Tribute, Plunder, and Social Order*

Brenton Kenkel†

October 19, 2017

Abstract

Under what conditions does the ruler of newly conquered territory benefit from social order as opposed to internal conflict? To answer, I develop a theory of the relationship between internal divisions and post-conquest governance by a rent-seeking ruler. I develop a formal model in which political factions in a conquered society face a tradeoff between internal competition, which shapes the distribution of benefits among groups in a zero-sum way, and resistance against expropriation, which is collectively beneficial. Although internal conflict reduces the likelihood of collective resistance, it also decreases the population's incentive to participate in economically productive activity. Therefore, a ruler who taxes the products of the society's labor will profit from eschewing "divide and rule" policies and promoting social order. Conversely, a ruler whose only objective is to control a fixed stock of wealth, such as natural resources, benefits from internal division.

*I thank Phil Arena, Jeff Arnold, Brett Benson, Cristina Bodea, Rob Carroll, Clifford Carrubba, Dave Conner, Tom Dolan, Mark Fey, Michael Gibilisco, Hein Goemans, Jeremy Kedziora, Jeff Marshall, Mattias Polborn, Kristopher Ramsay, Miguel Rueda, Peter Schram, Curt Signorino, Brad Smith, Matt Testerman, and Alan Wiseman for helpful discussions and comments.

†Assistant Professor, Department of Political Science, Vanderbilt University. Email: `brenton.kenkel@vanderbilt.edu`.

When an empire or an army gains control of new territory, does it prefer to promote internal order or sow chaos? Under what conditions does the ruler of newly conquered territory benefit from social order as opposed to internal conflict? In perhaps the most notable example of colonial conquest, the invasion of Mexico and subsequent overthrow of the Aztec empire by Hernán Cortés, we see how internal divisions might work to an outsider's benefit. On his march toward Tenochtitlan, Cortés identified groups that resented Aztec tribute policies and recruited them as allies (Burkholder and Johnson 2015, 53–55). By inflaming tensions within Mexico and bringing his new allies to the capital, Cortés was able to take Tenochtitlan in 1521 despite having landed in Mexico with only a few hundred Spaniards. Conquest in Mexico came at the expense of social order.

But the disintegration of internal peace is not a universal feature of conquest. Outside rulers of conquered territory sometimes promoted order rather than conflict.[1] For example, Henley (2004) shows how the Dutch East India Company acted to reduce looting and protect property rights within the northern Sulawesi region of present-day Indonesia. Warring parties regularly called on the Dutch to arbitrate their disputes, making the Company a kind of "stranger king" in Sulawesi society. And the Dutch found it in their interest to do so, as "any conflict quickly tended to interfere with the production and supply of the Minahasan rice which ... formed the Company's main economic interest in the area" (Henley 2004, 105). In stark contrast with the Mexican example, conquest in Sulawesi promoted internal accord.

The effect of conquest on social order is not just a matter of Mexican and Indonesian history—almost every contemporary state has at some time in its political history been governed by an outsider. With few exceptions, the countries of Africa, Asia, and the Americas were once European colonial possessions. Much of Europe was militarily occupied during the Napoleonic Wars of the nineteenth century or World War II in the twentieth, not to mention being subjects of intra-European conquest more than 500 years earlier (Bartlett 1993). Even though territorial conquest may be a relic of pre–Cold War history (Zacher 2001), it is well established that extractive policies from the distant past affect economic and political institutions in the present day (e.g., Acemoglu, Johnson and Robinson 2001; Dell 2010). If we understand why the empires of the past governed the way they

[1] This is not to suggest that conquest improved the overall well-being of the population. The claim is merely that colonial governments may have sought to reduce violent conflict between groups within the conquered population. Any positive effect of greater social order may well have been offset by the losses from expropriation, not to mention political repression, the spread of disease, and other negative effects of conquest not addressed here.

did, we can better understand the roots of contemporary social order and civil conflict.

In this paper, I develop a theory to explain why rulers of newly conquered territory sometimes benefit from social order and other times prefer internal conflict. The foundation of the theory is a formal model of the *extractive rule* of a *divided society*. By extractive rule, I mean the government's sole concern is the economic value it can extract from the territory; the government cares only about internal conflict insofar as such conflict affects its profits. Of course, not all governments have such kleptocratic motivations, but it is a reasonable assumption for colonial governance or a military occupation in the wake of conquest. By divided society, I mean that the conquered territory contains distinct political groupings that may engage in inefficient competition with one another, despite their common interest in resisting expropriation by the government. In the model, each separate political faction must divide its labor between economically productive activity, collective resistance against government expropriation, and internal competition (or conflict) that alters the distribution of output among the factions. A key parameter in the analysis is just how divided the society is—the number of different groups and the strength of their incentives to compete with each other rather than produce output or resist expropriation.

The key insight from the model is that the government's preference for social order depends critically on what kind of economic product it wants to extract from the conquered territory. I distinguish between two kinds of extraction: tribute and plunder. Tribute is the output of productive activity by members of the conquered society. If the population does not supply labor, there is no tribute to extract. Common forms of tribute include agricultural output (e.g., coffee, cotton, sugar), mining performed by the native population, and taxation of trading activity. Plunder, on the other hand, is extraction from some fixed stock whose value does not depend on contributions from the population. The archetypical example of plunder is capital-intensive natural resource extraction (e.g., offshore oil drilling, kimberlite diamond mining), whose relatively small labor force is often imported (Le Billon 2013). Other examples of plunder include taking land for settlement and sheer theft of existing valuables, like the room full of gold demanded for the ransom of the Inca emperor Atahualpa (Elliott 2007, 88).

Governments that extract tribute from the conquered territory benefit from promoting internal order. The more divided the society is, the less the ruler can profit from tribute. This finding runs contrary to the common idea embodied in the Roman maxim *divide et impera*, or divide and rule. The basic logic of divide-and-rule politics is that a ruler can secure her own position by pitting political factions against each other, thereby discouraging them from uniting to overthrow her authority (Olson 1965; Acemoglu, Verdier and Robinson 2004). Indeed, my model recovers exactly this dynamic—as the incentives for internal conflict increase, so does the tax rate the government can impose without provoking resistance in equilibrium. But this apparent benefit comes at an even greater cost. Internal conflict distracts the conquered population not only from resistance against the government, but also from producing the output that the government is trying to extract in the first place. As internal divisions and conflict increase, the government ends up with a slightly larger slice of a significantly smaller pie. A government that seeks tribute—like the Dutch East India Company in Sulawesi—benefits from raising the society's productivity by promoting social order.

For a government that extracts plunder, however, divide-and-rule politics are profitable. Internal chaos increases how much plunder a government can extract, so rulers who seek plunder will prefer not to promote social order. The reason for the difference between tribute- and plunder-based political economies is simple. The potential upside of internal conflict, namely that it distracts the conquered population from coordinating to drive out the new government, is present in both settings. However, the downside for a tribute-seeking government—that internal conflict also reduces the society's productivity—does not matter for the extraction of plunder, whose value is independent of the society's labor decisions. For a ruler whose goal is plunder, sowing internal discord among the conquered population is always beneficial.

I also analyze the effects of social division on the process of conquest itself, as opposed to post-conquest governance. The conquest process is similar to competition for plunder in that there is a fixed prize—namely, the rents that accrue to the government in the future. Following the usual logic of group size and collective action (Olson 1965), more fractionalized societies will find it harder to supply the collective effort necessary to repeal an outsider's attempt at conquest. Therefore, an outsider seeking to gain control is more likely to do so when the population is more internally divided. Whereas the idea of divide-and-rule applies only under certain conditions, divide-and-conquer holds broadly.

This paper both draws from and contributes to a wide literature on the political economy of appropriation and group conflict (Hirshleifer 1991; Skaperdas 1992; Grossman and Kim 1995; Azam 2002; Dal Bó and Dal Bó 2011; Silve 2017). These models study the tradeoff between production and appropriation in societies with imperfect protection of property rights. In order to study how appropriation within a society affects the policies of an extractive ruler (and vice versa), I extend these models of horizontal competition to have a vertical aspect.

In my model, there is a government that seeks to expropriate from the society as a whole, and the political factions within the society may devote labor to resisting this expropriation. An important result of the analysis is that the government's incentive to reduce internal appropriation, for example through more stringent protection of property rights, depends on the source of value the government seeks to expropriate.

My results on the political economy of expropriation and group conflict also relate to ongoing empirical research on the economic determinants of civil conflict. Dube and Vargas (2013) demonstrate that positive price shocks to agricultural commodities in Colombia decrease participation in civil conflict, while shocks to natural resource prices have the opposite effect. Their distinction between agricultural production and natural resource rents roughly corresponds to the distinction I draw between tribute and plunder. My results suggest that there might be similar effects even in entirely "horizontal" conflicts that do not seek to overthrow the central government, but for distinct reasons—the government will prefer to promote order when the main source of value is labor output, but exacerbate internal strife when its wealth comes from a fixed resource stock. My findings also suggest how colonial-era extractive policies might have contributed to both the economic (slower growth) and political (more civil conflict) dimensions of the "resource curse" (Ross 2004; Collier and Hoeffler 2005). Rulers seeking plunder in resource-rich colonies would profit from exacerbating internal conflict through arming the population or poor protection of property rights, the latter of which we would expect to reduce economic growth (Claessens and Laeven 2003).

Finally, this paper also contributes to the theoretical literature on "divide and rule" politics. Similar to this paper, existing work in this field has considered how social fractionalization affects a government's ability to extract rents from society (Acemoglu, Verdier and Robinson 2004; Debs 2007). But these models do not allow for the possibility of appropriation between different factions of society, or other forms of inefficient internal conflict. Group divisions merely serve to increase the government's bargaining power, by giving the government a credible threat to switch bases of support. As such, existing models are not well suited to study the key question of this paper—how rent-seeking policies by the government affect social order in a divided society.

The paper proceeds as follows. I lay out the theoretical model in the next section. The two subsequent sections characterize the political economy of tribute and plunder respectively. In the penultimate section, I extend the model to examine the process of conquest itself. I conclude with some broad takeaways and suggestions for further research. All formal proofs appear in the Appendix.

1 The Model

The players are the government, denoted G, and a set of N identical factions within society, denoted $\mathcal{N} = \{1, \ldots, N\}$. Let $i \in \mathcal{N}$ denote a generic faction.

The interaction proceeds in two stages. First, the government chooses a tax rate, $t \in [0, 1]$. Second, after observing the tax rate, each faction simultaneously allocates its labor among activities that affect the level and distribution of economic output. These are production, denoted p_i; resistance against the government, r_i; and internal competition, c_i. Each faction's allocation must meet the budget constraint,

$$\frac{p_i}{\pi^p} + \frac{r_i}{\pi^r} + \frac{c_i}{\pi^c} = \frac{L}{N}, \tag{1}$$

where $L > 0$ denotes the total size of the population and each $\pi^p, \pi^r, \pi^c > 0$ denotes the society's productivity in the given area.[2] For example, the greater π^r is, the less labor is required to produce the same amount of resistance. After these choices are made, the game ends and each player receives her payoffs.

Each player's goal, including the government's, is to capture as much economic output as possible. Production, the first potential outlet for labor, creates valuable output that the government and the factions compete over. I assume output simply equals the total amount of labor devoted to production,

$$f(p) = \sum_{i=1}^{N} p_i, \tag{2}$$

where $p = (p_1, \ldots, p_N)$ denotes the vector of each faction's production choice. The goal of each player is to consume as much of this output as possible. Accordingly, each player's utility will ultimately be a fraction of $f(p)$. The other choice variables—the tax rate, resistance, and internal competition—determine what these fractions are.

Resistance, the second way the factions can expend their labor, determines how much the government can actually collect in taxes. Given the nominal tax rate t and the resistance allocations $r = (r_1, \ldots, r_N)$, let $\tau(t, r)$ denote the effective tax rate—the proportion of economic output that goes to the government—and let $\bar{\tau} = 1 - \tau$ denote the share that remains to be divided among the factions. Resistance determines what proportion of the stated tax rate the government can

[2] I write each faction's labor as a fraction of L so that I can take comparative statics on the number of factions while holding fixed the total size of the society.

collect:

$$\tau(t, r) = t \times g\left(\sum_{i=1}^{N} r_i\right), \tag{3}$$

where $g : [0, \pi^r L] \to [0, 1]$ is a strictly decreasing function. Given total resistance $R = \sum_{i=1}^{N} r_i$, the function $g(R)$ may represent either the proportion of t that the government can collect or the probability that it collects t as opposed to nothing. As regularity conditions to ensure the existence of an equilibrium and ease its characterization, I assume g is twice continuously differentiable, convex, and log-concave.[3] Naturally, I also assume the government fully collects the announced tax rate if there is no resistance, so $g(0) = 1$. For example, the linear function $g(R) = 1 - R/\pi^r L$ satisfies these conditions.

Resistance consists of any activity that directly reduces the government's ability to extract the population's economic output. The most obvious example is anti-government violence, such as in the 1791 revolt against French rule in Saint-Domingue or the 1857 Indian mutiny against the British East India Company. But resistance can also take more subtle forms. Scott (2008) describes means of "everyday resistance" employed by peasants against elites, including acts as simple as dragging one's feet. There are also more overt forms of tax evasion, like the rampant smuggling of silver out of Spain's American colonies to circumvent the royal monopoly on bullion imports (Scammell 1989, 28).

Internal competition, the third and final outlet for the factions' labor, determines the share each group receives of what is left over after the government takes its cut. I model the internal competition over resources as a contest, in which each faction expends costly effort to increase its share of the pie. Following Hirshleifer (1991) and Skaperdas (1992), I assume the cost of participation in the contest is an opportunity cost—the more labor a faction spends increasing its own share of the pie through competition, the less it has to spend increasing the total size of the pie through production or reducing the effective tax rate through resistance. Given the competition allocations $c = (c_1, \ldots, c_N)$, a faction's share of post-tax output is given by the contest success function

$$\omega_i(c) = \frac{\phi(c_i)}{\sum_{j=1}^{N} \phi(c_j)}, \tag{4}$$

where $\phi : [0, \pi^c L/N] \to \mathbb{R}_+$ is strictly increasing. Factions that devote more effort to internal competition end up with larger shares of the output. If every

[3] Given the first condition, the latter two are equivalent to $0 \leq g''(R) \leq g'(R)^2/g(R)$ for all $R \in [0, \pi^r L]$.

faction spends the same amount, or they all spend nothing, they all end up with equal shares, $\omega_i(c) = 1/N$.[4] Again, as regularity conditions to ensure equilibrium existence, I assume ϕ is twice continuously differentiable and log-concave. For example, both of the most popular contest success functions satisfy these criteria: the ratio form, with $\phi(c_i) = c_i$, and the difference form, with $\phi(c_i) = \exp(c_i)$ (Hirshleifer 1989).

The contest determines the division of all of the post-tax output among the factions. In this sense the model features an environment with weak protection of property rights, in which possession is determined through appropriation (Skaperdas 1992).[5]

Each faction's utility is simply the amount of economic output it receives. This is a function of how much is produced, how much the government extracts through taxation, and the faction's standing in the internal competition. Together, these yield faction i's utility function,

$$u_i(t, p, r, c) = \omega_i(c) \times \bar{\tau}(t, r) \times f(p). \tag{5}$$

The multiplicative payoff structure is similar to that of Hirshleifer (1991) and Skaperdas (1992), which I extend to incorporate extraction by an outside force (here, via taxation) and collective resistance against that extraction.

The government in the model is rent-seeking or kleptocratic, insofar as its motivation is to increase revenues for its own consumption. Its utility is how much of the economic output it receives, accounting for reductions in the effective tax rate due to resistance:

$$u_G(t, p, r, c) = \tau(t, r) \times f(p). \tag{6}$$

The government does not use tax revenues to provide public goods or redistribute wealth within society. In addition, the government does not have any preferences over the distribution of post-tax consumption among the factions—internal competition does not directly enter its utility, though it matters indirectly insofar as it reduces production or resistance. Given the government's kleptocratic preferences and lack of connection to the internal factions, it is natural to think of it as an outside ruler of the society, such as a colonial power or a military occupier.

This model sets up stark strategic tradeoffs for both the factions and the government. Each faction must choose between increasing the total size of the post-tax pie, namely through production or resistance, and securing its own share of

[4] If $\phi(0) = 0$, in which case (4) is not well-defined at $c = 0$, let each $\omega_i(0) = 1/N$.

[5] For a model of competition with (endogenously) partial property rights, see Grossman and Kim (1995).

that pie through internal competition. They face a collective action problem, insofar as production and resistance are public goods while one's share of the internal competition is a private good.[6] I will consider under what circumstances the government can exploit this tradeoff for its own benefit. For the government, the key tradeoff comes in how it calibrates its extractive demand. Holding fixed the behavior of the factions, the government would always prefer a higher tax rate. But a high rate may in fact be counterproductive if it diverts social effort away from production and into resistance.

In analyzing the model, I focus on two parameters related to the idea of social order. The first is the number of factions, N. Each faction is treated as a unitary actor in the model—i.e., a faction is a political unit that has solved its internal collective action problems well enough to coordinate on a division of labor that maximizes the group's total welfare. Therefore, a greater number of factions corresponds to a more fractionalized society. As I show in the analysis below, the equilibrium level internal conflict increases with the level of fractionalization. A key question for the analysis is whether this internal conflict, and thus fractionalization itself, works to the benefit of the government. When is it more profitable to govern a fractionalized society rife with internal conflict, versus a more unified society that has better overcome its own collective action problems?

