Spolaore 2003) do not consider the possibility of violent civil conflict, a major concern in many real-world settings. The modeling framework developed in this paper can be further explored and extended to provide clear answers to questions of broad theoretical interest like these.

Appendix A. Proofs and Additional Results

All lemmas numbered with an "A" are supplemental results that do not appear in the main text.

A.1. **Existence and Uniqueness Proofs.** Where convenient, I use the first-order conditions for optimizing $\log u_i$ rather than those for u_i; the results, of course, are identical. For $i \in N$ let A_i denote $\{(c_i, p_i, r_i) \mid \frac{c_i}{\alpha_i^c} + \frac{p_i}{\alpha_i^p} + r_i = L_i\}$, and define $A = \bigtimes_{i \in N} A_i$.

Proposition 1. *In every group subgame $\Gamma(\tau)$, an equilibrium exists.*

Proof. I show that the group subgame meets the conditions of the Debreu–Glicksberg–Fan Theorem. First, each group's strategy space A_i is nonempty, compact, and convex. Second, the utility function u_i, given by (2.4), is continuous in (c, p, r). Finally, to show that u_i is quasiconcave in (c_i, p_i, r_i), examine the second partial derivatives of the log-utility function:

$$\frac{\partial^2 \log u_i}{\partial c_i^2}(c, p, r, \tau) = \frac{\partial^2 \log s_i(c)}{\partial c_i^2} < 0,$$
$$\frac{\partial^2 \log u_i}{\partial p_i^2}(c, p, r, \tau) = -\frac{1}{P^2} < 0,$$
$$\frac{\partial^2 \log u_i}{\partial r_i^2}(c, p, r, \tau) = -\frac{\tau(1 - \tau g(R))g''(R) + \tau^2 g'(R)^2}{(1 - \tau g(R))^2} < 0.$$

All of the cross-partials are zero, so $\log u_i$ is strictly concave in (c_i, p_i, r_i). Therefore, u_i is strictly quasiconcave in (c_i, p_i, r_i). □

Lemma A.1. *Let (c, p, r) be an equilibrium of some subgame $\Gamma(\tau)$. If $p_i > 0$ for some group i, then $\alpha_j^p > \alpha_i^p$ implies $r_j = 0$. Similarly, if $r_i > 0$, then $\alpha_j^p < \alpha_i^p$ implies $p_j = 0$.*

Proof. Consider any group $i \in N$. If $p_i > 0$, then the first-order conditions give

$$\alpha_i^p \frac{\partial \log u_i(c, p, r, \tau)}{\partial p_i} \geq \frac{\partial \log u_i(c, p, r, \tau)}{\partial r_i}$$

For any group j with $\alpha_j^p > \alpha_i^p$, this implies

$$\alpha_j^p \frac{\partial \log u_j(c, p, r, \tau)}{\partial p_j} > \frac{\partial \log u_j(c, p, r, \tau)}{\partial r_j},$$

and thus $r_j = 0$. An analogous argument shows that $r_i > 0$ and $\alpha_j^p < \alpha_i^p$ implies $p_j = 0$. □

Lemma A.2. *Let (c, p, r) and (c', p', r') be equilibria of $\Gamma(\tau)$ and $\Gamma(\tau')$ respectively.*

(a) If $P' \geq P$ and $R' \geq R$, with at least one of these holding strictly, then there exists $i \in N$ such that $c_i' < c_i$.

(b) If $P' = P$ and $R' = R$, then there exists $i \in N$ such that $c_i' \leq c_i$.

Proof. Let $\bar{\alpha}^p$ denote the lowest level of economic productivity by a group that produces under (c, p, r), so that $\bar{\alpha}^p \equiv \min\{\alpha_i^p \mid p_i > 0\}$. Partition the set of groups into those with $\alpha_i^p < \bar{\alpha}^p$, denoted N_1; those with $\alpha_i^p = \bar{\alpha}^p$, denoted N_2; and those with $\alpha_i^p > \bar{\alpha}^p$, denoted N_3. By Lemma A.1, we have $p_i = 0$ for all $i \in N_1$, and $r_i = 0$ for all $i \in N_3$.

To prove statement (a), suppose $c_i' \geq c_i$ for each group $i \in N$. Because $r_i = 0$ for all $i \in N_3$, the budget constraint gives $p_i' \leq p_i$ for all $i \in N_3$. Similarly, we have $r_i' \leq r_i$ for all $i \in N_1$. Now suppose $P' > P$. Because production cannot increase on N_3, this implies $\sum_{i \in N_1} p_i' > 0$ or else $\sum_{i \in N_2} p_i' > \sum_{i \in N_2} p_i$. In the first case, we have $p_j' > 0$ for some $j \in N_1$, and hence $r_j' < r_j$ by the budget constraint. Moreover, by

Lemma A.1, $p'_j > 0$ implies $r'_i = 0$ for all $i \in N_2 \cup N_3$. We thus have $R' = \sum_{i\in N_1} r'_i < \sum_{i\in N_1} r_i \leq R$. In the second case, note that

$$\sum_{i\in N_2} r'_i = \sum_{i\in N_2}\left[L_i - \frac{c'_i}{\alpha^c_i} - \frac{p'_i}{\bar{\alpha}^p}\right]$$
$$< \sum_{i\in N_2}\left[L_i - \frac{c_i}{\alpha^c_i} - \frac{p_i}{\bar{\alpha}^p}\right] = \sum_{i\in N_2} r_i.$$

Since $r'_i \leq r_i$ for all $i \in N_1$ and $r'_i = 0$ for all $i \in N_3$, this gives $R' < R$. An analogous argument shows that $R' > R$ implies $P' < P$. By contraposition, we have that if $P' \geq P$ and $R' \geq R$, with at least one holding strictly, then there exists $i \in N$ with $c'_i < c_i$.

To prove statement (b), suppose $c'_i > c_i$ for each group $i \in N$. It is immediate from the budget constraint that $p_i > p'_i$ for all $i \in N_3$ and that $r_i > r'_i$ for all $i \in N_1$. Suppose $p'_j > 0$ for some $j \in N_1$. By Lemma A.1, $r'_i = 0$ for all $i \in N_2 \cup N_3$, which in turn implies $R' < R$. Similarly, if $r'_j > 0$ for some $j \in N_3$, then $P' < P$. Finally, suppose $p'_i = 0$ for all $i \in N_1$ and $r'_i = 0$ for all $i \in N_3$. Since $c'_i > c_i$ for all i, it follows from the budget constraint that $\sum_{i\in N_2} p'_i < \sum_{i\in N_2} p_i$ or $\sum_{i\in N_2} r'_i < \sum_{i\in N_2} r_i$. In the former case, we have

$$P' = \sum_{i\in N_2\cup N_3} p'_i < \sum_{i\in N_2\cup N_3} p_i = P;$$

in the latter case, we have

$$R' = \sum_{i\in N_1\cup N_2} r'_i < \sum_{i\in N_1\cup N_2} r_i = R.$$

Therefore, by contraposition, if $P' = P$ and $R' = R$, then there exists $i \in N$ with $c'_i \leq c_i$. □

Let $F(c)$ denote the denominator of the contest success function given c; i.e., $F(c) = \sum_{i\in N} c_i$.

Lemma A.3. *Let c and c' be vectors of conflict allocations. If there exists $j \in N$ such that $c'_j < c_j$, then there exists $i \in N$ such that $c'_i < c_i$ and $s_i(c') \leq s_i(c)$.*

Proof. If $F(c') \geq F(c)$, then we have

$$s_j(c') = \frac{f(c'_j)}{F(c')} < \frac{f(c_j)}{F(c)} = s_j(c),$$

so we can set $i = j$. Now suppose $F(c') < F(c)$. Because the contest shares always sum to one, there exists $i \in N$ with $s_i(c') \leq s_i(c)$. It then follows from the definition of s_i that $c'_i < c_i$. □

Proposition 2. *All equilibria of $\Gamma(\tau)$ have the same outcome, $(c^*(\tau), P^*(\tau), R^*(\tau))$.*

Proof. Let (c, p, r) and (c', p', r') be equilibria of $\Gamma(\tau)$. Without loss of generality, let $P' \geq P$. To show that $P' = P$ and $R' = R$, I rule out each of the following three cases:

- $R' < R$
- $P' = P$ and $R' > R$, or $P' > P$ and $R' = R$
- $P' > P$ and $R' > R$

For the first case, suppose $R' < R$. Let i be any group such that $r'_i < r_i$; because $R' < R$, at least one such group must exist. Because $r_i > 0$, the first-order conditions give

$$\frac{-\tau g'(R)}{1 - \tau g(R)} \geq \frac{\alpha^p_i}{P}.$$

Since $R' < R$ and $P' \geq P$, the above inequality implies

$$\frac{-\tau g'(R')}{1 - \tau g(R')} > \frac{\alpha^p_i}{P'}.$$

Therefore, by the first-order conditions, $p_i' = 0$. Since $p_i' = 0$ and $r_i' < r_i$, the budget constraint gives $c_i' > c_i \geq 0$. Combining the first-order conditions for $c_i' > 0$ and $r_i > 0$ gives

$$\alpha_i^c \frac{f'(c_i')}{f(c_i')}(1 - s_i(c')) \geq \frac{-\tau g'(R')}{1 - \tau g(R')} > \frac{-\tau g'(R)}{1 - \tau g(R)} \geq \alpha_i^c \frac{f'(c_i)}{f(c_i)}(1 - s_i(c)).$$

Log-concavity of f gives $f'(c_i')/f(c_i') \leq f'(c_i)/f(c_i)$. Therefore, the above expression implies $s_i(c') < s_i(c)$, which in turn gives $F(c') > F(c)$ (since $c_i' > c_i$). As such, there must exist a group j such that $c_j' > c_j$ and $s_j(c') > s_j(c)$. Moreover, because we have seen that $r_i' < r_i$ implies $s_i(c') < s_i(c)$, it must be the case that $r_j' \geq r_j$. Since $c_j' > c_j$ and $r_j' \geq r_j$, it follows from the budget constraint that $p_j' < p_j$. Because $P' \geq P$, there must be some other group k with $p_k' > p_k$. We have already seen that $r_i' < r_i$ implies $p_i' = 0$, so it must be the case that $r_k' \geq r_k$. Since $p_k' > p_k$ and $r_k' \geq r_k$, the budget constraint implies $c_k' < c_k$. Because $F(c') > F(c)$, this in turn gives $s_k(c') < s_k(c)$. Combining these with the first-order conditions for $c_k > 0$ yields

$$\alpha_k^c \frac{f'(c_k')}{f(c_k')}(1 - s_k(c')) > \alpha_k^c \frac{f'(c_k)}{f(c_k)}(1 - s_k(c)) \geq \frac{\alpha_k^p}{P} \geq \frac{\alpha_k^p}{P'}.$$

This implies $p_k' = 0$, a contradiction. Therefore, $R' \geq R$.

Now suppose $P' = P$ and $R' > R$. The proof here proceeds similarly to the previous case. By Lemmas A.2(a) and A.3, there exists a group $i \in N$ with $c_i' < c_i$ and $s_i(c') \leq s_i(c)$. Combining the first-order conditions for $c_i > 0$ with $R' > R$ gives

$$\alpha_i^c \frac{f'(c_i')}{f(c_i')}(1 - s_i(c')) \geq \alpha_i^c \frac{f'(c_i)}{f(c_i)}(1 - s_i(c)) \geq \frac{-\tau g'(R)}{1 - \tau g(R)} > \frac{-\tau g'(R')}{1 - \tau g(R')},$$

so $r_i' = 0$. Since $c_i' < c_i$ and $r_i' = 0$, the budget constraint gives $p_i' > p_i$. Combining the first-order conditions for $c_i > 0$ and $p_i' > 0$ gives

$$\alpha_i^c \frac{f'(c_i)}{f(c_i)}(1 - s_i(c)) \geq \frac{\alpha_i^p}{P} = \frac{\alpha_i^p}{P'} \geq \alpha_i^c \frac{f'(c_i')}{f(c_i')}(1 - s_i(c')).$$

Recall that $c_i' < c_i$ and $s_i(c') \leq s_i(c)$ by hypothesis. The above expression thus implies $s_i(c') = s_i(c)$ and thus $F(c') < F(c)$. Because $p_i' > p_i$ and $P' = P$, there must exist another group j with $p_j' < p_j$. Using the first-order conditions for $p_j > 0$, we have

$$\frac{\alpha_j^p}{P'} = \frac{\alpha_j^p}{P} \geq \frac{-\tau g'(R)}{1 - \tau g(R)} > \frac{-\tau g'(R')}{1 - \tau g(R')},$$

so $r_j' = 0$. Since $p_j' < p_j$ and $r_j' = 0$, the budget constraint implies $c_j' > c_j$. Because $F(c') < F(c)$, this in turn gives $s_j(c') > s_j(c)$. However, combined with the first-order conditions for $c_j' > 0$, this gives

$$\alpha_j^c \frac{f'(c_j)}{f(c_j)}(1 - s_j(c)) > \alpha_j^c \frac{f'(c_j')}{f(c_j')}(1 - s_j(c')) \geq \frac{\alpha_j^p}{P'} = \frac{\alpha_j^p}{P}$$

and thus $p_j = 0$, a contradiction. Therefore, we cannot have $P' = P$ and $R' > R$. An analogous argument rules out the case in which $P' > P$ and $R' = R$.

To establish that $P' = P$ and $R' = R$, the last remaining case to rule out is $P' > P$ and $R' > R$. In this case, by Lemmas A.2(a) and A.3, there exists $i \in N$ such that $c_i' < c_i$ and $s_i(c') \leq s_i(c)$. Combining this with the first-order conditions for $c_i > 0$, we have

$$\alpha_i^c \frac{f'(c_i')}{f(c_i')}(1 - s_i(c')) \geq \alpha_i^c \frac{f'(c_i)}{f(c_i)}(1 - s_i(c)) \geq \frac{\alpha_i^p}{P} > \frac{\alpha_i^p}{P'},$$

so $p_i' = 0$, and

$$\alpha_i^c \frac{f'(c_i')}{f(c_i')}(1 - s_i(c')) \geq \alpha_i^c \frac{f'(c_i)}{f(c_i)}(1 - s_i(c)) \geq \frac{-\tau g'(R)}{1 - \tau g(R)} > \frac{-\tau g'(R')}{1 - \tau g(R')},$$

so $r_i' = 0$. Since $p_i' = r_i' = 0$, the budget constraint gives $c_i' \geq c_i$, a contradiction.

The last step is to show that $c' = c$. Suppose not, so $c_j' \neq c_j$ for some $j \in N$. Without loss of generality, let $c_j' > c_j$. I will first show that this implies $c_i' > c_i$ and $s_i(c') > s_i(c)$ for some $i \in N$. If $F(c') \leq F(c)$, then it is immediate that $s_j(c') > s_j(c)$, so we can set $i = j$. Otherwise, suppose $F(c') > F(c)$. By Lemma A.2(b), there exists $k \in N$ with $c_k' \leq c_k$ and thus $s_k(c') < s_k(c)$. Because the contest shares must sum to one, this in turn implies $s_i(c') > s_i(c)$ for some $i \in N$. Moreover, $F(c') > F(c)$ implies $c_i' > c_i$. Combining $c_i' > c_i$ and $s_i(c_i') > c_i$ with the first-order conditions for $c_i' > 0$ yields

$$\alpha_i^c \frac{f'(c_i)}{f(c_i)}(1 - s_i(c)) > \alpha_i^c \frac{f'(c_i')}{f(c_i')}(1 - s_i(c')) \geq \frac{-\tau g'(R')}{1 - \tau g(R')} = \frac{-\tau g'(R)}{1 - \tau g(R)},$$

so $r_i = 0$, and

$$\alpha_i^c \frac{f'(c_i)}{f(c_i)}(1 - s_i(c)) > \alpha_i^c \frac{f'(c_i')}{f(c_i')}(1 - s_i(c')) \geq \frac{\alpha_i^p}{P'} = \frac{\alpha_i^p}{P},$$

so $p_i = 0$. Since $p_i = r_i = 0$, the budget constraint gives $c_i \geq c_i'$, a contradiction. Therefore, $c' = c$. □

Lemma A.4. *The equilibrium outcome functions c^*, P^*, and R^* are continuous.*

Proof. I prove the argument for P^*; the other cases are analogous. Define the equilibrium correspondence $E : [0,1] \rightrightarrows A$ so that $(c,p,r) \in E(\tau)$ if and only if (c,p,r) is an equilibrium of the subgame $\Gamma(\tau)$. It follows from standard arguments (e.g., Fudenberg and Tirole 1991, pp. 30–32) that E has a closed graph. This implies E is upper hemicontinuous, as its codomain A is compact. It is easily verified that the function $P(c,p,r) = \sum_{i \in N} p_i$ is continuous and thus, viewed as a correspondence, upper hemicontinuous. Notice that P^* can be written as the composition of E and P,

$$P^*(\tau) = \{P(c,p,r) \mid (c,p,r) \in E(\tau)\} = (P \circ E)(\tau).$$

As a composition of upper hemicontinuous correspondences, P^* is upper hemicontinuous (Aliprantis and Border 2006, Theorem 17.23). It is singleton-valued by Proposition 2 and thus is continuous as a function. □