The second parameter I focus on is the factions' "productivity" in internal competition, π^c, which gauges how easily the factions can appropriate from each other. I will refer to π^c as competition effectiveness, since it reflects how effectively a faction can translate its labor into strength in the internal competition. The lower the value of π^c, the greater the opportunity cost of appropriating from other factions as opposed to engaging in production or resistance. Unlike the number of factions, which would be relatively difficult to manipulate directly, it is plausible that the government could shape the relative return to appropriation, at least at the margin. The introduction of new military technology, such as horses or firearms, might increase the ratio of effective strength to labor spent, thereby corresponding to an increase in π^c. On the other hand, stronger protection of the factions' property rights would correspond to a decrease in π^c. In the analysis of the model, I look for conditions under which the government prefers to increase (or decrease) the opportunity cost of internal conflict, and how these relate to the promotion of social order.

[6] If it is counterintuitive to think of production as a public good, remember that property rights are weak in the model—a group's claim on economic output depends only on what it can appropriate through internal competition.

2 Tribute

The model is a multistage game of complete information, so the natural solution concept is subgame perfect equilibrium. In this section, I solve for an equilibrium by backward induction.

In the model, the sole source of economic value is what the population produces. If the population does not produce anything, the government comes away with nothing. In this sense it is a model of tribute—expropriation of the output of the population's labor. The analysis in this section therefore concerns the interplay of fractionalization and social order in a tribute economy. In the next section, I modify the model, replacing endogenous production with an exogenous source of value, in order to examine what changes when the government's objective is plunder.

I focus on how taxation, fractionalization, and the relative labor cost of internal competition affect social order in equilibrium, and how this in turn affects how much the government can extract. In order to focus on the most substantively relevant cases of the model, I impose the following assumption throughout this section:

Assumption 1 (State of Nature)**.**

$$\frac{N-1}{N} > \frac{\phi(0)}{\phi'(0)\pi^c L}. \tag{7}$$

This assumption holds if and only if the baseline level of internal conflict—what would occur in equilibrium if the government imposed no taxes—is nonzero.[7] If the State of Nature assumption does not hold, then for every tax rate $t \in [0,1]$, the equilibrium of the subsequent labor allocation subgame entails each faction spending nothing on internal competition. These cases, in which internal conflict is effectively impossible regardless of the tax rate, are of relatively little substantive interest for the analysis of social order.

2.1 Division of Labor

Proceeding backward, I consider each faction's division of labor in the labor allocation subgame, after the government chooses a tax rate $t \in [0,1]$. Remember that each faction divides its labor between production, p_i; resistance, r_i; and internal competition, c_i.

[7] Assumption 1 holds only if $N > 1$, as the right-hand side of (7) is non-negative.

In an equilibrium of the labor allocation subgame, each faction's division of labor maximizes its own payoff, taking as fixed the other factions' actions. In an equilibrium labor allocation, each faction must devote its labor to the activity (or activities) with the greatest marginal benefit per unit of labor. If it expends labor on two or more activities, the marginal benefits from the two must be equal. For example, if there is an equilibrium (p, r, c) in which faction i expends labor on both production and internal competition, the following condition must hold:

$$\pi^p \frac{\partial u_i(t, p, r, c)}{\partial p_i} = \pi^c \frac{\partial u_i(t, p, r, c)}{\partial c_i}.$$

If the marginal benefit of production were greater than that of internal competition, then the faction could strictly benefit by shifting a bit of labor out of internal competition and into production, violating the condition of equilibrium.

The marginal benefit of one activity depends on the values of the others. For example, labor spent on internal competition, c_i, increases a faction's share of the output that is left over after taxation. Production and resistance increase the amount of post-tax output and, consequently, raise the marginal benefit of internal competition. Because of these interdependencies, the equilibrium labor allocation involves a mixture of activities. There cannot be an equilibrium with high production and no internal competition, because the return to internal competition would be too great for the factions to refrain. But there also cannot be an equilibrium with high internal competition and no production, because then the competition would have no benefit.

The benefits of resistance against government taxation depend not only on the level of production and a faction's share in the internal competition, but also on the tax rate itself. Naturally, the more the government demands, the more there is to be gained from resistance. In the extreme case of no taxes, $t = 0$, resistance has no effect on the outcome. Each faction's labor would be better spent either on increasing total output through production, or increasing its own share of output through internal competition. Therefore, the equilibrium division of labor following $t = 0$ will entail no resistance. More generally, when taxes are low enough, the marginal benefit of resistance remains too low to justify diverting effort from production or internal conflict, and there will be no resistance in equilibrium.

Without resistance, the equilibrium division of labor when taxes are low entails a mixture of production and internal competition. Because the factions are identical in terms of size and productivity, in equilibrium each devotes the same amount of labor to internal competition. As a result, each ends up with an equal share of the pie, $\omega_i(c) = 1/N$. The following proposition states the form of the equilibrium in this low-tax case.[8] I call this the baseline equilibrium, since it represents what would occur if the government imposed no taxes at all.

Proposition 1 (Baseline Equilibrium)**.** *There is a tax rate $\hat{t}_0 \in (0,1)$ such that $\sum_i r_i = 0$ in every equilibrium of the labor allocation subgame if and only if $t \leq \hat{t}_0$. Every subgame with $t \leq \hat{t}_0$ has the same unique equilibrium, in which $\sum_i p_i = \bar{P}_0 > 0$ and each $c_i = \bar{c}_0 > 0$.*

Even when the tax rate is zero, the equilibrium outcome is Pareto inefficient for the factions. Each faction devotes $\bar{c}_0 > 0$ to internal competition to end up with $1/N$ of the output. If every faction instead spent $c_i = 0$ and devoted all their labor to production, each would still have a share $1/N$, but now of a larger total surplus, leaving them strictly better off. A kind of prisoner's dilemma explains why this Pareto efficient allocation of labor is not sustainable as an equilibrium. If no faction plans to spend on the internal competition, then any single faction can obtain a large share by spending relatively little. Under Assumption 1, the temptation is large enough that every faction has an incentive to deviate from a strategy profile with no competition.

Within the set of tax rates below the baseline equilibrium cutoff, $t \leq \hat{t}_0$, the government benefits from greater social order—i.e., less internal conflict. To be clear, the government does not intrinsically care about internal conflict. However, when there is no resistance, any internal competition must come at the expense of production, to the government's detriment. For $t \leq \hat{t}_0$, the government's payoff is $t\bar{P}_0$. Therefore, to analyze how fractionalization and the opportunity costs of internal competition affect the government's profits, I take comparative statics on baseline equilibrium production with respect to these parameters.

The comparative statics on the number of factions, N, which I interpret as fractionalization, are straightforward—more factions means less production. The following remark states this result, which Figure 1 illustrates.[9]

Remark 1. *Total production in the baseline equilibrium, $\bar{P}_0$, is strictly decreasing in the number of factions, N.*

This result follows from the same logic as the classic finding that public good provision decreases with group size (Olson 1965). In the baseline equilibrium,

[8] All proofs are in the Appendix.

[9] All figures use the parameters $\pi^p = \pi^r = \pi^c = 1$ and $L = 2.5$, and the functional forms $g(R) = 1 - R/\pi^r L$ and $\phi(c_i) = \exp(c_i)$.

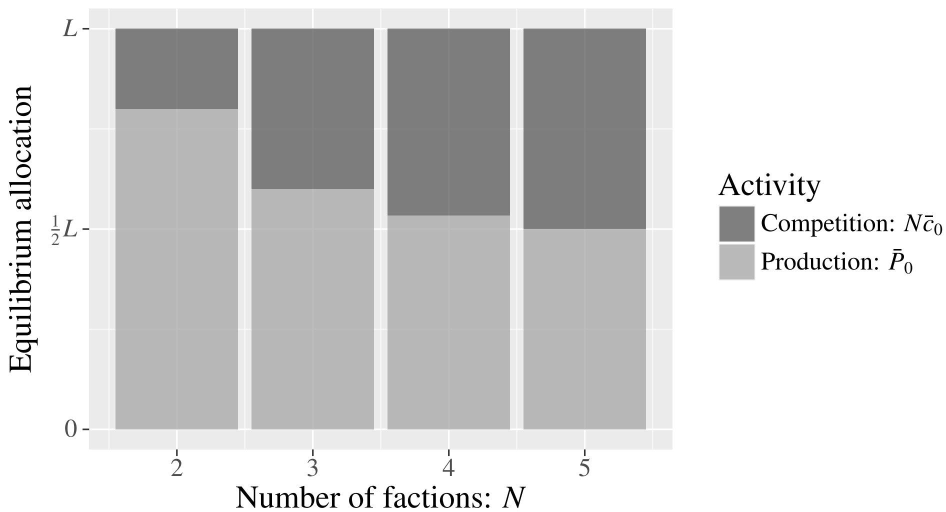

Figure 1. Division of labor in the baseline equilibrium (Proposition 1) as a function of the number of factions.

because of the internal competition among factions, each faction only recoups $1/N$ of what it produces. In other words, a faction only partially internalizes the benefits of its own production, less so as the number of factions increases. But each faction always fully internalizes the benefits of the labor it devotes to internal competition, regardless of the number of other factions. Therefore, as N increases, the relative return to internal competition increases, so the equilibrium division of labor entails less production. At least in the baseline equilibrium of a tribute economy, the government is better off when the society is less fractionalized.

The effects of competition effectiveness, π^c, on the baseline equilibrium are more complicated. Specifically, a marginal increase in π^c—which represents a decrease in the opportunity cost of expending labor on internal competition—has two cross-cutting effects. The first, which I call the *incentive effect*, is to raise the marginal benefit per unit of labor of internal competition. Since the marginal benefits of competition and production must be equal in the baseline equilibrium, on its own this would lead to greater competition and less production. However, the second effect of increasing π^c, which I call the *labor-saving effect*, works the other way. If a faction's competition effectiveness increases, it can achieve the same amount of effective strength, $\phi(c_i)$, with a smaller labor force. It may use some of the freed-up labor to even further push its advantage in the internal competition, but it may also devote some to production.

The overall effect of π^c on economic output, and thus the government's payoff in the baseline equilibrium, depend on whether the incentive effect or the labor-saving effect dominates. When the incentive effect is stronger, a marginal increase in π^c (e.g., arming the population or weakening protection of property rights) would lead to lower total production, reducing the amount the government can extract. The opposite is true when the labor-saving effect is stronger. The following condition characterizes the relative strength of the two effects. For any $C > 0$, I say the incentive effect outweighs the labor-saving effect at C if

$$\frac{d \log \phi(C)}{dC} + C \frac{d^2 \log \phi(C)}{dC^2} > 0,$$

and the labor-saving effect outweighs the incentive effect if the opposite inequality holds.[10]

Remark 2. *Total production in the baseline equilibrium, $\bar{P}_0$, strictly decreases with a marginal increase in competition effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

The aforementioned experience of the Dutch East India Company in the Sulawesi region of Indonesia illustrates how a change in competition effectiveness alters division of labor choices (Henley 2004). In the absence of a "stranger king" to mediate disputes, effort devoted to appropriation was relatively effective and therefore common. However, once the company began enforcing property rights, the return to labor spent raiding plunged, corresponding to a decrease in π^c. Consequently, Sulawesi groups spent relatively more effort on economic production.

The comparative statics results show that, at least in the baseline equilibrium when the government's objective is to extract tribute from the society, the government benefits from social order. Fractionalization always reduces the amount the government can extract in the baseline equilibrium. The government only profits from increasing competition effectiveness if the labor-saving effect is so strong that it causes the society to devote more labor to production and less to internal competition. However, these results are conditional on taxes being so low that the government does not face any resistance. When internal competition can only take place at the expense of economic output, of course the government prefers less internal competition. Before inferring from these results that the government

[10] In Lemma 12 in the Appendix, I verify that the incentive effect always outweighs the labor-saving effect for the "difference" contest success function ($\phi(C) = \theta \exp(\lambda C)$), and the two effects are exactly offsetting in case of a "ratio" contest success function ($\phi(C) = \theta C^{\lambda}$).

benefits from social order, I must show that the equilibrium tax rate results in no resistance, and thus the baseline division of labor.

In fact, the location of the cutpoint tax rate, $\hat{t}_0$, itself depends on fractionalization and competition effectiveness. This cutpoint—the most the government can demand in taxes without engendering resistance—is the point at which the factions are at the margin indifferent about diverting some labor away from their baseline equilibrium allocations and into resistance. In other words, it is the point at which the marginal returns to the baseline equilibrium levels of production ($\bar{P}_0$) and resistance (zero) are equal. Since there are diminishing marginal returns to production, this indifference occurs at a relatively low tax rate when $\bar{P}_0$ is relatively high. Therefore, factors that decrease baseline production (like fractionalization) thereby increase the cutpoint tax rate.

Remark 3. *The cutpoint tax rate,*

$$\hat{t}_0 = \frac{\pi^p}{\pi^p - \pi^r \bar{P}_0 g'(0)},$$

is strictly increasing in fractionalization, N. It strictly increases with a marginal increase in competition effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.

This finding complicates the picture of how social order affects the government's ability to extract from the population. On one hand, for any fixed tax rate below the threshold, a marginal increase in fractionalization or competition effectiveness (if it increases internal conflict at the expense of production) reduces the amount the government extracts. However, these same effects push the threshold upward. When the incentive for social strife increases, the economic pie shrinks, but the government can extract a greater portion of it without engendering resistance. I return to the interplay of these cross-cutting effects below when I analyze the government's equilibrium choice of tax rate.

For higher tax rates, the baseline equilibrium is no longer sustainable. Once the tax rate crosses the threshold, $t \geq \hat{t}_0$, the marginal return to resistance is too high for the factions to prefer spending nothing on resistance.

The exact equilibrium division of labor is different for every tax rate above the baseline cutpoint. At values just above the cutpoint, the equilibrium entails a mixture of all three activities—economic production, resistance against taxation, and internal competition over the post-tax output. As the tax rate increases, resistance increases gradually, while internal competition and production decrease. For extremely high tax rates, the collective benefit of reducing government extraction may overwhelm the factions' individual incentives to fight over the distribution of output, resulting in zero internal competition in equilibrium. However, even at the maximum tax rate, $t = 1$, there is always positive production in equilibrium. Every faction's payoff is a fraction of production; zero production would result in zero payoffs for everyone. The following proposition states the form of the equilibrium above the baseline; Figure 2 illustrates equilibrium labor allocations as a function of the tax rate and the number of factions.

Proposition 2 (Resistance Equilibrium)**.** *There is a tax rate $\hat{t}_1 > \hat{t}_0$ such that in every equilibrium of the labor allocation subgame with tax rate t:*

- *If $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = \tilde{P}_1(t) > 0$ (weakly decreasing in t), $\sum_i r_i = \tilde{R}_1(t) > 0$ (strictly increasing), and each $c_i = \tilde{c}_1(t) > 0$ (strictly decreasing).*
- *If $t \geq \hat{t}_1$, then $\sum_i p_i = \tilde{P}_2(t) > 0$ (strictly decreasing in t), $\sum_i r_i = \tilde{R}_2(t) > 0$ (strictly increasing), and each $c_i = 0$.*

A high tax rate in effect unifies the population, giving the factions an incentive to act in concert to reduce government expropriation. For example, we see excessive extraction by a colonial empire leading to unity both among the American colonies during the Stamp Act crisis of 1765 and between creoles and Indians during a contemporaneous tax revolt in Quito (Elliott 2007, 310–314). To see why high taxes unify the population, remember again that the marginal benefit of each activity depends on the others. When the stated tax rate is high, low resistance means the effective tax rate is high as well. Since the factions only consume the post-tax share of output, a high effective tax rate lowers the marginal return to additional production or to increasing a faction's own share through internal competition. Meanwhile, the marginal benefit of resistance increases with t. Therefore, in order for each faction to be spending its labor where the marginal benefit is greatest, production and internal conflict must both decrease with the tax rate.

2.2 Optimal Tax Rate

I now solve for the government's choice of tax rate. Given the equilibrium responses to each potential choice, as characterized above, the government faces a tradeoff. A higher tax rate has an obvious benefit—the government gets a greater share of the output, all else equal. But all else is not equal. Greater tax rates are met with greater resistance, reducing the government's effective share of output.