Proposition 3. *An equilibrium of the game exists.*

Proof. Consider a strategy profile in which the groups' strategies form an equilibrium in each group subgame $\Gamma(\tau)$. The existence of such a strategy profile is given by Proposition 1, so the only remaining step is to verify that an optimal tribute level exists. Because total production and resistance are continuous in τ (Lemma A.4), the conqueror's induced utility function (3.1) admits a maximizer on $[0,1]$. □

A.2. **Characterization of the Equilibrium.**

Lemma 2. *If the groups are symmetric, $s_i(c^*(\tau)) = \frac{1}{n}$ for all $i \in N$ and $\tau \in [0,1]$.*

Proof. Suppose not, so there is an equilibrium (c,p,r) of a subgame $\Gamma(\tau)$ in which $c_i > c_j$ for some $i,j \in N$. Combining $s_i(c) > s_j(c)$ with the first-order conditions for $c_i > 0$ gives

$$\alpha^c \frac{\partial \log u_j(c,p,r,\tau)}{\partial c_j} > \alpha^c \frac{\partial \log u_i(c,p,r,\tau)}{\partial c_i} \geq \alpha^p \frac{\partial \log u_i(c,p,r,\tau)}{\partial p_i} = \alpha^p \frac{\partial \log u_j(c,p,r,\tau)}{\partial p_j},$$

so $p_j = 0$, and

$$\alpha^c \frac{\partial \log u_j(c,p,r,\tau)}{\partial c_j} > \alpha^c \frac{\partial \log u_i(c,p,r,\tau)}{\partial c_i} \geq \frac{\partial \log u_i(c,p,r,\tau)}{\partial r_i} = \frac{\partial \log u_j(c,p,r,\tau)}{\partial r_j},$$

so $r_j = 0$. Since all groups face the same budget constraint, $p_j = r_j = 0$ implies $c_j \geq c_i$, a contradiction. $\square$

Lemma 3. *Let the groups be symmetric and let $\eta^* = \frac{f(0)}{\alpha^c L f'(0)}$. There is no internal conflict in any equilibrium of any subgame, i.e., $c_i^*(\tau) = 0$ for all $i \in N$ and $\tau \in [0,1]$, if and only if $\frac{n-1}{n} \leq \eta^*$.*

Proof. I begin by showing that $\frac{n-1}{n} \leq \eta^*$ implies no internal conflict in the equilibrium of any subgame. Let (c, p, r) be an equilibrium of a subgame $\Gamma(\tau)$ in which $c_i > 0$ for some $i \in N$. Symmetry implies $s_i(c) = \frac{1}{n}$ (Lemma 2), so the first-order conditions for $c_i > 0$ give

$$\alpha^c \frac{\partial \log u_i(c,p,r,\tau)}{\partial c_i} = \alpha^c \frac{f'(c_i)}{f(c_i)} \frac{n-1}{n} \geq \frac{\alpha^p}{P} = \alpha^p \frac{\partial \log u_i(c,p,r,\tau)}{\partial p_i} \tag{A.1}$$

Since $c_i > 0$, the budget constraint implies $P < \alpha^p L$. Moreover, log-concavity of f implies f/f' is weakly increasing. The inequality in (A.1) therefore yields

$$\frac{n-1}{n} \geq \frac{\alpha^p f(c_i)}{\alpha^c P f'(c_i)} \geq \frac{\alpha^p f(0)}{\alpha^c P f'(0)} > \frac{f(0)}{\alpha^c L f'(0)};$$

i.e., $\frac{n-1}{n} > \eta^*$.

The other half of the proof is to show that there is a subgame with internal conflict in equilibrium if $\frac{n-1}{n} > \eta^*$. I prove the result for the subgame $\Gamma(0)$. Let (c, p, r) be an equilibrium of $\Gamma(0)$, and suppose $c_i = 0$ for all $i \in N$. As shown in Section 3.2, $R^*(0) = 0$, so we have $p_i = \alpha^p L_i / n$ for all $i \in N$. But this, along with $\frac{n-1}{n} > \eta^*$, gives

$$\alpha^c \frac{\partial \log u_i(c,p,r,\tau)}{\partial c_i} = \alpha^c \frac{f'(0)}{f(0)} \frac{n-1}{n} > \frac{1}{L} = \alpha^p \frac{\partial \log u_i(c,p,r,\tau)}{\partial p_i},$$

contradicting the first-order conditions for $p_i > 0$. $\square$

Proposition 4. *If the groups are symmetric and $\frac{n-1}{n} \leq \eta^*$,*

(a) $R^*(\tau) = 0$ for all $\tau \leq (g(0) - L g'(0))^{-1} \equiv \bar{\tau}_0$,
(b) $R^*(\tau)$ solves $\zeta^{pr}(\alpha^p(L-R), R; \tau) = 0$ for all $\tau > \bar{\tau}_0$,
(c) $P^*(\tau) = \alpha^p(L - R^*(\tau))$ for all τ.

Proof. Suppose $\frac{n-1}{n} \leq \eta^*$, and let (c, p, r) be an equilibrium of a subgame $\Gamma(\tau)$. Because $c_i = 0$ for all $i \in N$ by Lemma 3, the budget constraint gives $P = \alpha^p(L - R)$, proving statement (c). Note also that $P > 0$ by Lemma 1. Therefore, using the first-order conditions, $R > 0$ implies

$$\zeta^{pr}(\alpha^p(L-R), R; \tau) = \alpha^p(1 - \tau g(R) + \tau(L-R)g'(R)) = 0.$$

Because g is strictly decreasing and convex, this expression is strictly increasing in R and strictly decreasing in τ. Notice that $\bar{\tau}_0$ is defined such that $\zeta^{pr}(\alpha^p L, 0; \bar{\tau}_0) = 0$. As such, if $\tau \leq \bar{\tau}_0$, then $R > 0$ gives

$$\zeta^{pr}(\alpha^p(L-R), R; \tau) > \zeta^{pr}(\alpha^p L, 0; \tau) \geq \zeta^{pr}(\alpha^p L, 0; \bar{\tau}_0) = 0,$$

a contradiction. This establishes statement (a). To prove statement (b), notice that the first-order conditions for $R = 0$ imply $\zeta^{pr}(\alpha^p L, 0; \tau) \geq 0$, which holds only if $\tau \leq \bar{\tau}_0$. Therefore, $\tau > \bar{\tau}_0$ implies $R > 0$, which in turn requires $\zeta^{pr}(\alpha^p(L-R), R; \tau) = 0$. $\square$

Proposition 5. *If the groups are symmetric, the social welfare–maximizing outcome of each subgame is the corresponding outcome given in Proposition 4.*

Proof. A social welfare optimum maximizes the sum of the groups' utilities,

$$\sum_{i \in N} u_i(c,p,r,\tau) = \left[\sum_{i \in N} s_i(c)\right] (1 - \tau g(R))P = (1 - \tau g(R))P,$$

subject to the budget constraint

$$\frac{\sum_{i\in N} c_i}{\alpha^c} + \frac{P}{\alpha^p} + R = L.$$

These are equivalent to the utility function and budget constraint of the sole group in the game when $n = 1$, the equilibrium outcome of which is given by Proposition 4. □

Proposition 6. *If the groups are symmetric and* $\frac{n-1}{n} > \eta^*$,

(a) $P^*(\tau)$ *solves* $\zeta^{pc}(P, 0; n) = 0$ *and* $R^*(\tau) = 0$ *for all* $\tau \le \bar\tau_1(n)$,
(b) $P^*(\tau)$ *and* $R^*(\tau)$ *solve* $\zeta^{pc}(P, R; n) = \zeta^{pr}(P, R; \tau) = 0$ *for all* $\tau \in (\bar\tau_1(n), \bar\tau_2(n))$,
(c) $R^*(\tau)$ *solves* $\zeta^{pr}(\alpha^p(L - R), R; \tau) = 0$ *and* $P^*(\tau) = \alpha^p(L - R^*(\tau))$ *for all* $\tau \ge \bar\tau_2(n)$
(d) $c_i^*(\tau) = \frac{\alpha^c}{n}(L - \frac{P^*(\tau)}{\alpha^p} - R^*(\tau))$ *for all* $i \in N$ *and all* τ, *with* $c_i^*(\tau) > 0$ *if and only if* $\tau < \bar\tau_2(n)$.

The cutpoints are $\bar\tau_1(n) \equiv \left(g(0) - \frac{\tilde P_1(n)}{\alpha^p} g'(0)\right)^{-1}$ *and* $\bar\tau_2(n) \equiv \left((g(L - \frac{\tilde P_2(n)}{\alpha^p}) - \frac{\tilde P_2(n)}{\alpha^p} g'(L - \frac{\tilde P_2(n)}{\alpha^p})\right)^{-1}$, *where* $\tilde P_1(n)$ *and* $\tilde P_2(n)$ *solve* $\zeta^{pc}(P, 0; n) = 0$ *and* $\zeta^{pc}(P, L - \frac{P}{\alpha^p}; n) = 0$ *respectively.*

Proof. Suppose $\frac{n-1}{n} > \eta^*$, and let (c, p, r) be an equilibrium of a subgame $\Gamma(\tau)$. Lemma 2 and the budget constraint give $c_i = \frac{\alpha^c}{n}(L - R - \frac{P}{\alpha^p}) \equiv \tilde c$ for all $i \in N$.

I begin by showing that $P \le \tilde P_1(n)$. Notice that $\zeta^{pc}(P, R; n)$ is strictly decreasing in both P and R, so $P > \tilde P_1(n)$ implies

$$\zeta^{pc}(P, R; n) < \zeta^{pc}(\tilde P_1(n), R; n) \le \zeta^{pc}(\tilde P_1(n), 0; n) = 0,$$

contradicting the first-order conditions for $P > 0$. Therefore, $P \le \tilde P_1(n)$. Moreover, note that $\frac{n-1}{n} > \eta^*$ implies $\zeta^{pc}(\alpha^p L, 0; n) < 0$; because ζ^{pc} is decreasing in P, this gives $\tilde P_1(n) < \alpha^p L$. Because $P \le \tilde P_1(n) < \alpha^p L$, not all labor can be devoted to production in equilibrium: we have $R > 0$ or $\tilde c > 0$.

Next, I show that $R > 0$ if and only if $\tau > \bar\tau_1(n)$. Recall that ζ^{pr} is strictly increasing in R and strictly decreasing in P and τ. Moreover, $\bar\tau_1(n)$ is defined so that $\zeta^{pr}(\tilde P_1(n), 0; \bar\tau_1(n)) = 0$. As such, if $\tau \le \bar\tau_1(n)$, then $R > 0$ gives

$$\zeta^{pr}(P, R; \tau) \ge \zeta^{pr}(\tilde P_1(n), R; \bar\tau_1(n)) > \zeta^{pr}(\tilde P_1(n), 0; \bar\tau_1(n)) = 0,$$

contradicting the first-order conditions for $P > 0$. Therefore, $\tau \le \bar\tau_1(n)$ implies $R = 0$. To prove the other direction, suppose $R = 0$. Because not all labor can be devoted to production, this implies $\tilde c > 0$. The first-order conditions for $P > 0$ and $\tilde c > 0$ imply $\zeta^{pc}(P, 0; n) = 0$, which in turn gives $P = \tilde P_1(n)$. Since ζ^{pr} is strictly decreasing in τ, the requirement that $\zeta^{pr}(\tilde P_1(n), 0; \tau) \ge 0$ is satisfied only if $\tau \le \bar\tau_1(n)$. Therefore, $R = 0$ implies $\tau \le \bar\tau_1(n)$.

The final step is to show that $\tilde c > 0$ if and only if $\tau < \bar\tau_2(n)$. Suppose $\tilde c > 0$. Recall that $\tilde P_2(n)$ is defined such that $\zeta^{pc}(\tilde P_2(n), L - \frac{\tilde P_2(n)}{\alpha^p}; n) = 0$, and $\bar\tau_2(n)$ is defined such that $\zeta^{pr}(\tilde P_2(n), L - \frac{\tilde P_2(n)}{\alpha^p}; \bar\tau_2(n)) = 0$. Since ζ^{pc} is strictly decreasing in both P and R, $P \le \tilde P_2(n)$ gives

$$\zeta^{pc}(P, R; n) \ge \zeta^{pc}(\tilde P_2(n), R; n) > \zeta^{pc}(\tilde P_2(n), L - \tfrac{\tilde P_2(n)}{\alpha^p}; n) = 0,$$

contradicting the first-order conditions. We thus must have $P > \tilde P_2(n)$. As such, $\tau \ge \bar\tau_2(n)$ gives

$$\zeta^{pr}(P, R; \tau) \le \zeta^{pr}(P, R; \bar\tau_2(n)) < \zeta^{pr}(\tilde P_2(n), R; \bar\tau_2(n)) < \zeta^{pr}(\tilde P_2(n), L - \tfrac{\tilde P_2(n)}{\alpha^p}; \bar\tau_2(n)) = 0,$$

a contradiction. Therefore, $\tilde c > 0$ implies $\tau < \bar\tau_2(n)$. To prove the other direction, suppose $\tau < \bar\tau_2(n)$. Notice that the expression

$$\zeta^{pc}(P, L - \tfrac{P}{\alpha^p}; n) = \frac{\alpha^p}{P} - \alpha^c \frac{n-1}{n} \frac{f'(0)}{f(0)}$$

is strictly decreasing in P. As such, if $\tilde c = 0$, then $P > \tilde P_2(n)$ gives

$$\zeta^{pc}(P, R; n) = \zeta^{pc}(P, L - \tfrac{P}{\alpha^p}; n) < \zeta^{pc}(\tilde P_2(n), L - \tfrac{\tilde P_2(n)}{\alpha^p}; n) = 0,$$

contradicting the first-order conditions for $P > 0$. We thus must have $P \leq \tilde{P}_2(n)$ if $\tilde{c} = 0$. However, this in turn gives

$$\zeta^{pr}(P, R; \tau) = \zeta^{pr}(P, L - \tfrac{P}{\alpha^p}; \tau) \geq \zeta^{pr}(\tilde{P}_2(n), L - \tfrac{\tilde{P}_2(n)}{\alpha^p}; \tau) > \zeta^{pr}(\tilde{P}_2(n), L - \tfrac{\tilde{P}_2(n)}{\alpha^p}; \bar{\tau}_2(n)) = 0,$$

contradicting the requirement that $\zeta^{pr}(P, R; \tau) = 0$ if $P > 0$ and $R > 0$. Therefore, $\tau < \bar{\tau}_2(n)$ implies $\tilde{c} > 0$.

To summarize, I have shown that $R > 0$ if and only if $\tau > \bar{\tau}_1(n)$ and that $\tilde{c} > 0$ if and only if $\tau < \bar{\tau}_2(n)$. The remaining statements in the proposition defining the values of $P^*(\tau)$ and $R^*(\tau)$ are immediate from the first-order conditions. □

Proposition 7. *It is an equilibrium for the conqueror to implement the highest tribute level at which there is no resistance,* $\tau^* = \max\{\tau \,|\, R^*(\tau) = 0\}$.

Proof. Under the conflict threshold, with $\frac{n-1}{n} \leq \eta^*$, we have $P^*(\tau^*) = \alpha^p L$, and thus $P^*(\tau) \leq P^*(\tau)$ for all τ by the budget constraint. Similarly, in the case in which $\frac{n-1}{n} > \eta^*$, I show in the proof of Proposition 6 that $P^*(\tau) \leq P^*(\tau^*) = \tilde{P}_1(n)$ for all τ. Therefore, if $\tau g(R^*(\tau)) \leq \tau^* g(0)$, then

$$u_C(\tau) = \tau g(R^*(\tau)) P^*(\tau) \leq \tau^* g(0) P^*(\tau^*) = u_C(\tau^*).$$

Now suppose $\tau g(R^*(\tau)) > \tau^* g(0)$. For any equilibrium (c, p, r) of the subgame $\Gamma(\tau)$, the first-order conditions for $P^*(\tau) > 0$ give

$$\alpha^p \frac{\partial \log u_i(c, p, r, \tau)}{\partial p_i} = \frac{\alpha^p}{P^*(\tau)} \geq \frac{-\tau g'(R^*(\tau))}{1 - \tau g(R^*(\tau))} = \frac{\partial \log u_i(c, p, r, \tau)}{\partial r_i}$$

In addition, both above and below the conflict threshold, we have $\tau^* = (g(0) - \frac{P^*(\tau^*)}{\alpha^p} g'(0))^{-1}$, which gives

$$1 - \tau^* g(0) = -\frac{\frac{P^*(\tau^*)}{\alpha^p} g'(0)}{g(0) - \frac{P^*(\tau^*)}{\alpha^p} g'(0)} = -\frac{g'(0)}{\alpha^p} \tau^* P^*(\tau^*).$$