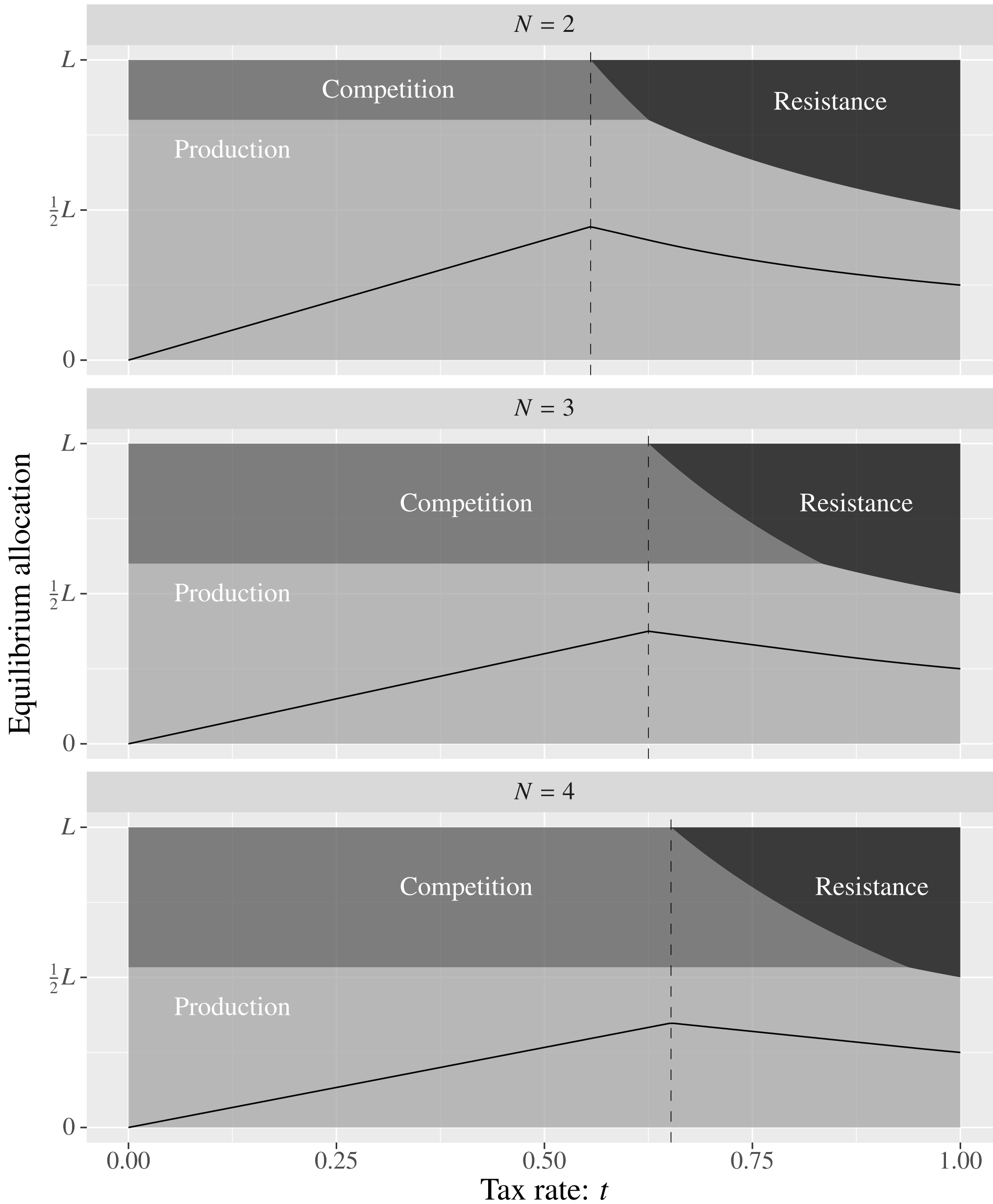

Figure 2. Equilibrium labor allocations, as described in Propositions 1 and 2, as a function of the tax rate and the number of factions. The solid curve is the government's payoff as a function of the tax rate, and the dashed line is the equilibrium tax rate.

Moreover, total output itself changes, shrinking as the factions devote more labor to resistance and thereby less to production when taxes are greater.

These tradeoffs create a "Laffer curve," such that the highest tax rate does not maximize the government's revenue. At low tax rates, namely those that result in the baseline equilibrium characterized by Proposition 1, the tax rate has no marginal effect on the factions' behavior. Therefore, the government's payoff strictly increases with the tax rate in this low range, as it receives a larger share of the same pie. Beyond that, however, greater taxes are self-defeating. As taxes increase above $\hat{t}_0$, the increase in resistance and the concomitant decrease in production are large enough to offset the gain the government might get from demanding a greater proportional share. As the following proposition states, the equilibrium tax rate is $\hat{t}_0$, the highest at which there is no resistance.

Proposition 3 (Optimal Tax Rate)**.** *There is an equilibrium in which the government chooses the greatest tax rate that engenders no resistance,* $t = \hat{t}_0$*. If* g *or* ϕ *is strictly log-concave, this is the unique equilibrium tax rate.*

To see why it is optimal for the government to avoid engendering resistance, consider a tax rate $t' > \hat{t}_0$ that does lead to resistance. This will result in an effective tax rate of $\tau(t', r) < t$, with some of the increase in resistance coming at the expense of production. It would be better for the government to make the announced tax rate equal the effective one, $t'' = \tau(t', r)$, and reap the gains of the additional production. Of course, if $t'' > \hat{t}_0$, there will still be positive resistance at t'' and the government will not recoup the full share. Nonetheless, as I show in the proof of this proposition in the Appendix, the increase in production by moving to a lower rate is great enough to be profitable for the government.

At a glance, this result might appear to imply that the government benefits from internal disorder. It is evident from Figure 2 that the equilibrium tax rate corresponds to a high point for internal competition. But this correlation is not exactly causal. Where internal competition is at its maximum, production is also at its maximum, and resistance is at its minimum. To analyze whether the government benefits from social order or disorder in a tribute economy, it would be more apt to imagine an exogenous shock to the incentives for internal conflict.

To better understand how social order affects the government's ability to extract economic surplus, I return to the main structural determinants of internal competition—fractionalization (N) and competition effectiveness (π^c). I showed above that these have cross-cutting effects on the baseline equilibrium. According to Proposition 3, the government's equilibrium payoff is the product of the cutpoint tax rate with baseline production: $\hat{t}_0 \bar{P}_0$. Fractionalization and competition effectiveness (when the incentive effect outweighs the labor-saving effect) increase the first term, per Remark 3, but decrease the second, per Remarks 1 and 2. In other words, as the structural incentives for internal violence increase, the government ends up with a large share of a smaller pie in equilibrium. The critical question is which of these effects dominates. On the whole, if the objective is to secure tribute from the population, would a government prefer to rule a society that is more or less prone to internal conflict? As the following proposition states, the answer is the latter—as the structural determinants of internal conflict increase, the government's equilibrium payoff decreases.

Proposition 4. *The government's equilibrium payoff is strictly decreasing in the number of factions, N. It strictly decreases with a marginal increase in competition effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

The most striking result here is that additional fractionalization always makes the government worse off. Why does the decrease in production always more than offset the increase in the equilibrium tax rate? Once again, the answer lies in the relative marginal benefits of production, resistance, and internal competition for the factions. As fractionalization increases, the return to production for an individual faction decreases, hence the reduction in baseline equilibrium production. Like production, resistance is a collective benefit at individual cost, so all else equal the return to resistance decreases with fractionalization at the same rate as production. However, the marginal benefit of resistance is also a function of how much the government extracts; the more the government would receive, the more there is for the population to take back through resistance. Therefore, in order for the factions to prefer not to spend labor on resistance—as must be the case on the equilibrium path—production must decrease enough with fractionalization that the government's overall payoff does not increase.

To state the logic a bit more precisely, at the equilibrium tax rate, the marginal benefits to the factions of production and resistance are exactly equal, satisfying the following equation:

$$\underbrace{\frac{\pi^p}{N}(1 - \hat{t}_0)}_{\pi^p(\partial u_i / \partial p_i)} = \underbrace{\frac{-\pi^r g'(0)}{N} \hat{t}_0 \bar{P}_0}_{\pi^r(\partial u_i / \partial r_i)}.$$

(Recall that g is the function that determines how much the effective tax rate shrinks with resistance.) The equilibrium return to production is a decreasing function of the tax rate, $\hat{t}_0$, as seen in the left-hand side of the equation above. As the number of factions increases, so does $\hat{t}_0$, resulting in a lower return to production. At the same time, the equilibrium return to resistance is an increasing function of the government's equilibrium payoff, $\hat{t}_0 \bar{P}_0$, as seen in the right-hand side above. Therefore, as the equilibrium tax rate increases with fractionalization (or with competition effectiveness, when the incentive effect is strongest), the government's overall payoff must decrease.

The upshot of Proposition 3 is that social order promotes the government's ability to extract tribute. As long as the government's objective is to profit from the fruits of the population's labor, it is better off governing a society where structural conditions are favorable to social order. If we think of these conditions as at least partially endogenous, this implies that it is in the government's interest to promote social order if the costs of doing so are low enough. For example, the government might seek to divert labor away from looting and into productive economic activity by better enforcing property rights. We have already seen this with the Dutch in Sulawesi (Henley 2004). Similarly, on the frontiers of Latin Christendom prior to Frankish conquest in the late Middle Ages, "direct predation ... was not the occasional excess of the lawless but the prime activity of the free adult male population" (Bartlett 1993, 303). By establishing free villages and improving the protection of property rights, the immigrant Frankish nobility was able to profit by directing labor into productive activity rather than banditry. In yet another example, the Mughal empire that preceded British rule on the Indian subcontinent "defined their task as to keep an ordered balance between the different forces which constituted Indian society" (Wilson 2016, 17).

In a tribute economy, in which the government expropriates the product of the population's labor, the logic of "divide and rule" appears inoperative. Internal conflict may distract the population from resistance against taxation, but it also reduces the incentives for economic production. On the whole, it is more profitable to control a society that is more unified, and therefore more productive. The benefits of social order might be difficult to detect from casual observation, however. The optimal policy choice for the government involves some level of internal competition among the factions—not because this competition is beneficial in itself, but because that policy happens also to be the one that maximizes production and minimizes resistance.

3 Plunder

The analysis so far has assumed the government's objective is to expropriate the product of the population's labor; if the population does not work, then there is no profit to be made. I showed that the government has an incentive to promote social order under these conditions. In this section, I consider an alternative kind of political economy, in which the main source of value—what the government and the factions want to appropriate—is a fixed stock whose value does not depend on labor inputs from the native population. In contrast with the form of tribute analyzed above, I call this kind of appropriation plunder. I find that whereas a tribute-seeking government benefits from social order, a plunder-seeking government benefits instead from chaos.

The clearest example of the kind of fixed stock of value that could be plundered is natural resources, particularly oil. Oil exploitation is capital-intensive, often takes place offshore, and can be conducted by workers imported from abroad (Le Billon 2013, 28–30). In the absence of mass resistance, a government can extract value from oil deposits regardless of indigenous labor contributions. Land itself may also play the role of a fixed resource that is valued in its own right. Whereas Spanish settlers in South America acquired property to be worked by native labor, English settlers in North America sought sparsely populated territories and fought to expel Indians where they settled (Elliott 2007, 36–38). In the terms I use here, the Spanish system of exploiting American Indian labor for mining and agriculture was tribute, whereas the English drive to dispossess American Indians of their land for English settlement was plunder.

When the government's objective is to expropriate from a fixed stock whose value does not depend on the population's labor, its strategic tradeoffs are significantly different than in the tribute case considered above. For a tribute-seeking government, internal conflict has cross-cutting effects: it reduces resistance, allowing the government to impose a higher effective tax rate, but it also reduces production. The situation is simpler for a plunder-seeking government. Since the value of the resources available to expropriate does not depend on the society's labor inputs, there is no longer a tradeoff between internal competition and the size of the pie. Internal conflict merely helps prevent the factions from coordinating to resist government expropriation. Consequently, a plunder-seeking government benefits from fractionalization and social disorder, and will profit from policies that heighten internal conflict.

To model the political economy of plunder, I make a simple change to the baseline model. In the original model, the total size of the pie was $f(p)$, the endogenous result of production by the factions. In the *plunder model*, the value of the economic product available for expropriation is fixed at the exogenous value $X > 0$.[11] The government's choice is still the tax rate $t \in [0, 1]$. With production out of the picture,[12] the factions now only choose to allocate their labor between resistance, r_i, and internal competition, c_i. The budget constraint for each faction is still given by (1), with p_i fixed to 0. Utility functions in the resource economy model are

$$u_i(t, r, c) = \omega_i(c) \times \bar{\tau}(t, r) \times X,$$
$$u_G(t, r, c) = \tau(t, r) \times X,$$

so as before all players' payoffs sum to the total value X. This model of internal appropriation with a fixed resource stock is similar to that of Hodler (2006), extended to include an extractive government and potential resistance to it.

The relationship between the tax rate and the equilibrium division of labor in a plunder economy mirrors that of the tribute economy studied above, except with production taken out. If taxes are low, the marginal benefit of resistance is too low for the factions to be willing to devote any labor to it. Consequently, they spend all their effort on internal competition. Conversely, if the stated tax rate is very high, the factions put all their labor toward resistance, so as to reduce the effective rate. In between these extremes, the equilibrium division of labor involves a mix of resistance and internal competition. Within this middle range, as the tax rate increases, the balance shifts toward more resistance and less internal competition. The following proposition summarizes these results, and Figure 3 illustrates them.

Proposition 5. *In the plunder model, every labor allocation subgame has a unique equilibrium. There exists a tax rate $\hat{t}_0^X \in (0, 1)$ such that each $r_i = 0$ in equilibrium if and only if $t \leq \hat{t}_0^X$. There exists $\hat{t}_1^X > \hat{t}_0^X$ such that each $c_i = 0$ in equilibrium if and only if $t \geq \hat{t}_1^X$. For $t \in (\hat{t}_1^X, \hat{t}_0^X)$, in equilibrium each $r_i = \tilde{R}_X(t)/N > 0$ (strictly increasing in t) and each $c_i = \tilde{c}_X(t) > 0$ (strictly decreasing).*

[11] Different actors might value the same stock differently. For example, control of oil fields might be more valuable in an absolute sense to the government than to rebel groups (Le Billon 2013, 29–30). The results of the analysis would not change if each actor valued the pie at a potentially different level $X_i > 0$, since this would simply entail the multiplication of each actor's utility function by a positive constant.

[12] The results would be similar substantively but more cumbersome analytically if economic output were the combination of exogenous resources and endogenous production, such as $f(p) = X + \sum_i p_i$.

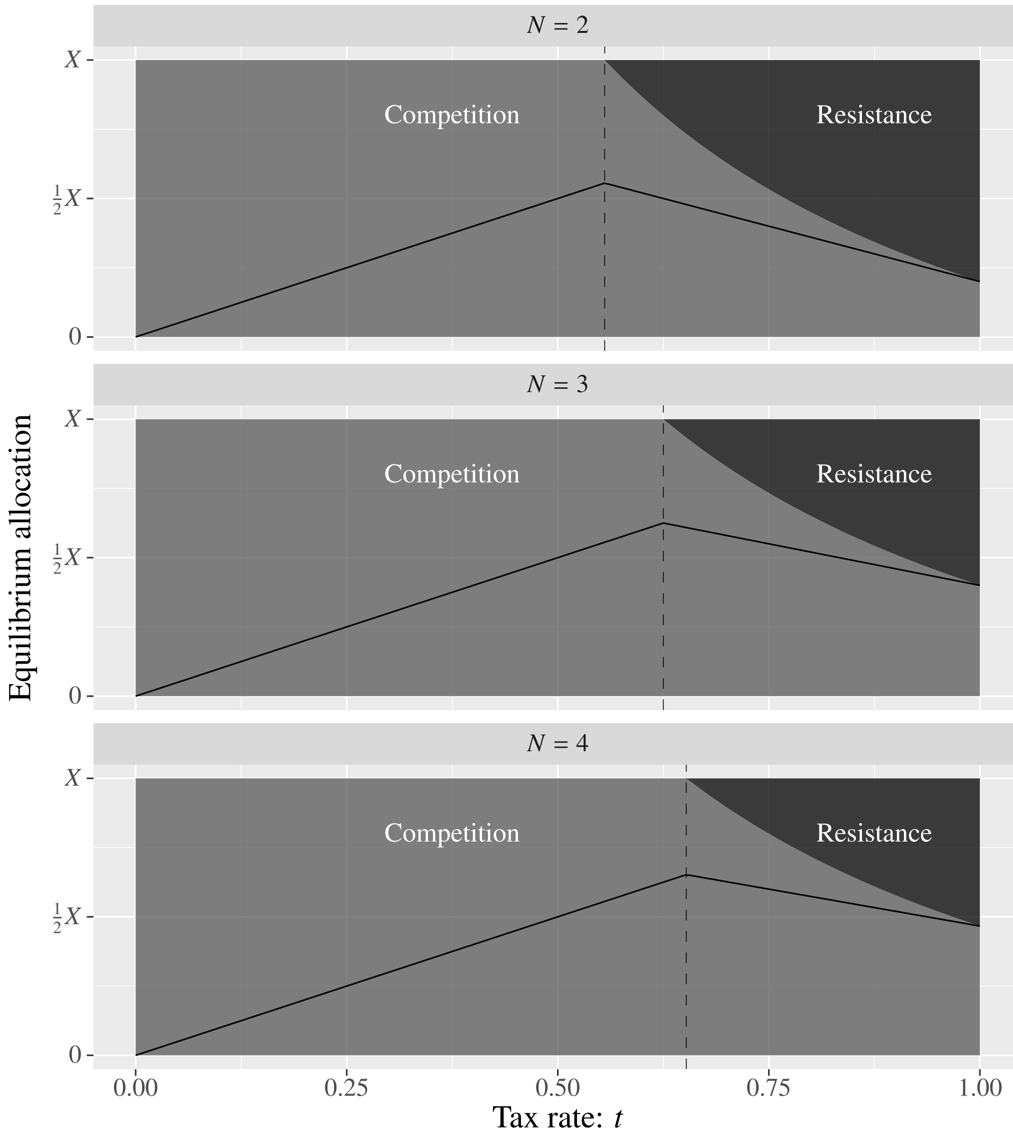

Figure 3. Equilibrium labor allocations in the plunder model, as described in Proposition 5, as a function of the tax rate and the number of factions. The solid curve is the government's payoff as a function of the tax rate, and the dashed line is the equilibrium tax rate. Parameters and functional forms are the same as in the previous figures, with $X = L = 2.5$.