Combining these, we have

$$\begin{aligned}
u_C(\tau) &= \tau g(R^*(\tau)) P^*(\tau) \\
&\leq -\frac{g(R^*(\tau))}{g'(R^*(\tau))} \alpha^p (1 - \tau g(R^*(\tau))) \\
&\leq -\frac{g(0)}{g'(0)} \alpha^p (1 - \tau g(R^*(\tau))) \\
&< -\frac{g(0)}{g'(0)} \alpha^p (1 - \tau^* g(0)) \\
&= \tau^* g(0) P^*(\tau^*) \\
&= u_C(\tau^*).
\end{aligned}$$
□

Proposition 8. *Suppose the groups are symmetric and let their number vary, so the equilibrium outcomes $P^*(\tau; n)$ and $R^*(\tau; n)$ depend on both tribute and the number of groups, as does the conqueror's induced utility function $u_C(\tau; n)$. Let $\tau^*(n)$ denote the equilibrium tribute level given by Proposition 7, $\tau^*(n) = \max\{\tau \,|\, R^*(\tau; n) = 0\}$. Take any $n', n'' \in \mathbb{N}$ such that $n'' > n'$.*

- *(a) Equilibrium tribute is weakly increasing, $\tau^*(n'') \geq \tau^*(n')$*
- *(b) Equilibrium production is weakly decreasing, $P^*(\tau^*(n''), n'') \leq P^*(\tau^*(n'), n')$,*
- *(c) The conqueror's equilibrium utility is weakly decreasing, $u_C(\tau^*(n''); n'') \leq u_C(\tau^*(n'); n')$,*
- *(d) Each of these inequalities is strict if $\frac{n''-1}{n''} > \eta^*$.*

Proof. If $\frac{n''-1}{n''} \leq \eta^*$, then $P^*(\tau^*(n''); n'') = P^*(\tau^*(n'); n') = \alpha^p L$. Otherwise, if $\frac{n''-1}{n''} > \eta^*$, Proposition 6 gives $P^*(\tau^*(n''); n'') = \tilde{P}_1(n'')$. Recall that $\tilde{P}_1(n)$ is defined as the solution to $\zeta^{pc}(P, 0; n) = 0$. Since ζ^{pc}

is strictly decreasing in both P and n, $n'' > n'$ implies $\tilde{P}(n'') < \tilde{P}(n')$. Therefore, $\frac{n''-1}{n''} > \eta^*$ implies $P^*(\tau^*(n''); n'') = \tilde{P}_1(n'') < \min\{\alpha^p L, \tilde{P}_1(n')\} = P^*(\tau^*(n'); n')$. In other words, equilibrium production is weakly decreasing in n, strictly so above the conflict threshold. Now recall that equilibrium tribute is

$$\tau^*(n) = \frac{1}{g(0) - \frac{P^*(\tau^*(n);n)}{\alpha^p} g'(0)},$$

which is strictly decreasing in $P^*(\tau^*(n); n)$. Therefore, $\tau^*(n'') \geq \tau^*(n')$, strictly so if $\frac{n''-1}{n''} > \eta^*$. Finally, the conqueror's utility in equilibrium is

$$u_C(\tau^*(n); n) = \frac{g(0)P^*(\tau^*(n); n)}{g(0) - \frac{P^*(\tau^*(n);n)}{\alpha^p} g'(0)} = \frac{g(0)}{\frac{g(0)}{P^*(\tau^*(n);n)} - \frac{g'(0)}{\alpha^p}},$$

which is strictly increasing in $P^*(\tau^*(n); n)$. Therefore, $u_C(\tau^*(n''); n'') \leq u_C^*(\tau^*(n'); n')$, strictly so if $\frac{n''-1}{n''} > \eta^*$. □

Proposition 9. *Suppose that for all $\tau \in (\tau', \tau'') \subseteq [0, 1]$, there is an equilibrium of $\Gamma(\tau)$ that is a specialist equilibrium. Then P^* and R^* are both strictly increasing on (τ', τ'').*

Proof. Take any $\tau \in (\tau', \tau'')$ and let (c, p, r) be a specialist equilibrium of $\Gamma(\tau)$. By definition of a specialist equilibrium, $p_W = \alpha_W^p (L_W - \frac{c_W}{\alpha_w^c})$ and $r_I = L_I - \frac{c_I}{\alpha_I^c}$. The first-order conditions imply that the following equations must be satisfied:

$$\begin{aligned}
\zeta_W^{pc}(c_I, c_W; \tau) &= \frac{\alpha_W^c}{\alpha_W^c L_W - c_W} - \alpha_W^c (\log f)'(c_W) s_I(c) \qquad = 0, \\
\zeta_I^{rc}(c_I, c_W; \tau) &= \frac{-\tau g'(L_I - \frac{c_I}{\alpha_I^c})}{1 - \tau g(L_I - \frac{c_I}{\alpha_I^c})} - \alpha_I^c (\log f)'(c_I) s_W(c) = 0.
\end{aligned}$$

The partial derivatives of the first equation are

$$\begin{aligned}
\frac{\partial \zeta_W^{pc}(c_I, c_W; \tau)}{\partial c_I} &= -\alpha_W^c (\log f)'(c_W)(\log f)'(c_I) s_I(c) s_W(c) && < 0, \\
\frac{\partial \zeta_W^{pc}(c_I, c_W; \tau)}{\partial c_W} &= \frac{\alpha_W^c}{(\alpha_W^c L_W - c_W)^2} - \alpha_W^c s_I(c) \left[(\log f)''(c_W) - (\log f)'(c_W)^2 s_W(c) \right] && > 0, \\
\frac{\partial \zeta_W^{pc}(c_I, c_W; \tau)}{\partial \tau} &= 0,
\end{aligned}$$

and the partial derivatives of the second equation are

$$\begin{aligned}
\frac{\partial \zeta_I^{rc}(c_I, c_W; \tau)}{\partial c_I} &= \frac{\tau g''(L_I - \frac{c_I}{\alpha_I^c})(1 - \tau g(L_I - \frac{c_I}{\alpha_I^c})) + (\tau g'(L_I - \frac{c_I}{\alpha_I^c}))^2}{\alpha_I^c (1 - \tau g(L_I - \frac{c_I}{\alpha_I^c}))^2} \\
&\quad - \alpha_I^c s_W(c) \left[(\log f)''(c_I) - (\log f)'(c_I)^2 s_I(c) \right] && > 0, \\
\frac{\partial \zeta_I^{rc}(c_I, c_W; \tau)}{\partial c_W} &= -\alpha_I^c (\log f)'(c_I)(\log f)'(c_W) s_I(c) s_W(c) && < 0, \\
\frac{\partial \zeta_I^{rc}(c_I, c_W; \tau)}{\partial \tau} &= \frac{-g'(L_I - \frac{c_I}{\alpha_I^c})}{(1 - \tau g(L_I - \frac{c_I}{\alpha_I^c}))^2} && > 0.
\end{aligned}$$

It is straightforward to verify that each of these expressions is continuous in the variables and that the Jacobian matrix is non-degenerate. Implicit differentiation then gives the result. □

A.3. **Extensions.**

A.3.1. *Natural resources.*

Lemma 4. *In the symmetric game with natural resources, if* $X \geq X^* \equiv \frac{\alpha^p}{\alpha^c}\frac{f(\alpha^c L/n)}{f'(\alpha^c L/n)}\frac{n}{n-1}$, *then* $P^*(\tau; X) = 0$ *for all* $\tau \in [0,1]$.

Proof. Suppose $X \geq X^*$ and let (c, p, r) be an equilibrium of $\Gamma(\tau, X)$ such that $P > 0$. Recall that $s_i(c) = \frac{1}{n}$ for all $i \in N$ by Lemma 2. Therefore, for all $i \in N$,

$$\alpha^p \frac{\partial \log u_i(c, p, r, \tau; X)}{\partial p_i} = \frac{\alpha^p}{P + X} < \frac{\alpha^p}{X^*} = \alpha^c \frac{f'(\frac{\alpha^c L}{n})}{f(\frac{\alpha^c L}{n})}\frac{n-1}{n} \leq \alpha^c \frac{\partial \log u_i(c, p, r, \tau; X)}{\partial c_i}.$$

The first-order conditions thus give $p_i = 0$ for all $i \in N$, contradicting $P > 0$. □

Lemma A.5. *Let the groups be symmetric and let* (c, p, r) *and* (c', p', r') *be equilibria of* $\Gamma(\tau; X)$ *and* $\Gamma(\tau; X')$, *respectively. If* $P' \geq P$, *then*

$$\max\left\{\alpha^c \frac{f'(c_i')}{f(c_i')}(1 - s_i(c')), \frac{-\tau g'(R')}{1 - \tau g(R')}\right\} \geq \max\left\{\alpha^c \frac{f'(c_i)}{f(c_i)}(1 - s_i(c)), \frac{-\tau g'(R)}{1 - \tau g(R)}\right\}$$

for all $i \in N$.

Proof. Recall that $c_i = c_j = \tilde{c}$ and $c_i' = c_j' = \tilde{c}'$ for all $i, j \in N$ by Lemma 2. We thus have $s_i(c) = s_i(c') = \frac{1}{n}$ for all $i \in N$. Now suppose $P' \geq P$. The result is trivial if $P = \alpha^p L$, as then $(c, p, r) = (c', p', r')$. Otherwise, the budget constraint implies that either $R > 0$ and $R' \leq R$, or $\tilde{c} > 0$ and $\tilde{c}' \leq \tilde{c}$. In the former case,

$$\frac{-\tau g'(R')}{1 - \tau g(R')} \geq \frac{-\tau g'(R)}{1 - \tau g(R)} = \max\left\{\alpha^c \frac{f'(c_i)}{f(c_i)}(1 - s_i(c)), \frac{-\tau g'(R)}{1 - \tau g(R)}\right\}.$$

In the latter case,

$$\alpha^c \frac{f'(\tilde{c}')}{f(\tilde{c}')}\frac{n-1}{n} \geq \alpha^c \frac{f'(\tilde{c})}{f(\tilde{c})}\frac{n-1}{n} = \max\left\{\alpha^c \frac{f'(c_i)}{f(c_i)}(1 - s_i(c)), \frac{-\tau g'(R)}{1 - \tau g(R)}\right\}.$$ □

Proposition 10. *In the symmetric game with natural resources, if* $X < X'$, *then for all* $\tau \in [0,1]$,

(a) $P^*(\tau; X) \geq P^*(\tau; X') \geq P^*(\tau; X) + X - X'$ *(the first holding strictly if* $0 < P^*(\tau; X) < \alpha^p L$*)*,
(b) $R^*(\tau; X) \leq R^*(\tau; X')$ *(with equality if* $P^*(\tau; X) = 0$*)*,
(c) $c_i^*(\tau; X) \leq c_i^*(\tau; X')$ *for all* $i \in N$ *(with equality if* $P^*(\tau; X) = 0$*)*.

Proof. Let (c, p, r) and (c', p', r') be equilibria of $\Gamma(\tau; X)$ and $\Gamma(\tau; X')$, respectively. Recall that $c_i = c_j = \tilde{c}$ and $c_i' = c_j' = \tilde{c}'$ for all $i, j \in N$ by Lemma 2. I begin by proving the first inequality in statement (a). If $P = \alpha^p L$, the result is immediate. Otherwise, if $P < \alpha^p L$, then $R > 0$ or $\tilde{c} > 0$. If $P' \geq P$, then the first-order conditions and Lemma A.5 give

$$\max\left\{\alpha^c \frac{f'(\tilde{c}')}{f(\tilde{c}')}\frac{n-1}{n}, \frac{-\tau g'(R')}{1 - \tau g(R')}\right\} \geq \max\left\{\alpha^c \frac{f'(\tilde{c})}{f(\tilde{c})}\frac{n-1}{n}, \frac{-\tau g'(R)}{1 - \tau g(R)}\right\} \geq \frac{\alpha^p}{P + X} > \frac{\alpha^p}{P' + X'},$$

so $P' = 0$, and thus $P = 0$ as well. To prove the second inequality in statement (a), suppose $P' + X' < P + X$. This implies $P > P' \geq 0$, so the first-order conditions and Lemma A.5 give

$$\frac{\alpha^p}{P' + X'} > \frac{\alpha^p}{P + X} \geq \max\left\{\alpha^c \frac{f'(\tilde{c})}{f(\tilde{c})}\frac{n-1}{n}, \frac{-\tau g'(R)}{1 - \tau g(R)}\right\} \geq \max\left\{\alpha^c \frac{f'(\tilde{c}')}{f(\tilde{c}')}\frac{n-1}{n}, \frac{-\tau g'(R')}{1 - \tau g(R')}\right\}.$$

This implies $R' = \tilde{c}' = 0$, a contradiction.

To prove statement (b), suppose $R > R'$. Since $P \geq P'$, the budget constraint gives $\tilde{c} < \tilde{c}'$. However, combined with the first-order conditions for $R > 0$, this implies

$$\frac{-\tau g'(R')}{1 - \tau g(R')} > \frac{-\tau g'(R)}{1 - \tau g(R)} \geq \alpha^c \frac{f'(\tilde{c})}{f(\tilde{c})}\frac{n-1}{n} \geq \alpha^c \frac{f'(\tilde{c}')}{f(\tilde{c}')}\frac{n-1}{n},$$

contradicting $\tilde{c}' > 0$. Notice that the same argument applies in reverse if $P = 0$, as then $P' \geq P$; therefore, $P = 0$ implies $R' = R$. The proof of statement (c) is similar. If $\tilde{c} > \tilde{c}'$, then the budget constraint gives $R < R'$; however, this implies

$$\alpha^c \frac{f'(\tilde{c}')}{f(\tilde{c}')}\frac{n-1}{n} \geq \alpha^c \frac{f'(\tilde{c})}{f(\tilde{c})}\frac{n-1}{n} \geq \frac{-\tau g'(R)}{1-\tau g(R)} > \frac{-\tau g'(R')}{1-\tau g(R')},$$

a contradiction. As before, the same argument can be applied in reverse if $P = 0$, so $P = 0$ implies $\tilde{c}' = \tilde{c}$. □

Proposition 11. *In the symmetric game with natural resources, there exists $X^{**} \in (0, X^*]$ such that*

(a) If $X < X^{**}$, it is an equilibrium for the conqueror to select $\bar{\tau}(X) \equiv \max\{\tau \mid R^*(\tau; X) = 0\}$,

(b) If $X > X^{**}$, $P^*(\tau^*; X) = 0$ for every equilibrium tribute level τ^*.

Proof. The proof of Proposition 7 may be applied, *mutatis mutandis*, to show that $\bar{\tau}(X)$ is optimal for the conqueror within the set of tribute levels τ such that $P^*(\tau; X) > 0$. Therefore, for any X such that there is an equilibrium with $P^*(\tau; X) > 0$, it is an equilibrium for the conqueror to select $\bar{\tau}(X)$. Now suppose that there is some X such that every equilibrium entails no production; i.e., there exists τ such that $u_C(\tau; X) > u_C(\bar{\tau}(X); X)$ (and thus $P^*(\tau; X) = 0$). I claim that this implies $u_C(\tau; X') > u_C(\bar{\tau}(X'); X')$ for all $X' > X$. Note that $R^*(\tau; X') = R^*(\tau; X)$ and $\bar{\tau}(X') \leq \bar{\tau}(X)$ by Proposition 10. As in the original symmetric game, $R^*(\tau; X) = 0$ implies $P^*(\tau'; X) = P^*(\tau; X)$ for all $\tau' \leq \tau$. As such, because $\bar{\tau}(X') \leq \bar{\tau}(X)$,

$$P^*(\bar{\tau}(X'); X') \leq P^*(\bar{\tau}(X'); X) = P^*(\bar{\tau}(X); X).$$

In addition, note that $u_C(\tau; X) > u_C(\bar{\tau}(X); X)$ implies $\tau g(R^*(\tau; X)) \geq \bar{\tau}(X) g(0)$. Taking a difference in the utility differences therefore yields

$$\begin{aligned}
&[u_C(\tau; X') - u_C(\bar{\tau}(X'); X')] - [u_C(\tau; X) - u_C(\bar{\tau}(X); X)] \\
&\quad = \tau g(R^*(\tau; X'))X' - \bar{\tau}(X')g(0)(X' + P^*(\bar{\tau}(X'), X')) \\
&\qquad - \tau g(R^*(\tau; X))X + \bar{\tau}(X)g(0)(X + P^*(\bar{\tau}(X); X)) \\
&\quad = \tau g(R^*(\tau; X))(X' - X) - \bar{\tau}(X')g(0)(X' + P^*(\bar{\tau}(X'); X')) + \bar{\tau}(X)g(0)(X + P^*(\bar{\tau}(X); X)) \\
&\quad \geq \tau g(R^*(\tau; X))(X' - X) - \bar{\tau}(X)g(0)[(X' - X) + (P^*(\bar{\tau}(X'); X') - P^*(\bar{\tau}(X); X))] \\
&\quad \geq [\tau g(R^*(\tau; X)) - \bar{\tau}(X)g(0)](X' - X) \\
&\quad \geq 0;
\end{aligned}$$

i.e., $u_C(\tau; X') - u_C(\bar{\tau}(X'); X') \geq u_C(\tau; X) - u_C(\bar{\tau}(X); X) > 0$, as claimed. □