As in the tribute model, excessive taxation effectively unifies the population in anti-government resistance. As the tax rate increases beyond the lower cutpoint, $\hat{t}_0^X$, internal competition decreases as the factions spend a greater proportion of their labor on resistance. However, the other drawback of a high tax rate that I found in the tribute model—less output due to decreases in production—does not carry over here. Since the source of value here is exogenously fixed and does not depend on the population's labor, a high tax rate does not cause the pie to shrink. Consequently, unlike in the tribute model, it is not necessarily true that there will be no resistance on the equilibrium path.[13] Depending on the specific technology of resistance, the government may be able to attain an effective tax rate $\tau > \hat{t}_0^X$ even after accounting for the population's equilibrium response. When the government expropriates only from what the population produces, any effective tax rate so high would have to result in production so low that the tradeoff would not be worth it to the government, as I showed above. However, when there is a fixed source of wealth, such as natural resources, the value of the taxable goods does not change with the tax rate.

Whereas a tribute-seeking government always chooses the tax rate that results in maximal internal conflict (given equilibrium responses by the factions), a plunder-seeking government may not do so. But this is not because the incentives for social order are stronger in a plunder economy than a tribute economy. Instead, it simply reflects how the government's strategic tradeoffs change when it no longer values production by the population. Higher tax rates now have only one drawback (greater resistance), compared to two (that and less production) in a tribute economy. Therefore, a plunder-seeking government might be more willing to incur resistance than a tribute-seeking government would be. It just so happens, because the factions have a limited labor pool, that greater resistance is correlated with lower internal conflict.

In fact, looking at the structural parameters that drive internal competition, it is clear that plunder-seeking governments prefer chaos over social order. Their incentives are therefore opposite those of tribute-seeking governments. For example, consider fractionalization, as represented by the number of factions, N. In an economy based on tribute, fractionalization leads to lower production (Remark 1) and thereby reduces how much the government can extract (Proposition 4), even though it also reduces the incentives for collective resistance. But when the source of value for the government is exogenously fixed, the negative effect of fractional-

[13] In Figure 3 the equilibrium tax rate is $\hat{t}_0^X$, as in the original model, but this is an artifact of the particular functional forms used to make the figures.

ization on the population's economic activity is irrelevant, while its negative effect on resistance remains. Therefore, for a plunder-seeking government, additional fractionalization is a net benefit.

The story is similar for competition effectiveness, parameterized here by π^c, though again complicated by its cross-cutting effects on the factions' division of labor. Remember that π^c represents how easily the factions can translate their labor into effective strength in the internal competition, so it is greater when the population is more heavily armed or when property rights protection is weaker. Greater π^c represents the marginal benefit per unit of labor to internal competition, increasing the incentive to engage in internal competition, i.e., the incentive effect. However, greater π^c also allows a faction to achieve a greater effective strength at a lower labor cost, i.e., the labor-saving effect. When the incentive effect dominates the labor-saving effect, the result of a marginal increase in π^c is that the factions devote more labor to internal competition and less to other activities. In the case of a plunder economy, that means less labor on resistance, which works to the government's benefit; by contrast, in a tribute economy, more internal competition means less production, to the government's detriment. When an increase in π^c results in a decline in social order—i.e., when the incentive effect dominates the labor-saving effect—a plunder-seeking government gains, while a tribute-seeking government loses. The following proposition summarizes how a plunder-seeking government benefits from structural conditions that favor social disorder.

Proposition 6. *In the plunder model, the government's equilibrium payoff is increasing in the number of factions, N. If there is a unique equilibrium tax rate t^*, the government's equilibrium payoff is locally increasing in competition effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at the corresponding equilibrium level of internal competition.*

Proposition 6, in combination with the results of the previous section, shows that the profitability of "divide and rule" in extractive politics depends critically on what kind of goods the government wants to extract. If the main source of value is the product of the population's labor, as in the tribute model, then fractionalization and internal disorder decrease the incentive to produce, ultimately reducing the profits of expropriation. But if the source of contention between the government and the population is some fixed pool of goods or resources, as in the plunder model, the opposite logic prevails. In this case, internal conflict does not reduce the value of the pie, but it does keep the population distracted from resistance against the government.

The major empirical implication of these results is that the relationship between social order and the policies and profitability of extractive governance should be conditional on the nature of what is being extracted. All else equal, when a colonial power or military occupier seeks tribute—i.e., to extract from the output of indigenous labor—we should expect it to impose policies that reduce conflict and promote internal order, at least at the margins. By reducing appropriation among various ethnic groups or political factions, these policies raise the overall productivity of the governed population, which is profitable for the government. Moreover, we should expect extractive governance to be more profitable and stable in polities where structural conditions favor internal order. For example, imperial regimes funded by tribute should be less willing to expend resources to gain or maintain control over internally divided societies.

When the object of government extraction is a fixed stock of value, such as a natural resource, we should expect these relationships to go the other direction. A plunder-seeking government will, at the margin, prefer policies that increase internal disorder and thereby reduce anti-government resistance, such as lax enforcement of competing groups' property rights. Colonies or occupations whose main objective is to secure control of some existing resource will be more successful when the population is more divided, or local conditions otherwise favor internal chaos. The answer to "Does it pay to divide and rule?" is "It depends"—specifically, on what the ruler wants to get out of the society.

4 Conquest

The results so far have concerned the governance of a society that has already been conquered. The models above take as given that there is a government in place with the power to collect taxes (if resistance is sufficiently low). I now briefly consider conquest itself—the process that determines who becomes the government, prior to the selection of the tax rate and subsequent division of society's labor. When an outside force seeks to usurp authority, is it more likely to succeed when the population is more divided?

Whereas internal fractionalization is only *conditionally* beneficial for post-conquest governance, namely when the government's objective is plunder, it is *unconditionally* beneficial for an outsider seeking to gain control in the first place. The more divided the society is, the more likely an outsider is to prevail in a contest for control. This follows for much the same reason that in the post-conquest stage modeled above, fractionalization increases the tax rate the government can impose without engendering resistance (Remark 3). Resistance against an outsider's attempt to take control is effectively a public good. As the number of factions increases, the incentive to provide this public good rather than to fend for oneself decreases (Olson 1965). Therefore, an outsider can more easily take control of a divided society than a unified one. Whether it pays to divide and rule is complicated, but whether it pays to divide and conquer is not.

In the *conquest model*, a set of N factions compete with each other and with an outsider, denoted O, for the ability to be the government in the future. The incremental value of being the government is $v(N) > 0$, which may increase with N (as in a plunder economy) or decrease (as in a tribute economy). Each faction has L/N units of labor, which it may divide between two activities. It contributes an amount $s_i \geq 0$ to preventing the outsider from taking over control. The faction also contributes an amount $d_i \geq 0$ to influence its own chance of becoming the government, conditional on the outsider being successfully repelled. Each faction's budget constraint is[14]

$$s_i + d_i = \frac{L}{N}. \tag{8}$$

The success of the attempted conquest depends on how much the factions spend to combat the outsider. I assume the outsider's military strength is a fixed value, $\bar{s}_O > 0$, so the outsider is not a strategic player here. The assumption that the outsider's strength is exogenous is of course a simplification, but it is plausible in situations where the outsider marshals its forces before fully understanding the internal political situation—such as in Cortés's incursion into the Mexican mainland, and other early maritime colonial ventures.[15] The probability that the outsider becomes the government is

$$\frac{\bar{s}_O}{\bar{s}_O + \chi(\sum_{i=1}^N s_i)}, \tag{9}$$

where $\chi : [0, L] \to \mathbb{R}_+$ represents the translation of society's labor into its strength against the outsider. In case the outsider's attempted conquest fails, the probability that faction i becomes the government is

$$\frac{\psi(d_i)}{\sum_{j=1}^N \psi(d_j)}, \tag{10}$$

[14]The assumption of unit productivity for each activity is without loss of generality. The model here with functional forms $\chi(S) = \tilde{\chi}(\pi^s S)$ and $\psi(D) = \tilde{\psi}(\pi^d D)$ is isomorphic to a model with the common budget constraint $s_i/\pi^s + d_i/\pi^d = L/N$ and functional forms $\tilde{\chi}$ and $\tilde{\psi}$.

[15]With some additional restrictions on the parameters of the model, the results of the conquest game would be essentially the same if the outsider's strength were chosen endogenously.

where $\psi : [0, L/N] \to \mathbb{R}_+$ represents the translation of an individual faction's labor into its proportional chance of success against other factions. As with the function ϕ in the original model, I assume χ and ψ are strictly increasing and log-concave.

The factions simultaneously choose how to allocate their labor, subject to the budget constraint (8). A faction's utility function is

$$u_i(s, d) = \frac{\psi(d_i)}{\sum_{j=1}^{N} \psi(d_j)} \times \frac{\psi(\sum_{j=1}^{N} s_j)}{\bar{s}_O + \psi(\sum_{j=1}^{N} s_j)} \times v(N), \tag{11}$$

where $s = (s_1, \ldots, s_N)$ and $d = (d_1, \ldots, d_N)$.

The strategic tradeoff for the factions here is analogous to the tradeoff between resistance and internal competition in the tribute and plunder models. In particular, the marginal benefit of each activity—the internal struggle and the effort against the outsider—is dependent on the value of the other. There is nothing to be gained from improving one's own position vis-a-vis the other factions if the outsider is sure to prevail. But there is also no profit in fighting the outsider if some other faction is bound to become the government instead. The equilibrium division of labor will therefore generally consist of a mix of the two activities. Critically, though, the relative marginal benefit of fighting the outsider declines as the number of factions increases. When the number of factions is large, any individual faction's chance of becoming the government if the outsider loses is small, which in turn reduces its incentive to contribute to the collective effort against the government. Consequently, as the following result states, the outsider is more likely to win the more divided the society is.

Proposition 7. *In the conquest model, the probability that the outsider wins is increasing in the number of factions, N.*

To be clear, unlike some of the earlier results, Proposition 7 does not address how fractionalization affects the outsider's overall welfare in equilibrium. In particular, if the outsider's ultimate objective is tribute, there is no guarantee that the increase in the chance of winning due to greater fractionalization would offset the decrease in $v(N)$.

Proposition 7, in combination with the earlier results, implies that a society that is easy to conquer may nevertheless be difficult to govern. Specifically, fractionalization benefits a tribute-seeking government in the conquest stage, but not in the governance stage. The American invasion and subsequent occupation of Iraq is an obvious illustration of this conclusion (Gordon and Trainor 2006). The logic is somewhat distinct—the American occupiers had a direct preference to promote social order, whereas the government in the model cares about social order only insofar as doing so makes tribute extraction more profitable. In any case, if the outsider would prefer minimal internal conflict upon becoming the government, the structural conditions for successful conquest are opposite those of successful rule. Only for a plunder-seeking government, which benefits from internal chaos even while governing, does fractionalization have the same effect on ease of conquest and the profitability of rule.

The conquest model captures how Cortés benefited from internal divisions in Aztec society. He was able to conquer with significantly less military support than would have been necessary otherwise, because the incumbent regime also had to contend with its internal enemies (Elliott 2007; Burkholder and Johnson 2015). The Dutch East India Company similarly exploited internal divisions when initially establishing its foothold in present-day Indonesia (Scammell 1989, 20). In pure military competition, the political economy issues that arise in tribute extraction—namely, the tradeoff between internal conflict and economic productivity—are sidelined. Only after conquest is done does internal fractionalization become a potential problem for an outside ruler.

5 Conclusion

I have characterized the political economy of extractive governance in a divided society, primarily with applications to new rulers of conquered territory. The main result is that the profitability of divide-and-rule politics depends on the nature of the economic product the ruler wishes to extract. When it is the output of the population's labor, or tribute, the ruler is better off when society is less deeply divided. The opposite is true for a government that seeks to expropriate from an exogenously fixed source of value, or plunder. Additionally, regardless of the ultimate extractive aims, internal fractionalization increases the likelihood of successful conquest in the first place.

Besides its contributions to the literature on the political economy of colonialism and civil conflict, the model here may also have applications in the study of state formation. The idea of a profit-seeking government lines up with Tilly's (1985) conception of early states. My results suggest that it sometimes may be profitable for such a state to refrain from establishing total sovereignty—to allow some banditry to take place within its sphere of ostensible authority. If the proto-state's main revenue source is the output of its subjects, then eliminating internal appropriation and establishing full sovereignty would indeed be optimal if possible. But if it benefits mainly from control of some fixed resource, such as control of an economically or strategically important waterway, the early state may be better off tolerating some infighting so as to minimize the chance of a serious competitor emerging from within.

My results also speak to the literature on international conflict. The most influential theory of conflict posits that war is the result of bargaining failure (Fearon 1995). Although international relations theorists have made tremendous progress identifying how and why bargaining might break down, the theoretical literature has relatively little to say about the issues that states bargain over. The model here provides a novel explanation of why some territory is more valuable than others—an important question, in light of the centrality of territorial disputes as a cause of war (Goertz and Diehl 1992). In particular, there is an important interaction between natural resource wealth and social fractionalization. Internal divisions should increase the value of resource-rich territory, but decrease the value of territory with relatively few natural resources. Consequently, in the empirical record, we should expect fractionalization to have differential effects on the probability of a crisis breaking out.

There are numerous fruitful directions for further research, particularly since the model I have developed here is easily extensible. One important question, in line with much of the literature on production and appropriation (e.g., Hirshleifer 1991), is how inequality among factions affects the government's policy choices and incentives. Here I have assumed symmetric factions in order to be able to take comparative statics on fractionalization (and to keep the model tractable); incorporating inequality would allow for a more complete analysis of internally stratified societies. Asymmetric extensions to the model may also be useful for examining relations between insurgents and civilians in the midst of civil conflict (Humphreys and Weinstein 2006).

References

Acemoglu, Daron, Simon Johnson and James A. Robinson. 2001. "The Colonial Origins of Comparative Development: An Empirical Investigation." *American Economic Review* 91(5):1369–1401.

Acemoglu, Daron, Thierry Verdier and James A Robinson. 2004. "Kleptocracy and Divide-and-Rule: A Model of Personal Rule." *Journal of the European Economic Association* 2(2-3):162–192.

Aliprantis, Charalambos D. and Kim C. Border. 2006. *Infinite Dimensional Analysis: A Hitchhiker's Guide*. 3 ed. Berlin: Springer.

Azam, Jean-Paul. 2002. "Looting and conflict between ethnoregional groups: Lessons for state formation in Africa." *Journal of Conflict Resolution* 46(1):131–153.

Bartlett, Robert. 1993. *The Making of Europe: Conquest, Colonization and Cultural Change 950–1350*. Princeton University Press.

Burkholder, Mark A. and Lyman L. Johnson. 2015. *Colonial Latin America*. 9 ed. New York: Oxford University Press.

Claessens, Stijn and Luc Laeven. 2003. "Financial development, property rights, and growth." *The Journal of Finance* 58(6):2401–2436.

Collier, Paul and Anke Hoeffler. 2005. "Resource rents, governance, and conflict." *Journal of conflict resolution* 49(4):625–633.

Dal Bó, Ernesto and Pedro Dal Bó. 2011. "Workers, warriors, and criminals: social conflict in general equilibrium." *Journal of the European Economic Association* 9(4):646–677.

Debs, Alexandre. 2007. Divide and Rule and the Media. In *On Dictatorships*. Massachusetts Institute of Technology pp. 84–100.

Dell, Melissa. 2010. "The persistent effects of Peru's mining mita." *Econometrica* 78(6):1863–1903.

Dube, Oeindrila and Juan F Vargas. 2013. "Commodity price shocks and civil conflict: Evidence from Colombia." *The Review of Economic Studies* 80(4):1384–1421.

Elliott, John Huxtable. 2007. *Empires of the Atlantic World: Britain and Spain in America, 1492–1830*. Yale University Press.

Fearon, James D. 1995. "Rationalist explanations for war." *International organization* 49(3):379–414.

Fudenberg, Drew and Jean Tirole. 1991. *Game Theory*. Cambridge, Mass.: MIT Press.

Goertz, Gary and Paul F. Diehl. 1992. *Territorial Changes and International Conflict*. Routledge.

Gordon, Michael R and Bernard E Trainor. 2006. *Cobra II: The inside story of the invasion and occupation of Iraq*. Vintage.

Grossman, Herschel I and Minseong Kim. 1995. "Swords or plowshares? A theory of the security of claims to property." *Journal of Political Economy* 103(6):1275–1288.

Henley, David. 2004. "Conflict, justice, and the stranger-king indigenous roots of colonial rule in Indonesia and elsewhere." *Modern Asian Studies* 38(1):85–144.

Hirshleifer, Jack. 1989. "Conflict and rent-seeking success functions: Ratio vs. difference models of relative success." *Public choice* 63(2):101–112.

Hirshleifer, Jack. 1991. "The paradox of power." *Economics & Politics* 3(3):177–200.

Hodler, Roland. 2006. "The Curse of Natural Resources in Fractionalized Countries." *European Economic Review* 50(6):1367–1386.

Humphreys, Macartan and Jeremy M Weinstein. 2006. "Handling and manhandling civilians in civil war." *American Political Science Review* 100(3):429–447.

Le Billon, Philippe. 2013. *Wars of Plunder: Conflicts, profits and the politics of resources*. Oxford University Press.

Olson, Mancur. 1965. *The Logic of Collective Action*. Cambridge: Harvard University Press.

Reny, Philip J. 1999. "On the existence of pure and mixed strategy Nash equilibria in discontinuous games." *Econometrica* 67(5):1029–1056.

Ross, Michael L. 2004. "What do we know about natural resources and civil war?" *Journal of peace research* 41(3):337–356.

Scammell, Geoffrey V. 1989. *The first imperial age: European overseas expansion 1500-1715*. Routledge.

Scott, James C. 2008. *Weapons of the weak: Everyday forms of peasant resistance*. Yale University Press.

Silve, Arthur. 2017. "Asset Complementarity, Resource Shocks, and the Political Economy of Property Rights." Forthcoming in *Journal of Conflict Resolution*.

Skaperdas, Stergios. 1992. "Cooperation, conflict, and power in the absence of property rights." *The American Economic Review* pp. 720–739.

Tilly, Charles. 1985. War Making and State Making as Organized Crime. In *Bringing the State Back In*, ed. Peter B. Evans, Dietrich Rueschemeyer and Theda Skocpol. Cambridge University Press chapter 5, pp. 169–191.

Wilson, Jon. 2016. *The Chaos of Empire: The British Raj and the Conquest of India*. PublicAffairs.

Zacher, Mark W. 2001. "The territorial integrity norm: International boundaries and the use of force." *International Organization* 55(2):215–250.

A Appendix

Contents

Throughout the appendix, let $\Phi(c) = \sum_i \phi(c_i)$. We have $\log \omega_i(c) = \log \phi(c_i) - \log \Phi(c)$ and thus

$$\begin{aligned}\frac{\partial \log \omega_i(c)}{\partial c_i} &= \frac{\phi'(c_i)}{\phi(c_i)} - \frac{\phi'(c_i)}{\Phi(c)} \\ &= \frac{\phi'(c_i)}{\phi(c_i)}\left(1 - \frac{\phi(c_i)}{\Phi(c)}\right) \\ &= \hat{\phi}'(c_i)(1 - \omega_i(c)),\end{aligned}$$

where $\hat{\phi} = \log \phi$. Because ϕ is strictly increasing and log-concave, $\hat{\phi}' > 0$ and $\hat{\phi}'' \leq 0$.

A.1 Equilibrium Existence and Uniqueness

For the existence and uniqueness results, I consider a more general version of the model presented in the text. I allow groups to be asymmetric in their size and productivities, which entails generalizing each faction i's budget constraint (1) to

$$\frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} = L_i, \tag{12}$$

where $\pi_i^p, \pi_i^r, \pi_i^c, L_i > 0$. In addition, the results here do not depend on Assumption 1.

Let $\Gamma(t)$ denote the subgame that follows the government's selection of t, in which the factions simultaneously decide how to allocate their labor. Let $\sigma_i = (p_i, r_i, c_i)$ be a strategy for faction i in the subgame, and let

$$\Sigma_i = \left\{ (p_i, r_i, c_i) \;\middle|\; \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} = L_i \right\}$$

denote the strategy space. Let $\sigma = (\sigma_1, \ldots, \sigma_N)$ and $\Sigma = \bigtimes_{i=1}^N \Sigma_i$.

I begin by proving that a Nash equilibrium exists in each subgame. The task is complicated by the potential discontinuity of the factions' payoffs, namely at $c = 0$ when $\phi(0) = 0$. I rely on Reny's (1999) conditions for the existence of pure strategy equilibria in a discontinuous game. The key condition is *better-reply security*—informally, that at least one player can assure a strict benefit by deviating from any non-equilibrium strategy profile, even if the other players make slight deviations.

Lemma 1. $\Gamma(t)$ *is better-reply secure.*

Proof. Let $U^t : \Sigma \to \mathbb{R}^N_+$ be the vector payoff function for the factions in $\Gamma(t)$, so that $U^t(\sigma) = (u_1(t,\sigma), \ldots, u_N(t,\sigma))$. Take any convergent sequence in the graph of U^t, call it $(\sigma^k, U^t(\sigma^k)) \to (\sigma^*, U^*)$, such that σ^* is not an equilibrium of $\Gamma(t)$. Because production and the effective tax rate are continuous in (p, r), we have

$$U_i^* = w_i^* \times \bar{\tau}(t, r^*) \times f(p^*)$$

for each i, where $w_i^* \geq 0$ and $\sum_{i=1}^N w_i^* = 1$. I must show there is a player i who can secure a payoff $\bar{U}_i > U_i^*$ at σ^*; i.e., there exists $\bar{\sigma}_i \in \Sigma_i$ such that $u_i(t, \bar{\sigma}_i, \sigma'_{-i}) \geq \bar{U}_i$ for all σ'_{-i} in a neighborhood of σ^*_{-i} (Reny 1999, 1032).

If $N = 1$ or $\Phi(c^*) > 0$, then U^t is continuous in a neighborhood of σ^*, so the conclusion is immediate. If $\bar{\tau}(t, r^*) \times f(p^*) = 0$, then each $U_i^* = 0$ and each faction can assure a strictly greater payoff by deviating to a strategy with positive production, resistance, and competition. For the remaining cases, suppose $N > 1$, $\bar{\tau}(t, r^*) \times f(p^*) > 0$, and $\Phi(c^*) = 0$, the latter of which implies $c^* = 0$ and $\phi(0) = 0$. Since $N > 1$, there is a faction i such that $w_i^* < 1$. Take any $\epsilon \in (0, (1 - w_i^*)/2)$ and any $\delta_1 > 0$ such that

$$\bar{\tau}(t, r') \times f(p') \geq (w_i^* + 2\epsilon) \times \bar{\tau}(t, r^*) \times f(p^*)$$

for all σ' in a δ_1-neighborhood of σ^*. Since $w_i^* + 2\epsilon < 1$ and $\bar{\tau}(t, r) \times f(p)$ is continuous in (p, r), such a δ_1 exists. Then let $\bar{\sigma}_i = (\bar{p}_i, \bar{r}_i, \bar{c}_i)$ be any strategy in a δ_1-neighborhood of σ_i^* such that $\bar{c}_i > 0$. Because $c^*_{-i} = 0$ and ϕ is continuous, there exists $\delta_2 > 0$ such that

$$\omega_i(\bar{c}_i, c'_{-i}) = \frac{\phi(\bar{c}_i)}{\phi(\bar{c}_i) + \sum_{j \in \mathcal{N} \setminus \{i\}} \phi(c'_j)} \geq \frac{w_i^* + \epsilon}{w_i^* + 2\epsilon}$$

for all σ'_{-i} in a δ_2-neighborhood of σ^*_{-i}. Therefore, for all σ'_{-i} in a $\min\{\delta_1, \delta_2\}$-neighborhood of σ^*_{-i}, we have

$$u_i(t, \bar{\sigma}_i, \sigma'_{-i}) \geq (w_i^* + \epsilon) \times \bar{\tau}(t, r^*) \times f(p^*) > U_i^*,$$

establishing the claim. □

The other main condition for equilibrium existence is that each faction's utility function be quasiconcave in its own actions. I prove this by showing that the logarithm of a faction's utility function is concave in its actions.

Lemma 2. $\Gamma(t)$ *is log-concave.*

Proof. Take any (p, r, c) such that $u_i(t, p, r, c) > 0$, and let $P = \sum_j p_j$ and $R = \sum_j r_j$. First, assume $\sum_{j \neq i} \phi(c_j) > 0$, so that u_i is continuously differentiable in (p_i, r_i, c_i). We have

$$\begin{aligned}
\frac{\partial \log u_i(t, p, r, c)}{\partial p_i} &= \frac{1}{P}, \\
\frac{\partial \log u_i(t, p, r, c)}{\partial r_i} &= \frac{-tg'(R)}{1 - tg(R)}, \\
\frac{\partial \log u_i(t, p, r, c)}{\partial c_i} &= \hat{\phi}'(c_i)(1 - \omega_i(c)),
\end{aligned}$$

and therefore

$$\begin{aligned}
\frac{\partial^2 \log u_i(t, p, r, c)}{\partial p_i^2} &= \frac{-1}{P^2} < 0, \\
\frac{\partial^2 \log u_i(t, p, r, c)}{\partial r_i^2} &= \frac{-tg''(R)(1 - tg(R)) - (tg'(R))^2}{(1 - tg(R))^2} \leq 0, \\
\frac{\partial^2 \log u_i(t, p, r, c)}{\partial c_i^2} &= \hat{\phi}''(c_i)(1 - \omega_i(c)) - \hat{\phi}'(c_i)\frac{\partial \omega_i(c)}{\partial c_i} \leq 0, \\
\frac{\partial^2 \log u_i(t, p, r, c)}{\partial p_i \partial r_i} &= \frac{\partial^2 \log u_i(t, p, r, c)}{\partial p_i \partial c_i} = \frac{\partial^2 \log u_i(t, p, r, c)}{\partial r_i \partial c_i} = 0,
\end{aligned}$$

so $\log u_i$ is concave in (p_i, r_i, c_i). By the same token, $\bar{\tau}(t, r) \times f(p)$ is log-concave in (p, r) regardless of whether $\sum_{j \neq i} \phi(c_j) > 0$.

Now assume $\sum_{j \neq i} \phi(c_j) = 0$. Take any (p'_i, r'_i, c'_i) such that $u_i(t, p', r', c') > 0$, where $(p', r', c') = ((p'_i, p_{-i}), (r'_i, r_{-i}), (c'_i, c_{-i}))$. Take any $\alpha \in [0, 1]$, and let $(p^\alpha, r^\alpha, c^\alpha) = \alpha(p, r, c) + (1 - \alpha)(p', r', c')$. If $c_i = c'_i = 0$, then $\omega_i(c^\alpha) = \omega_i(c) = \omega_i(c') = 1/N$ and thus

$$\begin{aligned}
\log u_i(t, p^\alpha, r^\alpha, c^\alpha) &= \log \frac{1}{N} + \log \bar{\tau}(t, r^\alpha) + \log f(p^\alpha) \\
&\geq \log \frac{1}{N} + \alpha \left(\log \bar{\tau}(t, r) + \log f(p)\right) \\
&\quad + (1 - \alpha) \left(\log \bar{\tau}(t, r') + \log f(p')\right) \\
&= \alpha \log u_i(t, p, r, c) + (1 - \alpha) \log u_i(t, p', r', c'),
\end{aligned}$$

where the inequality follows from the log-concavity of $\bar{\tau}(t,r) \times f(p)$ in (p,r). If $c_i > 0$ and $c'_i = 0$, then $\omega_i(c^\alpha) = \omega_i(c) = 1$, $\omega_i(c') = 1/N$, and thus

$$\begin{aligned}
\log u_i(t, p^\alpha, r^\alpha, c^\alpha) &= \log \bar{\tau}(t, r^\alpha) + \log f(p^\alpha) \\
&\geq \alpha \left(\log \bar{\tau}(t,r) + \log f(p)\right) \\
&\quad + (1-\alpha)\left(\log \frac{1}{N} + \log \bar{\tau}(t,r') + \log f(p')\right) \\
&= \alpha \log u_i(t,p,r,c) + (1-\alpha) \log u_i(t,p',r',c').
\end{aligned}$$

The same argument holds in case $c_i = 0$ and $c'_i > 0$. It is easy to see that the same conclusion holds if $c_i > 0$ and $c'_i > 0$, in which case $\omega_i(c^\alpha) = \omega_i(c) = \omega_i(c') = 1$. Therefore, $\log u_i$ is concave in (p_i, r_i, c_i). □

Equilibrium existence follows immediately from the two preceding lemmas.

Proposition 8. $\Gamma(t)$ *has a pure strategy equilibrium.*

Proof. The strategy space Σ is compact, each payoff function u_i is bounded on Σ, and $\Gamma(t)$ is better-reply secure (Lemma 1) and quasiconcave (Lemma 2). Therefore, a pure strategy equilibrium exists (Reny 1999, Theorem 3.1). □

I now turn to the question of uniqueness. I show that although $\Gamma(t)$ may have multiple equilibria, these equilibria are identical in terms of three essential characteristics: total production, $\sum_i p_i$; total resistance, $\sum_i r_i$; and the vector of individual expenditures on internal competition, c.

To prove essential uniqueness, I must characterize the equilibrium more fully than I have up to this point. The following result rules out equilibria in which (1) a faction's share in the internal competition is zero or (2) a faction could raise its share to one by an infinitesimal change in strategy.

Lemma 3. *If* $N > 1$, *then each* $\phi(c_i) > 0$ *in any equilibrium of* $\Gamma(t)$.

Proof. Assume $N > 1$, and let (p, r, c) be a strategy profile of $\Gamma(t)$ in which $c_i = 0$ for some $i \in \mathcal{N}$. The claim holds trivially if $\phi(0) > 0$, so assume $\phi(0) = 0$. If $\Phi(c) > 0$ or $\bar{\tau}(t,r) \times f(p) = 0$, then $u_i(t,p,r,c) = 0$. But i could ensure a strictly positive payoff with any strategy that allocated nonzero labor to production, resistance, and competition, so (p,r,c) is not an equilibrium. Conversely, suppose $\Phi(c) = 0$, which implies $c_j = 0$ for all $j \in \mathcal{N}$, and $\bar{\tau}(t,r) \times f(p) > 0$. Then $u_i(t,p,r,c) = (\bar{\tau}(t,r) \times f(p))/N$. But i could obtain a payoff arbitrarily close to

$\bar{\tau}(t,r) \times f(p)$ by diverting an infinitesimal amount of labor away from production or resistance and into internal competition, so (p,r,c) is not an equilibrium. $\square$

This result is important because it implies the game is continuously differentiable in the neighborhood of any equilibrium. Equilibria can therefore be characterized in terms of first-order conditions.

Lemma 4. *(p',r',c') is an equilibrium of $\Gamma(t)$ if and only if, for each $i \in \mathcal{N}$,*

$$p_i'\left(\pi_i^p \frac{\partial \log f(p')}{\partial p_i} - \mu_i\right) = 0, \tag{13}$$

$$r_i'\left(\pi_i^r \frac{\partial \log \bar{\tau}(t,r')}{\partial r_i} - \mu_i\right) = 0, \tag{14}$$

$$c_i'\left(\pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} - \mu_i\right) = 0, \tag{15}$$

$$\frac{p_i'}{\pi_i^p} + \frac{r_i'}{\pi_i^r} + \frac{c_i'}{\pi_i^c} - L_i = 0, \tag{16}$$

where

$$\mu_i = \max\left\{\pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t,r')}{\partial r_i}, \pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i}\right\}.$$

Proof. In equilibrium, each faction's strategy must solve the constrained maximization problem

$$\begin{aligned}
\max_{p_i,r_i,c_i} \quad & \log u_i(t,p,r,c) \\
\text{s.t.} \quad & \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} - L_i = 0, \\
& p_i \geq 0, r_i \geq 0, c_i \geq 0.
\end{aligned}$$

It follows from Lemma 3 that each u_i is C^1 in (p_i,r_i,c_i) in a neighborhood of any equilibrium. This allows use of the Karush–Kuhn–Tucker conditions to characterize solutions of the above problem. The "only if" direction holds because (13)–(16) are the first-order conditions for the problem and the linearity constraint qualification holds. The "if" direction holds because $\log u_i$ is concave in (p_i,r_i,c_i), per Lemma 2. $\square$

A weak welfare optimality result follows almost immediately from this equilibrium characterization. If (p',r',c') is an equilibrium of $\Gamma(t)$, then there is no other equilibrium (p'', r'', c'') such that $c'' = c'$ and $\bar{\tau}(t, r'') \times f(p'') > \bar{\tau}(t, r') \times f(p')$. In other words, taking as fixed the factions' allocations toward internal competition, there is no inefficient misallocation of labor between production and resistance.

Corollary 1. *If (p', r', c') is an equilibrium of $\Gamma(t)$, then (p', r') solves*

$$\begin{aligned} \max_{p,r} \quad & \log \bar{\tau}(t, r) + \log f(p) \\ s.t. \quad & \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} = L_i - \frac{c_i'}{\pi_i^c}, \qquad && i = 1, \dots, N, \\ & p_i \geq 0, r_i \geq 0, && i = 1, \dots, N. \end{aligned}$$

Proof. This is a C^1 concave maximization problem with linear constraints, so the Karush–Kuhn–Tucker first-order conditions are necessary and sufficient for a solution. The result then follows from Lemma 4. □

I next prove that if post-tax output is weakly greater in one equilibrium of $\Gamma(t)$ than another, then each of the two individual components (production and the factions' total share) is weakly greater. The proof relies on the fact that if $c_i' \leq c_i''$ and $\omega_i(c') \leq \omega_i(c'')$, then

$$\frac{\partial \log \omega_i(c')}{\partial c_i} = \hat{\phi}'(c_i')(1 - \omega_i(c')) \geq \hat{\phi}'(c_i'')(1 - \omega_i(c'')) = \frac{\partial \log \omega_i(c'')}{\partial c_i}.$$

If in addition $\omega_i(c') < \omega_i(c'')$, the inequality is strict.

Lemma 5. *If (p', r', c') and (p'', r'', c'') are equilibria of $\Gamma(t)$ such that $\bar{\tau}(t, r') \times f(p') \geq \bar{\tau}(t, r'') \times f(p'')$, then $\bar{\tau}(t, r') \geq \bar{\tau}(t, r'')$ and $f(p') \geq f(p'')$.*

Proof. Suppose the claim of the lemma does not hold, so there exist equilibria such that $\bar{\tau}(t, r') \times f(p') \geq \bar{\tau}(t, r'') \times f(p'')$ but $\bar{\tau}(t, r') < \bar{\tau}(t, r'')$. Together, these inequalities imply $f(p') > f(p'')$. (The proof in case $\bar{\tau}(t, r') > \bar{\tau}(t, r'')$ and $f(p') < f(p'')$ is analogous.)

I will first establish that $p_i' > 0$ implies $r_i'' = 0$. Per Lemma 4 and the log-concavity of f and $\bar{\tau}$, $p_i' > 0$ implies

$$\pi_i^p \frac{\partial \log f(p'')}{\partial p_i} > \pi_i^p \frac{\partial \log f(p')}{\partial p_i} \geq \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} > \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i}.$$

Therefore, again by Lemma 4, $r_i'' = 0$.