Example 1. Let $f(c_i) = \exp(c_i)$ and $g(R) = 1 - \frac{R}{L}$, so the production threshold is $X^* = \frac{\alpha^p}{\alpha^c}\frac{n}{n-1}$. Suppose $n \geq 2$ and $X \geq \frac{2\alpha^c}{\alpha^p} \geq X^*$, so there is no production in any equilibrium of any subgame. Let τ^* be an optimal tribute level. It is obvious that $\tau^* \geq \max\{\tau \mid R^*(\tau; X) = 0\}$ and that $R^*(\tau^*; X) < L$ (since $g(L) = 0$). This implies the groups' marginal returns to conflict and resistance are equal at τ^*. The conqueror's payoff from selecting τ^* is therefore

$$u_C^*(\tau^*; X) = \tau^* g(R^*(\tau^*; X))X = \left(1 - \frac{\tau^*}{\alpha^c L}\frac{n}{n-1}\right)X,$$

which is increasing in n. □

A.3.2. *Commitment.* To reduce the notational burden, let $\sigma^t = (c^t, p^t, r^t)$. A subgame-perfect equilibrium consists of a value $\sigma^1 \in A$ and functions $\tau^1 : A \to [0, 1]$, $\sigma^2 : A \times [0, 1] \to A$, and $\tau^2 : A^2 \times [0, 1] \to [0, 1]$.

Lemma A.6. *In any equilibrium of the symmetric two-stage game,*

(a) The function τ^2 is identically one,

(b) *Each group's equilibrium payoff in the second stage, $u_i^2(\sigma^2(\sigma^1,\tau^1),1;\sigma^1,\tau^1)$, is weakly increasing in $(1-\tau^1 g(R^1))P^1$.*

Proof. Observe that u_C is strictly increasing in τ^2. None of the other equilibrium quantities may depend on it, as it is chosen last, giving statement (a). This implies that the second-stage game is isomorphic to the $\tau = 1$ subgame of the natural resources game with $X = \beta(1-\tau^1 g(R^1))P^1$. Statement (b) then follows from Proposition 10. □

Lemma A.7. *In any equilibrium of the symmetric two-stage game in which $\tau^1(\sigma^1) = 1$, $(1-g(R^1))P^1 \geq (1-g(R^*(1)))P^*(1)$.*

Proof. Let (σ^2,τ^1,τ^2) be an equilibrium of the subgame that follows the groups' first-stage allocations. Take any σ^1 such that $\tau^1(\sigma^1) = 1$ and $(1-g(R^1))P^1 < (1-g(R^*(1)))P^*(1)$, where $P^*(1)$ and $R^*(1)$ are equilibrium totals for $\Gamma(1)$, the $\tau = 1$ subgame of the original symmetric game.

I claim that at least one group has an incentive to deviate to a strategy that would raise $(1-g(R^1))P^1$. That is, there exists an allocation σ' and a group i such that $\sigma'_j = \sigma^1_j$ for all $j \neq i$, $u_i^1(\sigma',1) > u_i^1(\sigma^1,1)$, and $(1-g(R'))P' > (1-g(R^1))P^1$. First, suppose $P^1 \leq P^*(1)$ and $R^1 \leq R^*(1)$, at least one strictly so. Then, by the budget constraint and Lemma A.3, there exists $j \in N$ such that $c_j^1 > c_j^*(1)$ and $s_j(c^1) \geq s_j(c^*(1))$. Combining this with the first-order conditions for an equilibrium of $\Gamma(1)$ gives

$$\begin{aligned}
\alpha^c \frac{\partial \log u_i^1(\sigma^1,1)}{\partial c_i} &= \alpha^c \frac{f'(c_i^1)}{f(c_i^1)}(1-s_i(c^1)) \\
&< \alpha^c \frac{f'(c_i^*(1))}{f(c_i^*(1))}(1-s_i(c^*(1))) \\
&\leq \frac{\alpha^p}{P^*(1)} \\
&\leq \frac{\alpha^p}{P^1} \\
&= \alpha^p \frac{\partial \log u_i^1(\sigma^1,1)}{\partial p_i}.
\end{aligned}$$

Therefore, letting $(c'_i,p'_i,r'_i) = (c_i^1 - \alpha^c\epsilon, p_i^1 + \alpha^p\epsilon, r_i^1)$, we have $u_i^1(\sigma',1) > u_i^1(\sigma^1,1)$ for any sufficiently small $\epsilon > 0$. Moreover, $P' > P^1$ and $R' = R^1$ implies $(1-g(R'))P' > (1-g(R^1))P^1$. Next, suppose $P^1 > P^*(1)$, in which case $R^1 < R^*(1)$. Let $(c'_i,p'_i,r'_i) = (c_i^1, p_i^1 - \alpha^p\epsilon, r_i^1 + \epsilon)$ for any group i such that $p_i^1 > 0$, with $0 < \epsilon \leq \min\{R^*(1) - R^1, \frac{1}{\alpha^p}(P^1 - P^*(1))\}$. Strict quasiconcavity implies $(1-g(R'))P' > (1-g(R^1))P^1$. Moreover, because $s_i(c') = s_i(c^1)$, this gives $u_i^1(\sigma',1) > u_i^1(\sigma^1,1)$. An analogous argument can be used in the remaining case, where $P^1 < P^*(1)$ and $R^1 > R^*(1)$.

The final step is to show that the proposed deviation by group i would be profitable. The deviation strictly increases i's first-stage payoff, as we have $u_i^1(\sigma',\tau^1(\sigma')) \geq u_i^1(\sigma',1) > u_i^1(\sigma^1,1)$. It will now suffice to show that the deviation weakly improves i's second-stage payoff. Because $(1-g(R'))P' > (1-g(R^1))P^1$, Lemma A.6 gives

$$u_i^2(\sigma^2(\sigma',\tau^1(\sigma')),1;\sigma',\tau^1(\sigma')) \geq u_i^2(\sigma^2(\sigma',1),1;\sigma',1) \geq u_i^2(\sigma^2(\sigma^1,1),1;\sigma^1,1). \qquad \square$$

Proposition 12. *Suppose $g(L) > 0$. There exists $\beta^* > 0$ such that if $\beta > \beta^*$, there is no equilibrium of the symmetric two-stage game in which the conqueror selects $\tau^1 = 1$ on the equilibrium path.*

Proof. I prove the result for

$$\beta^* = \max \left\{ \frac{\frac{\alpha^p}{\alpha^c} \frac{f(\alpha^c L/n)}{f'(\alpha^c L/n)} \frac{n}{n-1}}{(1-g(R^*(1)))P^*(1)}, \frac{1}{g(L)} \right\}.$$

Suppose $\beta > \beta^*$ and let $(\sigma^1, \tau^1, \sigma^2, \tau^2)$ be an equilibrium in which $\tau^1(\sigma^1) = 1$. Recall that the group choices in the second stage are isomorphic to the $\tau = 1$ subgame of the natural resources game with $X = \beta(1 - \tau^1 g(R^1))P^1$. Since $(1 - g(R^1))P^1 \geq (1 - g(R^*(1)))P^*(1)$ by Lemma A.7, $\beta > \beta^*$ implies $\beta(1 - g(R^1))P^1 > X^*$. Lemma 4 then gives $P^2(\sigma^1, 1) = P^2(\sigma^1, 1 - \epsilon) = 0$ for all sufficiently small $\epsilon > 0$. This in turn implies $R^2(\sigma^1, 1) = R^2(\sigma^1, 1 - \epsilon)$ for all sufficiently small $\epsilon > 0$, via Proposition 10. Differentiating the conqueror's payoff with respect to τ^1 gives

$$\begin{aligned}
\frac{du_C(\sigma^1, 1, \sigma^2(\sigma^1, 1), 1)}{d\tau^1} &= \frac{\partial u_C^1(\sigma^1, 1)}{\partial \tau^1} + \frac{\partial u_C^2(\sigma^2(\sigma^1, 1), 1; \sigma^1, 1)}{\partial \tau^1} \\
&= g(R^1)P^1 - \beta g(R^2(\sigma^1, 1))g(R^1)P^1 \\
&\leq g(R^1)P^1[1 - \beta g(L)] \\
&< 0,
\end{aligned}$$

contradicting the assumption of equilibrium. □

A.3.3. *Varying tribute.*

Lemma A.8. *Let there be $n = 2$ groups in the symmetric game with varying tribute. In any equilibrium of any subgame such that $\tau_1 > \tau_2$, $c_1 \geq c_2$, $p_1 \leq p_2$, and $r_1 \geq r_2$.*

Proof. Let (c, p, r) be an equilibrium of some subgame. If $c_2 = 0$, it is trivial that $c_1 \geq 0$. Otherwise, if $c_2 > 0$, the first-order conditions give

$$\begin{aligned}
\alpha^c \frac{f'(c_1)}{f(c_1)} s_1(c)s_2(c)(P - (\tau \cdot p)g(R)) &\geq \alpha^c \frac{f'(c_2)}{f(c_2)} s_1(c)s_2(c)(P - (\tau \cdot p)g(R)) \\
&\geq \max\left\{\alpha^p s_2(c)(1 - \tau_2 g(R)), s_2(c)(-(\tau \cdot p)g'(R))\right\} \\
&> \max\left\{\alpha^p s_1(c)(1 - \tau_1 g(R)), s_1(c)(-(\tau \cdot p)g'(R))\right\},
\end{aligned}$$

so $p_1 = r_1 = 0$; the budget constraint then gives $c_1 \geq c_2$.