Next, I establish that $\Phi(c'') > \Phi(c')$. Since $f(p') > f(p'')$, there is a faction $i \in \mathcal{N}$ such that $p'_i > p''_i$. As this implies $r''_i = 0$, the budget constraint gives $c''_i > c'_i$. If $\Phi(c'') \leq \Phi(c')$, then $\omega_i(c'') > \omega_i(c')$ and thus by Lemma 4

$$\pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} > \pi_i^c \frac{\partial \log \omega_i(c'')}{\partial c_i} \geq \pi_i^p \frac{\partial \log f(p'')}{\partial p_i} > \pi_i^p \frac{\partial \log f(p')}{\partial p_i}.$$

But this implies $p'_i = 0$, a contradiction. Therefore, $\Phi(c'') > \Phi(c')$.

Using these intermediate results, I can now establish the main claim by contradiction. Since $\bar{\tau}(t, r'') > \bar{\tau}(t, r')$, there is a faction $j \in \mathcal{N}$ such that $r''_j > r'_j$. This implies $p'_j = 0$, so the budget constraint gives $c''_j < c'_j$. Since $\Phi(c'') > \Phi(c')$, this in turn gives $\omega_j(c'') < \omega_j(c')$ and thus

$$\pi_j^c \frac{\partial \log \omega_j(c'')}{\partial c_j} > \pi_j^c \frac{\partial \log \omega_j(c')}{\partial c_j} \geq \pi_j^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j} > \pi_j^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j}.$$

But this implies $r''_j = 0$, a contradiction. □

I can now state and prove the essential uniqueness of the equilibrium of each labor allocation subgame.

Proposition 9. *If (p', r', c') and (p'', r'', c'') are equilibria of $\Gamma(t)$, then $f(p') = f(p'')$, $\bar{\tau}(t, r') = \bar{\tau}(t, r'')$, and $c' = c''$.*

Proof. First I prove that $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$. Suppose not, so that, without loss of generality, $\bar{\tau}(t, r') \times f(p') > \bar{\tau}(t, r'') \times f(p'')$. Then Lemma 5 implies $\bar{\tau}(t, r') \geq \bar{\tau}(t, r'')$ and $f(p') \geq f(p'')$, at least one strictly so, and thus

$$\max\left\{\pi_i^p \frac{\partial \log f(p'')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i}\right\} \geq \max\left\{\pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i}\right\}$$

for all $i \in \mathcal{N}$, strictly so for some $j \in \mathcal{N}$. Since $\bar{\tau}(t, r'') \times f(p'') < \bar{\tau}(t, r') \times f(p')$, it follows from Corollary 1 that the set $\mathcal{N}^+ = \{i \in \mathcal{N} \mid c''_i > c'_i\}$ is nonempty. For any $i \in \mathcal{N}^+$ such that $\omega_i(c'') > \omega_i(c')$,

$$\begin{aligned}
\pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} &> \pi_i^c \frac{\partial \log \omega_i(c'')}{\partial c_i} \\
&\geq \max\left\{\pi_i^p \frac{\partial \log f(p'')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i}\right\} \\
&\geq \max\left\{\pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i}\right\}.
\end{aligned}$$

But this implies $p'_i = r'_i = 0$, contradicting $c''_i > c'_i$. So $\omega_i(c'') \leq \omega_i(c')$ for all $i \in \mathcal{N}^+$. Since $\mathcal{N}^+$ is nonempty and the competition shares are increasing in c_i and sum to one, this can hold only if $\mathcal{N}^+ = \mathcal{N}$ and $\omega_i(c'') = \omega_i(c')$ for all $i \in \mathcal{N}$. This implies

$$\begin{aligned}\pi^c_j \frac{\partial \log \omega_j(c')}{\partial c_j} &\geq \pi^c_j \frac{\partial \log \omega_j(c'')}{\partial c_j} \\ &\geq \max\left\{\pi^p_j \frac{\partial \log f(p'')}{\partial p_j}, \pi^r_j \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j}\right\} \\ &> \max\left\{\pi^p_j \frac{\partial \log f(p')}{\partial p_j}, \pi^r_j \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j}\right\},\end{aligned}$$

which in turn implies $p'_j = r'_j = 0$, contradicting $c''_j > c'_j$. I conclude that $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$ and thus, by Lemma 5, $\bar{\tau}(t, r') = \bar{\tau}(t, r'')$ and $f(p') = f(p'')$.

Next, I prove that $c' = c''$. Suppose not, so $c' \neq c''$. Without loss of generality, suppose $\Phi(c') \geq \Phi(c'')$. Since $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$ yet $c' \neq c''$, by Corollary 1 there exists $i \in \mathcal{N}$ such that $c'_i > c''_i$ and $j \in \mathcal{N}$ such that $c'_j < c''_j$. It follows from $\Phi(c') \geq \Phi(c'')$ that $\omega_j(c') < \omega_j(c'')$ and therefore

$$\begin{aligned}\pi^c_j \frac{\partial \log \omega_j(c')}{\partial c_j} &> \pi^c_j \frac{\partial \log \omega_j(c'')}{\partial c_j} \\ &\geq \max\left\{\pi^p_j \frac{\partial \log f(p'')}{\partial p_j}, \pi^r_j \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j}\right\} \\ &= \max\left\{\pi^p_j \frac{\partial \log f(p')}{\partial p_j}, \pi^r_j \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j}\right\}.\end{aligned}$$

But this implies $p'_j = r'_j = 0$, contradicting $c''_j > c'_j$. □

Proposition 9 allows me to write the equilibrium values of total production, total resistance, and individual conflict allocations as functions of the tax rate. For each $t \in [0, 1]$, let $P^*(t) = P$ if and only if there is an equilibrium (p, r, c) of $\Gamma(t)$ such that $\sum_i p_i = P$. Let the functions $R^*(t)$ and $c^*(t)$, the latter of which is vector-valued, be defined analogously.

The only remaining step to prove the existence of an equilibrium in the full game is to show that an optimal tax rate exists. An important consequence of Proposition 9 is that the optimal tax rate (if one exists) does not depend on the equilibrium that is selected in each labor allocation subgame, since the government's payoff depends only on total production and resistance. The main step toward proving the existence of an optimal tax rate is to show that total production and resistance are continuous in t.

Lemma 6. *P^*, R^*, and c^* are continuous.*

Proof. Define the equilibrium correspondence $E : [0,1] \rightrightarrows \Sigma$ by

$$E(t) = \{(p, r, c) \,|\, (p, r, c) \text{ is an equilibrium of } \Gamma(t)\}.$$

Standard arguments (e.g., Fudenberg and Tirole 1991, 30–32) imply that E has a closed graph.[16] This in turn implies E is upper hemicontinuous, as its codomain, Σ, is compact. Let $F : \Sigma \to \mathbb{R}_+^{N+2}$ be the function defined by $F(p, r, c) = (\sum_i p_i, \sum_i r_i, c)$. Since F is continuous as a function, it is upper hemicontinuous as a correspondence. Then we can write the functions in the lemma as the composition of F and E:

$$(P^*(t), R^*(t), c^*(t)) = \{F(p, r, c) \,|\, (p, r, c) \in E(t)\} = (F \circ E)(t).$$

As the composition of upper hemicontinuous correspondences, (P^*, R^*, c^*) is upper hemicontinuous (Aliprantis and Border 2006, Theorem 17.23). Then, as an upper hemicontinuous correspondence that is single-valued (per Proposition 9), (P^*, R^*, c^*) is continuous as a function. □

Continuity of total production and resistance in the tax rate imply that the government's payoff is continuous in the tax rate, so an equilibrium exists.

Proposition 10. *There is a pure strategy equilibrium.*

Proof. For each labor allocation subgame $\Gamma(t)$, let $\sigma^*(t)$ be a pure strategy equilibrium of $\Gamma(t)$. Proposition 8 guarantees the existence of these equilibria. By Proposition 9, the government's payoff from any $t \in [0, 1]$ is

$$u_G(t, \sigma^*(t)) = t \times g(R^*(t)) \times P^*(t).$$

This expression is continuous in t, per Lemma 6, and therefore attains its maximum on the compact interval $[0, 1]$. A maximizer t^* exists, and the pure strategy profile $(t^*, (\sigma^*(t))_{t\in[0,1]})$ is an equilibrium. □

[16] The only complication in applying the usual argument is that the model is discontinuous at $c = 0$ in case $\phi(0) = 0$. However, by the same arguments as in the proof of Lemma 1, if $\phi(0) = 0$ there cannot be a sequence $(t^k, (p^k, r^k, c^k))$ in the graph of E such that $c^k \to 0$.

A.2 Proof of Propositions 1 and 2

In these and all remaining proofs, I consider the special symmetric case of the model discussed in the text, in which each $\pi_i^p = \pi^p$, $\pi_i^r = \pi^r$, $\pi_i^c = \pi^c$, and $L_i = L/N$. An important initial result for the symmetric case is that in every equilibrium of every labor allocation subgame, every faction spends the same amount on internal competition.

Lemma 7. *If the game is symmetric and* (p, r, c) *is an equilibrium of* $\Gamma(t)$*, then* $c_i = c_j$ *for all* $i, j \in \mathcal{N}$.

Proof. Consider an equilibrium in which $c_i > c_j$. This implies $\omega_i(c) > \omega_j(c)$ and therefore

$$\begin{aligned}
\pi^c \frac{\partial \log \omega_j(c)}{\partial c_j} &> \pi^c \frac{\partial \log \omega_i(c)}{\partial c_i} \\
&\geq \max\left\{\pi^p \frac{\partial \log f(p)}{\partial p_i}, \pi^r \frac{\partial \log \bar{\tau}(t,r)}{\partial r_i}\right\} \\
&= \max\left\{\pi^p \frac{\partial \log f(p)}{\partial p_j}, \pi^r \frac{\partial \log \bar{\tau}(t,r)}{\partial r_j}\right\}.
\end{aligned}$$

Lemma 4 then gives $p_j = r_j = 0$. But since i and j have the same budget constraint, this contradicts $c_i > c_j$. □

In the equilibrium of each labor allocation subgame, total production, total resistance, and the amount each faction spends on conflict (the same for all factions by Lemma 7) solve some subset of the following system of equations. These equations give the conditions for equal marginal benefits per unit of labor across activities, as well as the budget constraint (1). I write them as functions of t as well as the exogenous parameters $\pi = (\pi^p, \pi^r, \pi^c, L, N)$ to allow for comparative statics via implicit differentiation:

$$Q^{pr}(P, R, C; t, \pi) = \pi^p(1 - tg(R)) + \pi^r t P g'(R) = 0, \tag{17}$$

$$Q^{pc}(P, R, C; t, \pi) = \frac{\pi^p}{P} - \frac{N-1}{N}\pi^c \hat{\phi}'(C) = 0, \tag{18}$$

$$Q^{rc}(P, R, C; t, \pi) = \frac{\pi^r t g'(R)}{1 - tg(R)} + \frac{N-1}{N}\pi^c \hat{\phi}'(C) = 0, \tag{19}$$

$$Q^{b}(P, R, C; t, \pi) = L - \frac{P}{\pi^p} - \frac{R}{\pi^r} - \frac{NC}{\pi^c} = 0. \tag{20}$$

The condition (19) is redundant when (17) and (18) both hold, but I use it later in the plunder model.

The quantities defined in Propositions 1 and 2 are as follows. $(\bar{P}_0, \bar{c}_0)$ is the solution to the system

$$Q^{pc}(\bar{P}_0, 0, \bar{c}_0; t, \pi) = \frac{\pi^p}{\bar{P}_0} - \frac{N-1}{N}\pi^c \hat{\phi}'(\bar{c}_0) = 0, \tag{21}$$

$$Q^{b}(\bar{P}_0, 0, \bar{c}_0; t, \pi) = L - \frac{\bar{P}_0}{\pi^p} - \frac{N\bar{c}_0}{\pi^c} = 0. \tag{22}$$

$(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t))$ is the solution to the system

$$Q^{pr}(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = \pi^p(1 - tg(\tilde{R}_1(t)) + \pi^r t\tilde{P}_1(t)g'(\tilde{R}_1(t)) = 0, \tag{23}$$

$$Q^{pc}(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = \frac{\pi^p}{\tilde{P}_1(t)} - \frac{N-1}{N}\pi^c \hat{\phi}'(\tilde{c}_1(t)) = 0, \tag{24}$$

$$Q^{b}(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = L - \frac{\tilde{P}_1(t)}{\pi^p} - \frac{\tilde{R}_1(t)}{\pi^r} - \frac{N\tilde{c}_1(t)}{\pi^c} = 0. \tag{25}$$

$(\tilde{P}_2(t), \tilde{R}_2(t))$ is the solution to the system

$$Q^{pr}(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi) = \pi^p(1 - tg(\tilde{R}_2(t)) + \pi^r t\tilde{P}_2(t)g'(\tilde{R}_2(t)) = 0, \tag{26}$$

$$Q^{b}(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi) = L - \frac{\tilde{P}_2(t)}{\pi^p} - \frac{\tilde{R}_2(t)}{\pi^r} = 0. \tag{27}$$

The first cutpoint tax rate is

$$\hat{t}_0 = \frac{\pi^p}{\pi^p - \pi^r \bar{P}_0 g'(0)}. \tag{28}$$

Lemma 8 below shows that $\bar{P}_0 > 0$ and therefore, since $g'(0) < 0$, that $\hat{t}_0 < 1$. The second cutpoint tax rate is

$$\hat{t}_1 = \frac{\pi^p}{\pi^p g(\bar{R}_1) - \pi^r \bar{P}_1 g'(\bar{R}_1)}, \tag{29}$$

where

$$\bar{P}_1 = \frac{N}{N-1}\frac{\pi^p}{\pi^c}\frac{1}{\hat{\phi}'(0)}, \tag{30}$$

$$\bar{R}_1 = \pi^r\left(L - \frac{\bar{P}_1}{\pi^p}\right). \tag{31}$$

The next three lemmas give conditions on the tax rate under which there is positive production, resistance, and internal competition in the equilibrium of the labor allocation subgame. Jointly, these lemmas constitute the bulk of the proof of Propositions 1 and 2. The proofs rely on the following equalities:

$$\begin{aligned}
\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, 0)}{\partial r_i} &= -\pi^r \frac{\hat{t}_0 g'(0)}{1 - \hat{t}_0} &&= \frac{\pi^p}{\bar{P}_0}, \\
\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, (\bar{R}_1/N)\mathbf{1}_N)}{\partial r_i} &= -\pi^r \frac{\hat{t}_1 g'(\bar{R}_1)}{1 - \hat{t}_1 g(\bar{R}_1)} &&= \frac{\pi^p}{\bar{P}_1}
\end{aligned}$$

for all $i \in \mathcal{N}$, where $\mathbf{1}_N$ is the N-vector each of whose elements equals one.

Lemma 8. *If the game is symmetric, Assumption 1 holds, and (p, r, c) is an equilibrium of $\Gamma(t)$, then $0 < \sum_i p_i \leq \bar{P}_0 < \pi^p L$.*

Proof. Assumption 1 implies

$$Q^{pc}(\pi^p L, 0, 0; 0, \pi) = \frac{1}{L} - \frac{N-1}{N} \pi^c \hat{\phi}'(0) < 0.$$

Since Q^{pc} is decreasing in P and weakly increasing in C, this gives $\bar{P}_0 < \pi^p L$.

Let $P = \sum_i p_i$, and suppose $P > \bar{P}_0$. The budget constraint and Lemma 7 then give $c_i = C < \bar{c}_0$ for each $i \in \mathcal{N}$. But then we have

$$\pi^c \frac{\partial \log \omega_i(c)}{\partial c_i} \geq \frac{N-1}{N} \pi^c \hat{\phi}'(\bar{c}_0) = \frac{\pi^p}{\bar{P}_0} > \pi^p \frac{\partial \log f(p)}{\partial p_i}$$

for each $i \in \mathcal{N}$. By Lemma 4, this implies each $p_i = 0$, a contradiction. Therefore, $P \leq \bar{P}_0$.

Finally, since $P = 0$ implies each $u_i(t, p, r, c) = 0$, but any faction can assure itself a positive payoff with any $(p_i, r_i, c_i) \gg 0$, in equilibrium $P > 0$. □

Lemma 9. *If the game is symmetric, Assumption 1 holds, and (p, r, c) is an equilibrium of $\Gamma(t)$, then $\sum_i r_i > 0$ if and only if $t > \hat{t}_0$.*

Proof. Let $P = \sum_i p_i$ and $R = \sum_i r_i$. To prove the "if" direction, suppose $t > \hat{t}_0$ and $R = 0$. Since $P \leq \bar{P}_0$, this implies each $c_i = C > 0$; the first-order conditions of Lemma 4 then give $P = \bar{P}_0$ and $C = \bar{c}_0$. It follows that

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, r)}{\partial r_i} = \frac{\pi^p}{\bar{P}_0} = \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

This implies each $p_i = 0$, a contradiction.

To prove the "only if" direction, suppose $t \leq \hat{t}_0$ and $R > 0$. For each $i \in \mathcal{N}$,

$$\pi^r \frac{\partial \log \bar{\tau}(t,r)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, 0)}{\partial r_i} = \frac{\pi^p}{\bar{P}_0} \leq \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

This implies each $r_i = 0$, a contradiction. □

Lemma 10. *If the game is symmetric and Assumption 1 holds, then $\hat{t}_1 > \hat{t}_0$. If, in addition, (p, r, c) is an equilibrium of $\Gamma(t)$, then each $c_i > 0$ if and only if $t < \hat{t}_1$.*

Proof. To prove that $\hat{t}_1 > \hat{t}_0$, note that

$$\bar{P}_1 = \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(0)} \leq \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(\bar{c}_0)} = \bar{P}_0$$

by log-concavity of ϕ. This implies $\bar{R}_1 > 0$, so $g(\bar{R}_1) < g(0) = 1$ and $g'(0) \leq g'(\bar{R}_1) < 0$. Therefore,

$$\pi^p - \pi^r \bar{P}_0 g'(0) > \pi^p g(\bar{R}_1) - \pi^r \bar{P}_1 g'(\bar{R}_1) > 0,$$

which implies $\hat{t}_1 > \hat{t}_0$.

Let $P = \sum_i p_i$ and $R = \sum_i r_i$. To prove the "if" direction of the second statement, suppose $t \geq \hat{t}_1$ and some $c_i > 0$. By Lemma 7, $c_j = c_i = C > 0$ for each $j \in \mathcal{N}$. Since $P > 0$ by Lemma 8, the first-order conditions give

$$P = \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(C)} \geq \bar{P}_1.$$

The budget constraint then gives $R < \bar{R}_1$ and thus

$$\pi^r \frac{\partial \log \bar{\tau}(t,r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, (\bar{R}_1/N)\mathbf{1}_N)}{\partial r_i} = \frac{\pi^p}{\bar{P}_1} \geq \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

But this implies each $p_i = 0$, a contradiction.

To prove the "only if" direction, suppose $t < \hat{t}_1$ and each $c_i = 0$. The first-order conditions then give $P \leq \bar{P}_1$, so $R \geq \bar{R}_1 > 0$ by the budget constraint. This in turn gives

$$\pi^p \frac{\partial \log f(p)}{\partial p_i} = \frac{\pi^p}{P} \geq \frac{\pi^p}{\bar{P}_1} \geq \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(t,r)}{\partial r_i}$$

for each $i \in \mathcal{N}$. But this implies each $r_i = 0$, a contradiction. □

The last thing we need to prove the propositions is how the labor allocations change with the tax rate when $t > \hat{t}_0$.

Lemma 11. *Let the game be symmetric and Assumption 1 hold. For all $t \in (\hat{t}_0, \hat{t}_1)$,*

$$\frac{d\tilde{P}_1(t)}{dt} = \frac{-(N-1)\pi^p\pi^c\hat{\phi}''(\tilde{c}_1(t))}{N\pi^r t\Delta_1(t)} \leq 0, \tag{32}$$

$$\frac{d\tilde{R}_1(t)}{dt} = \frac{-\pi^p\left(N\pi^p/\pi^c\tilde{P}_1(t)^2 - (N-1)\pi^c\hat{\phi}''(\tilde{c}_1(t))/N\pi^p\right)}{t\Delta_1(t)} > 0, \tag{33}$$

$$\frac{d\tilde{c}_1(t)}{dt} = \frac{(\pi^p)^2}{\pi^r t\tilde{P}_1(t)^2\Delta_1(t)} < 0, \tag{34}$$

where

$$\begin{aligned}\Delta_1(t) = &\left(\pi^p tg'(\tilde{R}_1(t)) - \pi^r t\tilde{P}_1(t)g''(\tilde{R}_1(t))\right)\left(\frac{N\pi^p}{\pi^c\tilde{P}_1(t)^2} - \frac{N-1}{N}\frac{\pi^c}{\pi^p}\hat{\phi}''(\tilde{c}_1(t))\right)\\ &- \frac{N-1}{N}\pi^c tg'(\tilde{R}_1(t))\hat{\phi}''(\tilde{c}_1(t))\\ &< 0.\end{aligned} \tag{35}$$

For all $t > \hat{t}_1$,

$$\frac{d\tilde{P}_2(t)}{dt} = \frac{-\pi^p}{\pi^r t\Delta_2(t)} < 0, \tag{36}$$

$$\frac{d\tilde{R}_2(t)}{dt} = \frac{1}{t\Delta_2(t)} > 0, \tag{37}$$

where

$$\Delta_2(t) = \frac{\pi^r}{\pi^p}t\tilde{P}_2(t)g''(\tilde{R}_2(t)) - 2tg'(\tilde{R}_2(t)) > 0. \tag{38}$$

Proof. Throughout the proof, let $\eta = (N-1)/N$.

First consider $t \in (\hat{t}_0, \hat{t}_1)$. To reduce clutter in what follows, I omit the evaluation point $(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi)$ from all partial derivative expressions. The Jacobian of the system of equations that defines $(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t))$ is

$$\mathbf{J}_1(t) = \begin{bmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R & \partial Q^{pr}/\partial C \\ \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial R & \partial Q^{pc}/\partial C \\ \partial Q^b/\partial P & \partial Q^b/\partial R & \partial Q^b/\partial C \end{bmatrix}$$

$$= \begin{bmatrix} \pi^r t g'(\tilde{R}_1(t)) & \pi^r t \tilde{P}_1(t) g''(\tilde{R}_1(t)) - \pi^p t g'(\tilde{R}_1(t)) & 0 \\ -\pi^p / \tilde{P}_1(t)^2 & 0 & -\eta \pi^c \hat{\phi}''(\tilde{c}_1(t)) \\ -1/\pi^p & -1/\pi^r & -N/\pi^c \end{bmatrix}.$$

It is easy to verify that $|\mathbf{J}_1(t)| = \Delta_1(t) < 0$. Notice that

$$\begin{aligned} \frac{\partial Q^{pr}}{\partial t} &= \pi^r \tilde{P}_1(t) g'(\tilde{R}_1(t)) - \pi^p g(\tilde{R}_1(t)) \\ &= \pi^r \left(-\frac{\pi^p (1 - t g(\tilde{R}_1(t)))}{\pi^r t g'(\tilde{R}_1(t))} \right) g'(\tilde{R}_1(t)) - \pi^p g(\tilde{R}_1(t)) \\ &= -\frac{\pi^p}{t}. \end{aligned}$$

Then, by the implicit function theorem and Cramer's rule,

$$\begin{aligned} \frac{d\tilde{P}_1(t)}{dt} &= \frac{\begin{vmatrix} -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial R & \partial Q^{pr}/\partial C \\ -\partial Q^{pc}/\partial t & \partial Q^{pc}/\partial R & \partial Q^{pc}/\partial C \\ -\partial Q^{b}/\partial t & \partial Q^{b}/\partial R & \partial Q^{b}/\partial C \end{vmatrix}}{|\mathbf{J}_1(t)|} \\ &= \frac{-\eta \pi^p \pi^c \hat{\phi}''(\tilde{c}_1(t))}{\pi^r t \Delta_1(t)} \\ &\leq 0, \\ \frac{d\tilde{R}_1(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial C \\ \partial Q^{pc}/\partial P & -\partial Q^{pc}/\partial t & \partial Q^{pc}/\partial C \\ \partial Q^{b}/\partial P & -\partial Q^{b}/\partial t & \partial Q^{b}/\partial C \end{vmatrix}}{|\mathbf{J}_1(t)|} \\ &= \frac{-\pi^p \left(N\pi^p/\pi^c \tilde{P}_1(t)^2 - \eta \pi^c \hat{\phi}''(\tilde{c}_1(t))/\pi^p \right)}{t \Delta_1(t)} \\ &> 0, \\ \frac{d\tilde{c}_1(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R & -\partial Q^{pr}/\partial t \\ \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial R & -\partial Q^{pc}/\partial t \\ \partial Q^{b}/\partial P & \partial Q^{b}/\partial R & -\partial Q^{b}/\partial t \end{vmatrix}}{|\mathbf{J}_1(t)|} \\ &= \frac{(\pi^p)^2}{\pi^r t \tilde{P}_1(t)^2 \Delta_1(t)} \\ &< 0, \end{aligned}$$

as claimed.

Now consider $t > \hat{t}_1$. Again to reduce clutter in what follows, I omit the evaluation point $(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi)$ from all partial derivative expressions. The Jacobian of the system of equations that defines $(\tilde{P}_2(t), \tilde{R}_2(t))$ is

$$\begin{aligned}
\mathbf{J}_2(t) &= \begin{bmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R \\ \partial Q^{b}/\partial P & \partial Q^{b}/\partial R \end{bmatrix} \\
&= \begin{bmatrix} \pi^r t g'(\tilde{R}_2(t)) & \pi^r t \tilde{P}_2(t) g''(\tilde{R}_2(t)) - \pi^p t g'(\tilde{R}_2(t)) \\ -1/\pi^p & -1/\pi^r \end{bmatrix}.
\end{aligned}$$

It is easy to verify that $|\mathbf{J}_2(t)| = \Delta_2(t) > 0$. As before, $\partial Q^{pr}/\partial t = -\pi^p/t$. So by the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{d\tilde{P}_2(t)}{dt} &= \frac{\begin{vmatrix} -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial R \\ -\partial Q^{b}/\partial t & \partial Q^{b}/\partial R \end{vmatrix}}{|\mathbf{J}_2(t)|} \\
&= \frac{-\pi^p}{\pi^r t \Delta_2(t)} \\
&< 0, \\
\frac{d\tilde{R}_2(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & -\partial Q^{pr}/\partial t \\ \partial Q^{b}/\partial P & -\partial Q^{b}/\partial t \end{vmatrix}}{|\mathbf{J}_2(t)|} \\
&= \frac{1}{t \Delta_2(t)} \\
&> 0,
\end{aligned}$$

as claimed. □

The proofs of Propositions 1 and 2 follow almost immediately from these lemmas. I prove them jointly.

Proposition 1 (Baseline Equilibrium)**.** *There is a tax rate $\hat{t}_0 \in (0, 1)$ such that $\sum_i r_i = 0$ in every equilibrium of the labor allocation subgame if and only if $t \leq \hat{t}_0$. Every subgame with $t \leq \hat{t}_0$ has the same unique equilibrium, in which $\sum_i p_i = \bar{P}_0 > 0$ and each $c_i = \bar{c}_0 > 0$.*

Proposition 2 (Resistance Equilibrium)**.** *There is a tax rate $\hat{t}_1 > \hat{t}_0$ such that in every equilibrium of the labor allocation subgame with tax rate t:*

- *If $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = \tilde{P}_1(t) > 0$ (weakly decreasing in t), $\sum_i r_i = \tilde{R}_1(t) > 0$ (strictly increasing), and each $c_i = \tilde{c}_1(t) > 0$ (strictly decreasing).*

- *If $t \geq \hat{t}_1$, then $\sum_i p_i = \tilde{P}_2(t) > 0$ (strictly decreasing in t), $\sum_i r_i = \tilde{R}_2(t) > 0$ (strictly increasing), and each $c_i = 0$.*

Proof. For fixed t, every equilibrium of $\Gamma(t)$ has the same total production, total resistance, and individual competition allocations, per Proposition 9. Consider any $t \in [0, 1]$ and let (p, r, c) be an equilibrium of $\Gamma(t)$.

If $t \leq \hat{t}_0$, then $\sum_i p_i = P > 0$, $\sum_i r_i = 0$, and each $c_i = C > 0$ by Lemmas 8–10. The first-order conditions (Lemma 4) imply that P and C solve $Q^{pc}(P, 0, C; t, \pi) = Q^b(P, 0, C; t, \pi) = 0$; therefore, $P = \bar{P}_0$ and $C = \bar{c}_0$. Since each $r_i = 0$, each $p_i = \pi^p(L/N - \bar{c}_0/\pi^c) = \bar{P}_0/N$, so the equilibrium is unique.

Similarly, if $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = P > 0$, $\sum_i r_i = R > 0$, and each $c_i = C > 0$ by Lemmas 8–10. The first-order conditions then imply that these solve the system (17)–(20); therefore, $P = \tilde{P}_1(t)$, $R = \tilde{R}_1(t)$, and $C = \tilde{c}_1(t)$. The comparative statics on $\tilde{P}_1$, $\tilde{R}_1$, and $\tilde{c}_1$ follow from Lemma 11.

Finally, if $t \geq \hat{t}_1$, then $\sum_i p_i = P > 0$, $\sum_i r_i = R > 0$, and each $c_i = 0$ by Lemmas 8–10. The first-order conditions then imply that P and R solve $Q^{pr}(P, R, 0; t, \pi) = Q^b(P, R, 0; t, \pi) = 0$; therefore, $P = \tilde{P}_2(t)$ and $R = \tilde{R}_2(t)$. The comparative statics on $\tilde{P}_2$ and $\tilde{R}_2$ follow from Lemma 11. □

A.3 Proof of Proposition 3

Proposition 3 (Optimal Tax Rate)**.** *There is an equilibrium in which the government chooses the greatest tax rate that engenders no resistance, $t = \hat{t}_0$. If g or ϕ is strictly log-concave, this is the unique equilibrium tax rate.*

Proof. As in the proof of Lemma 11, let $\eta = (N - 1)/N$.

For each $t \in [0, 1]$, fix an equilibrium $(p(t), r(t), c(t))$ of $\Gamma(t)$. By Propositions 1, 2, and 9, the government's induced utility function is

$$u_G^*(t) = u_G(t, p(t), r(t), c(t)) = \begin{cases} t\bar{P}_0 & t \leq \hat{t}_0, \\ tg(\tilde{R}_1(t))\tilde{P}_1(t) & \hat{t}_0 < t < \hat{t}_1, \\ tg(\tilde{R}_2(t))\tilde{P}_2(t) & t \geq \hat{t}_1. \end{cases}$$

It is immediate from the above expression that $u_G^*(t) < u_G^*(\hat{t}_0)$ for all $t < \hat{t}_0$.

Now consider $t \in (\hat{t}_0, \hat{t}_1)$. By Lemma 11,

$$\begin{aligned}
\frac{du_G^*(t)}{dt} &= g(\tilde{R}_1(t))\tilde{P}_1(t) + tg'(\tilde{R}_1(t))\frac{d\tilde{R}_1(t)}{dt}\tilde{P}_1(t) + tg(\tilde{R}_1(t))\frac{d\tilde{P}_1(t)}{dt} \\
&= g(\tilde{R}_1(t))\tilde{P}_1(t) - \frac{\pi^p g'(\tilde{R}_1(t))\tilde{P}_1(t)\left(N\pi^p/\pi^c\tilde{P}_1(t)^2 - \eta\pi^c\hat{\phi}''(\tilde{c}_1(t))/\pi^p\right)}{\Delta_1(t)} \\
&\quad - \frac{\eta\pi^p\pi^c g(\tilde{R}_1(t))\hat{\phi}''(\tilde{c}_1(t))}{\pi^r\Delta_1(t)},
\end{aligned}$$

where $\Delta_1(t)$ is defined by (35). To reduce clutter in what follows, let $\tilde{P} = \tilde{P}_1(t)$, $\tilde{R} = \tilde{R}_1(t)$, and $\tilde{c} = \tilde{c}_1(t)$. Since $\Delta_1(t) < 0$, the sign of the above expression is the same as that of

$$\begin{aligned}
& g'(\tilde{R})\tilde{P}\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) + \frac{\eta\pi^p\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})}{\pi^r} - g(\tilde{R})\tilde{P}\Delta_1(t) \\
&= \tilde{P}g'(\tilde{R})\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) + \frac{\eta\pi^p\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})}{\pi^r} \\
&\quad - \tilde{P}g(\tilde{R})\left(\pi^p tg'(\tilde{R}) - \pi^r t\tilde{P}g''(\tilde{R})\right)\left(\frac{N\pi^p}{\pi^c\tilde{P}^2} - \eta\frac{\pi^c}{\pi^p}\hat{\phi}''(\tilde{c})\right) \\
&\quad + \eta\pi^c t\tilde{P}g(\tilde{R})g'(\tilde{R})\hat{\phi}''(\tilde{c}) \\
&= \tilde{P}\left(g'(\tilde{R}) - \frac{g(\tilde{R})\left(\pi^p tg'(\tilde{R}) - \pi^r t\tilde{P}g''(\tilde{R})\right)}{\pi^p}\right)\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) \\
&\quad + \eta\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})\left(\frac{\pi^p}{\pi^r} + t\tilde{P}g'(\tilde{R})\right) \\
&= \frac{\tilde{P}}{g'(\tilde{R})}(1 - tg(\tilde{R}))\left(g'(\tilde{R})^2 - g(\tilde{R})g''(\tilde{R})\right)\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) \\
&\quad + \eta\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})\left(\frac{\pi^p}{\pi^r}tg(\tilde{R})\right).
\end{aligned}$$

The first term is weakly negative, strictly so if g is strictly log-concave. The second term is weakly negative, strictly so if ϕ is strictly log-concave. Therefore, $du_G^*(t)/dt \leq 0$ for all $t \in (\hat{t}_0, \hat{t}_1)$, strictly so if g or ϕ is strictly log-concave. This implies $u_G^*(\hat{t}_0) \geq u_G^*(t)$ for all $t \in (\hat{t}_0, \hat{t}_1]$, strictly so if g or ϕ is strictly log-concave.