Now suppose $r_2 > r_1$. The first-order conditions for $r_2 > 0$ give

$$\alpha^c s_2(c)(-(\tau \cdot p)g'(R)) \geq \alpha^p s_2(c)(1 - \tau_2 g(R)).$$

This in turn gives

$$\alpha^c s_1(c)(-(\tau \cdot p)g'(R)) \geq \alpha^p s_1(c)(1 - \tau_2 g(R)) > \alpha^p s_1(c)(1 - \tau_1 g(R)),$$

so $p_1 = 0$. Since $r_1 < r_2$ and $p_1 \leq p_2$, the budget constraint implies $c_1 > c_2$. The first-order conditions for $c_1 > 0$ give

$$\begin{aligned}
\alpha^c \frac{f'(c_2)}{f(c_2)} s_1(c)s_2(c)(P - (\tau \cdot p)g(R)) &\geq \alpha^c \frac{f'(c_1)}{f(c_1)} s_1(c)s_2(c)(P - (\tau \cdot p)g(R)) \\
&\geq s_1(c)(-(\tau \cdot p)g'(R)) \\
&> s_2(c)(-(\tau \cdot p)g'(R)),
\end{aligned}$$

so $r_2 = 0$, a contradiction. Therefore, $r_1 \geq r_2$. Finally, via the budget constraint, $c_1 \geq c_2$ and $r_1 \geq r_2$ imply $p_1 \leq p_2$. □

Lemma A.9. *If $\tau_1 \geq \tau_2$ and $p_1 \leq p_2$, then $\tau \cdot p \leq \frac{\tau_1 + \tau_2}{2} P$.*

Proof. Under the given conditions,

$$\frac{\tau_1+\tau_2}{2}P - \tau \cdot p = \frac{\tau_1-\tau_2}{2}(p_2-p_1) \geq 0. \qquad \square$$

Proposition 13. *Let there be $n=2$ groups in the symmetric game with varying tribute, and suppose f'/f is convex. There is no equilibrium that gives the conqueror a greater payoff than its equilibrium payoff in the game with common tribute.*

Proof. Let $\tau = (\tau_1, \tau_2)$ be fixed, with $\tau_1 \geq \tau_2$, and let (c, p, r) be an equilibrium of the corresponding subgame. Define τ^* as in Proposition 7; i.e., as the policy of accommodation in the game with common tribute. My first task is to show that $P \leq P^*(\tau^*)$. This is trivial if $\frac{1}{2} = \frac{n-1}{n} \leq \eta^*$, as then $P^*(\tau^*) = \alpha^p L$ by Proposition 4. Suppose otherwise, so $\frac{1}{2} > \eta^*$ and $P^*(\tau^*) = \tilde{P}_1(2)$. If $p_1 = 0$, so that $P = p_2$, the first-order conditions for $p_2 > 0$ give

$$\alpha^p s_2(c)(1-\tau_2 g(R)) \geq \alpha^c \frac{f'(c_2)}{f(c_2)} s_2(c) s_1(c)(1-\tau_2 g(R))P.$$

Because $s_1(c) \geq \frac{1}{2}$ and $c_2 \leq \alpha^c(\frac{L}{2} - \frac{P}{\alpha^p}) < \frac{\alpha^c}{2}(L - \frac{P}{\alpha^p})$, this implies

$$\begin{aligned}
\frac{\alpha^p}{P} - \frac{\alpha^c}{2}\frac{f'(\frac{\alpha^c}{2}(L-\frac{P}{\alpha^p}))}{f(\frac{\alpha^c}{2}(L-\frac{P}{\alpha^p}))} &\geq \frac{\alpha^p}{P} - \alpha^c s_1(c)\frac{f'(c_2)}{f(c_2)} \\
&\geq 0 \\
&= \frac{\alpha^p}{\tilde{P}_1(2)} - \frac{\alpha^c}{2}\frac{f'(\frac{\alpha^c}{2}(L-\frac{\tilde{P}_1(2)}{\alpha^p}))}{f(\frac{\alpha^c}{2}(L-\frac{\tilde{P}_1(2)}{\alpha^p}))}
\end{aligned}$$

which gives $P \leq \tilde{P}_1(2)$. On the other hand, if $p_1 > 0$, the first-order conditions for $p_1 > 0$ and $p_2 > 0$ (the latter of which follows from Lemma A.8) give

$$\alpha^p s_1(c)(1-\tau_1 g(R)) \geq \alpha^c \frac{f'(c_1)}{f(c_1)} s_1(c) s_2(c)(P - (\tau \cdot p)g(R)),$$
$$\alpha^p s_2(c)(1-\tau_2 g(R)) \geq \alpha^c \frac{f'(c_2)}{f(c_2)} s_2(c) s_1(c)(P - (\tau \cdot p)g(R)).$$

Because f'/f is convex, summing these and applying Lemma A.9 gives

$$\begin{aligned}
2\alpha^p\left(1 - \frac{\tau_1+\tau_2}{2}g(R)\right) &\geq \alpha^c\left(s_1(c)\frac{f'(c_2)}{f(c_2)} + s_2(c)\frac{f'(c_1)}{f(c_1)}\right)(P - (\tau \cdot p)g(R)) \\
&\geq \alpha^c\left(\frac{1}{2}\frac{f'(c_2)}{f(c_2)} + \frac{1}{2}\frac{f'(c_1)}{f(c_1)}\right)\left(1 - \frac{\tau_1+\tau_2}{2}g(R)\right)P \\
&\geq \alpha^c\frac{f'(\frac{c_1+c_2}{2})}{f(\frac{c_1+c_2}{2})}\left(1 - \frac{\tau_1+\tau_2}{2}g(R)\right)P \\
&\geq \alpha^c\frac{f'(\frac{\alpha^c}{2}(L-\frac{P}{\alpha^p}))}{f(\frac{\alpha^c}{2}(L-\frac{P}{\alpha^p}))}\left(1 - \frac{\tau_1+\tau_2}{2}g(R)\right)P.
\end{aligned}$$

Simplifying and rearranging terms gives

$$\frac{\alpha^p}{P} - \frac{\alpha^c}{2}\frac{f'(\frac{\alpha^c}{2}(L-\frac{P}{\alpha^p}))}{f(\frac{\alpha^c}{2}(L-\frac{P}{\alpha^p}))} \geq 0,$$

which, as in the previous case, implies $P \leq \tilde{P}_1(2)$.

From here, the proof is similar to that of Proposition 7. Suppose there is a group i such that $p_i > 0$ and $\tau_i g(R) > \tau^* g(0)$. The first-order conditions for $p_i > 0$ give

$$\alpha^p s_i(c)(1-\tau_i g(R)) \geq s_i(c)(-(\tau \cdot p) g'(R))$$

and thus

$$\begin{aligned} u_C(c,p,r,\tau_1,\tau_2) &= (\tau \cdot p) g(R) \\ &\leq -\frac{g(R)}{g'(R)} \alpha^p (1-\tau_i g(R)) \\ &< -\frac{g(0)}{g'(0)} \alpha^p (1-\tau^* g(0)) \\ &= \tau^* g(0) P^*(\tau^*). \end{aligned}$$

Otherwise, if $\tau_i g(R) \leq \tau^* g(0)$ for each group i such that $p_i > 0$,

$$u_C(c,p,r,\tau_1,\tau_2) = (\tau \cdot p) g(R) \leq \tau^* g(0) P \leq \tau^* g(0) P^*(\tau^*). \qquad \square$$

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