Finally, consider $t > \hat{t}_1$. Again by Lemma 11,

$$\frac{du_G^*(t)}{dt} = g(\tilde{R}_2(t))\tilde{P}_2(t) + tg'(\tilde{R}_2(t))\frac{d\tilde{R}_2(t)}{dt}\tilde{P}_2(t) + tg(\tilde{R}_2(t))\frac{d\tilde{P}_2(t)}{dt}$$

$$= g(\tilde{R}_2(t))\tilde{P}_2(t) + \frac{g'(\tilde{R}_2(t))\tilde{P}_2(t)}{\Delta_2(t)} - \frac{\pi^p g(\tilde{R}_2(t))}{\pi^r \Delta_2(t)},$$

where $\Delta_2(t)$ is defined by (38). To reduce clutter in what follows, let $\tilde{P} = \tilde{P}_2(t)$ and $\tilde{R} = \tilde{R}_2(t)$. Since $\Delta_2(t) > 0$, the sign of the above expression is the same as that of

$$\begin{aligned}
&\tilde{P}g(\tilde{R})\Delta_2(t) + \tilde{P}g'(\tilde{R}) - \frac{\pi^p g(\tilde{R})}{\pi^r} \\
&\quad = \tilde{P}g(\tilde{R})\left(\frac{\pi^r t\tilde{P}g''(\tilde{R})}{\pi^p} - 2tg'(\tilde{R})\right) + \tilde{P}g'(\tilde{R}) - \frac{\pi^p g(\tilde{R})}{\pi^r} \\
&\quad = \frac{\pi^p}{\pi^r t g'(\tilde{R})^2}\left((1 - tg(\tilde{R}))^2\left(g(\tilde{R})g''(\tilde{R}) - g'(\tilde{R})^2\right) - \left(tg(\tilde{R}g'(\tilde{R}))\right)^2\right) \\
&\quad < \frac{\pi^p(1 - tg(\tilde{R}))^2\left(g(\tilde{R})g''(\tilde{R}) - g'(\tilde{R})^2\right)}{\pi^r t g'(\tilde{R})^2} \\
&\quad \leq 0.
\end{aligned}$$

Therefore, $u^*_G(\hat{t}_0) \geq u^*_G(\hat{t}_1) > u^*_G(t)$ for all $t > \hat{t}_1$.

Combining these findings, $u^*_G(\hat{t}_0) \geq u^*_G(t)$ for all $t \in [0,1] \setminus \{\hat{t}_0\}$, strictly so if g or ϕ is strictly log-concave. Therefore, there is an equilibrium in which $t = \hat{t}_0$, and every equilibrium has this tax rate if g or ϕ is strictly log-concave. □

A.4 Proof of Remarks 1 and 2

The comparative statics in both remarks come from the same system of equations, so I prove them jointly.

Remark 1. *Total production in the baseline equilibrium, $\bar{P}_0$, is strictly decreasing in the number of factions, N.*

Remark 2. *Total production in the baseline equilibrium, $\bar{P}_0$, strictly decreases with a marginal increase in competition effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

Proof. I will treat N as if it were continuous in order to obtain comparative statics by implicit differentiation. Throughout the proof I write $\bar{P}_0$ and $\bar{c}_0$ as functions of (N, π^c).

Recall that $(\bar{P}_0(N,\pi^c), \bar{c}_0(N,\pi^c))$ is defined as the solution to (21) and (22). To reduce clutter in what follows, I omit the evaluation point $(\bar{P}_0(N,\pi^c), 0, \bar{c}_0(N,\pi^c); t, \pi)$ from all partial derivative expressions. The Jacobian of the system is

$$\mathbf{J}_0 = \begin{bmatrix} \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial C \\ \partial Q^b/\partial P & \partial Q^b/\partial C \end{bmatrix} = \begin{bmatrix} -\pi^p/\bar{P}_0(N,\pi^c)^2 & -(N-1)\pi^c\hat{\phi}''(\bar{c}_0(N,\pi^c))/N \\ -1/\pi^p & -N/\pi^c \end{bmatrix},$$

with determinant

$$|\mathbf{J}_0| = \frac{N\pi^p}{\pi^c\bar{P}_0(N,\pi^c)^2} - \frac{N-1}{N}\frac{\pi^c}{\pi^p}\hat{\phi}''(\bar{c}_0(N,\pi^c)) > 0.$$

By the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{\partial \bar{P}_0(N,\pi^c)}{\partial N} &= \frac{\begin{vmatrix} -\partial Q^{pc}/\partial N & \partial Q^{pc}/\partial C \\ -\partial Q^b/\partial N & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_0|} \\
&= \frac{\begin{vmatrix} \pi^c\hat{\phi}'(\bar{c}_0(N,\pi^c))/N^2 & -(N-1)\pi^c\hat{\phi}''(\bar{c}_0(N,\pi^c))/N \\ \bar{c}_0(N,\pi^c)/\pi^c & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_0|} \\
&= \frac{1}{|\mathbf{J}_0|}\left(\frac{N-1}{N}\bar{c}_0(N,\pi^c)\hat{\phi}''(\bar{c}_0(N,\pi^c)) - \frac{\hat{\phi}'(\bar{c}_0(N,\pi^c))}{N}\right) \\
&< 0,
\end{aligned}$$

as claimed. Similarly,

$$\begin{aligned}
\frac{\partial \bar{P}_0(N,\pi^c)}{\partial \pi^c} &= \frac{\begin{vmatrix} -\partial Q^{pc}/\partial \pi^c & \partial Q^{pc}/\partial C \\ -\partial Q^b/\partial \pi^c & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_0|} \\
&= \frac{\begin{vmatrix} (N-1)\hat{\phi}'(\bar{c}_0(N,\pi^c))/N & -(N-1)\pi^c\hat{\phi}''(\bar{c}_0(N,\pi^c))/N \\ -N\bar{c}_0(N,\pi^c)/(\pi^c)^2 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_0|} \\
&= -\frac{N-1}{\pi^c|\mathbf{J}_0|}\left(\hat{\phi}'(\bar{c}_0(N,\pi^c)) + \bar{c}_0(N,\pi^c)\hat{\phi}''(\bar{c}_0(N,\pi^c))\right),
\end{aligned}$$

which is negative if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0(N,\pi^c)$. □

I also prove the claim in footnote 10.

Lemma 12. *Let $\theta, \lambda > 0$. If $\phi(C) = \theta \exp(\lambda C)$, then the incentive effect outweighs the labor-saving effect for all $C \geq 0$. If $\phi(C) = \theta C^{\lambda}$, then the incentive and labor-saving effects are exactly offsetting for all $C > 0$.*

Proof. First consider the difference contest success function, $\phi(C) = \theta \exp(\lambda C)$. Then $\hat{\phi}(C) = \log \theta + \lambda C$, $\hat{\phi}'(C) = \lambda$, and $\hat{\phi}''(C) = 0$ for all $C \geq 0$. Therefore,

$$\hat{\phi}'(C) + C\hat{\phi}''(C) = \lambda > 0.$$

Now consider the ratio contest success function, $\phi(C) = \theta C^{\lambda}$. Then $\hat{\phi}(C) = \log \theta + \lambda \log C$, $\hat{\phi}'(C) = \lambda / C$, and $\hat{\phi}''(C) = -\lambda / C^2$. Therefore,

$$\hat{\phi}'(C) + C\hat{\phi}''(C) = -\frac{\lambda}{C^2} + C\frac{\lambda}{C} = 0. \qquad \square$$

A.5 Proof of Proposition 4

Proposition 4. *The government's equilibrium payoff is strictly decreasing in the number of factions, N. It strictly decreases with a marginal increase in competition effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

Proof. By Proposition 3, the government's equilibrium payoff is

$$\hat{t}_0 \bar{P}_0 = \frac{\pi^p \bar{P}_0}{\pi^p - \pi^r \bar{P}_0 g'(0)} = \frac{\pi^p}{(\pi^p / \bar{P}_0) - \pi^r g'(0)}.$$

This expression is strictly increasing in $\bar{P}_0$. Since N and π^c only enter through the equilibrium value of $\bar{P}_0$, the claim follows from Remark 1 and Remark 2. $\square$

A.6 Proof of Proposition 5

Let $\Gamma_X(t)$ denote the labor allocation subgame with tax rate t in the plunder model. The quantities defined in Proposition 5 are as follows. $(\tilde{R}_X(t), \tilde{c}_X(t))$ is the solution to the system

$$Q^{pr}(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi) = \frac{\pi^r t g'(\tilde{R}_X(t))}{1 - t g(\tilde{R}_X(t))} + \frac{N-1}{N} \pi^c \hat{\phi}'(\tilde{c}_X(t)) = 0, \tag{39}$$

$$Q^b(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi) = L - \frac{\tilde{R}_X(t)}{\pi^r} - \frac{N \tilde{c}_X(t)}{\pi^c} = 0. \tag{40}$$

The cutpoint tax rates are

$$\hat{t}_0^X = \frac{\eta\pi^c\hat{\phi}'(\pi^c L/N)}{\eta\pi^c\hat{\phi}'(\pi^c L/N) - \pi^r g'(0)}, \tag{41}$$

$$\hat{t}_1^X = \frac{\eta\pi^c\hat{\phi}'(0)}{\eta\pi^c g(\pi^r L)\hat{\phi}'(0) - \pi^r g'(\pi^r L)}, \tag{42}$$

where $\eta = (N-1)/N$. Similar to the cutpoints in the original model,

$$\begin{aligned}
\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} &= \eta\pi^c\hat{\phi}'(\pi^c L/N) = \pi^c \frac{\partial \log \omega_i((\pi^c L/N)\mathbf{1}_N)}{\partial c_i}, \\
\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1^X, (\pi^r L/N)\mathbf{1}_N)}{\partial r_i} &= \eta\pi^c\hat{\phi}'(0) \qquad = \pi^c \frac{\partial \log \omega_i(0)}{\partial c_i}
\end{aligned}$$

for each $i \in \mathcal{N}$.

Proposition 5. *In the plunder model, every labor allocation subgame has a unique equilibrium. There exists a tax rate $\hat{t}_0^X \in (0,1)$ such that each $r_i = 0$ in equilibrium if and only if $t \leq \hat{t}_0^X$. There exists $\hat{t}_1^X > \hat{t}_0^X$ such that each $c_i = 0$ in equilibrium if and only if $t \geq \hat{t}_1^X$. For $t \in (\hat{t}_1^X, \hat{t}_0^X)$, in equilibrium each $r_i = \tilde{R}_X(t)/N > 0$ (strictly increasing in t) and each $c_i = \tilde{c}_X(t) > 0$ (strictly decreasing).*

Proof. The existence and essential uniqueness results from the original game, Proposition 8 and Proposition 9, carry over to the plunder model. So does Lemma 7, showing that all individual allocations toward competition are equal under symmetry. Therefore, $\Gamma_X(t)$ has an equilibrium, and there exists $c_X^*(t)$ such that each $c_i = c_X^*(t)$ in every equilibrium of $\Gamma_X(t)$. The budget constraint then implies each $r_i = \pi^r(L/N - c_X^*(t)/\pi^c)$ in every equilibrium of $\Gamma_X(t)$, so the equilibrium is unique.

Let (r, c) be the equilibrium of $\Gamma_X(t)$. Let $R = \sum_i r_i$ and $C = c_1$, so by Lemma 7 each $c_i = C$. If $t \leq \hat{t}_0^X$ and $R > 0$, then the first-order conditions give

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} = \pi^c \frac{\partial \log \omega_i((\pi^c L/N)\mathbf{1}_N)}{\partial c_i} \leq \pi^c \frac{\partial \log \omega_i(c)}{\partial c_i}$$

for each $i \in \mathcal{N}$. But this implies each $r_i = 0$, contradicting $R > 0$. Therefore, if $t \leq \hat{t}_0^X$, then $R = 0$. Similarly, if $t > \hat{t}_0^X$ and $R = 0$, then each $c_i = \pi^c L/N$ and thus

$$\pi^c \frac{\partial \log \omega_i(c)}{\partial c_i} = \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i}.$$

But this implies each $c_i = 0$, a contradiction. Therefore, if $t > \hat{t}_0^X$, then $R > 0$. The proof that $C > 0$ if and only if $t < \hat{t}_1^X$ is analogous.

For $t \in (\hat{t}_0^X, \hat{t}_1^X)$, the first-order conditions imply that R and C solve $Q^{rc}(0, R, C; t, \pi) = Q^b(0, R, C; t, \pi) = 0$; therefore, $R = \tilde{R}_X(t)$ and $C = \tilde{c}_X(t)$. To reduce clutter in what follows, I omit the evaluation point $(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi)$ from all partial derivative expressions. The Jacobian of the system defining $(\tilde{R}_X(t), \tilde{c}_X(t))$ is

$$\begin{aligned}
\mathbf{J}_X &= \begin{bmatrix} \partial Q^{rc}/\partial R & \partial Q^{rc}/\partial C \\ \partial Q^b/\partial R & \partial Q^b/\partial C \end{bmatrix} \\
&= \begin{bmatrix} \pi^r t \dfrac{g''(\tilde{R}_X(t)) - tg(\tilde{R}_X(t))^2 \hat{g}''(\tilde{R}_X(t))}{(1 - tg(\tilde{R}_X(t)))^2} & \eta \pi^c \hat{\phi}''(\tilde{c}_X(t)) \\ -1/\pi^r & -N/\pi^c \end{bmatrix}
\end{aligned}$$

where $\eta = (N-1)/N$ and $\hat{g} = \log g$. Its determinant is

$$|\mathbf{J}_X| = \frac{\pi^c}{\pi^r}\left(\eta \hat{\phi}''(\tilde{c}_X(t)) - N\pi^r t \frac{g''(\tilde{R}_X(t)) - tg(\tilde{R}_X(t))^2 \hat{g}''(\tilde{R}_X(t))}{(1 - tg(\tilde{R}_X(t)))^2}\right) < 0.$$

By the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{d\tilde{R}_X(t)}{dt} &= \frac{\begin{vmatrix} -\partial Q^{rc}/\partial t & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial t & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X|} \\
&= \frac{\begin{vmatrix} -\pi^r g'(\tilde{R}_X(t))/(1 - tg(\tilde{R}_X(t)))^2 & \eta \pi^c \hat{\phi}''(\tilde{c}_X(t)) \\ 0 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X|} \\
&= \frac{N\pi^r g'(\tilde{R}_X(t))}{\pi^c |\mathbf{J}_X|} \\
&> 0,
\end{aligned}$$

as claimed. The budget constraint then implies $d\tilde{c}_X(t)/dt < 0$, as claimed. □

A.7 Proof of Proposition 6

Before proving the proposition, I separately derive the comparative statics of $\hat{t}_0^X$ and $\tilde{R}_X(t)$ in N and π^c.

Lemma 13. *In the plunder model, the lower cutpoint $\hat{t}_0^X$ is strictly increasing in the number of factions, N. It is locally decreasing in the effectiveness of competition, π^c, if and only if*

$$\hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N}\hat{\phi}''\left(\frac{\pi^c L}{N}\right) \geq 0.$$

Proof. Recall that

$$\hat{t}_0^X = \frac{((N-1)/N)\pi^c\hat{\phi}'(\pi^c L/N)}{((N-1)/N)\pi^c\hat{\phi}'(\pi^c L/N) - \pi^r g'(0)}.$$

Since $g'(0) < 0$, $(N-1)/N$ is strictly increasing in N, and $\hat{\phi}'(\pi^c L/N)$ is weakly increasing in N, $\hat{t}_0^X$ is strictly increasing in N. Notice that

$$\frac{\partial}{\partial \pi^c}\left[\pi^c\hat{\phi}'\left(\frac{\pi^c L}{N}\right)\right] = \hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N}\hat{\phi}''\left(\frac{\pi^c L}{N}\right),$$

so $\hat{t}_0^X$ is locally increasing in π^c if and only if the above expression is positive. □

Lemma 14. *In the plunder model, for fixed $t \in (\hat{t}_0^X, \hat{t}_1^X)$, total resistance, $\tilde{R}_X(t)$, is strictly decreasing in the number of factions, N. It is locally decreasing in the effectiveness of competition, π^c, if and only if*

$$\hat{\phi}'(\tilde{c}_X(t)) + \tilde{c}_X(t)\hat{\phi}''(\tilde{c}_X(t)) \geq 0.$$

Proof. As in the proof of Remark 1, I will treat N as if it were continuous in order to obtain comparative statics by implicit differentiation. Throughout the proof I write $\tilde{R}_X(t)$ and $\tilde{c}_X(t)$ as functions of (N, π^c).

I first consider comparative statics in N. To reduce clutter in what follows, I omit the evaluation point $(0, \tilde{R}_X(t; N, \pi^c), \tilde{c}_X(t; N, \pi^c); t, \pi)$ from all partial derivative expressions. By the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial N} &= \frac{\begin{vmatrix} -\partial Q^{rc}/\partial N & \partial Q^{rc}/\partial C \\ -\partial Q^{b}/\partial N & \partial Q^{b}/\partial C \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|} \\
&= \frac{\begin{vmatrix} -\pi^c\hat{\phi}'(\tilde{c}_X(t; N, \pi^c))/N^2 & ((N-1)/N)\pi^c\hat{\phi}''(\tilde{c}_X(t; N, \pi^c)) \\ \tilde{c}_X(t; N, \pi^c)/\pi^c & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|}
\end{aligned}$$

$$= \frac{\hat{\phi}'(\tilde{c}_X(t;N,\pi^c)) - (N-1)\tilde{c}_X(t;N,\pi^c)\hat{\phi}''(\tilde{c}_X(t;N,\pi^c))}{N|\mathbf{J}_X(t;N,\pi^c)|}$$
$$< 0,$$

as claimed, where $|\mathbf{J}_X(t;N,\pi^c)| < 0$ is defined as in the proof of Proposition 5.

I now consider comparative statics in π^c. Again by the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{\partial \tilde{R}_X(t;N,\pi^c)}{\partial \pi^c} &= \frac{\begin{vmatrix} -\partial Q^{rc}/\partial \pi^c & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial \pi^c & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X(t;N,\pi^c)|} \\
&= \frac{\begin{vmatrix} -((N-1)/N)\hat{\phi}'(\tilde{c}_X(t;N,\pi^c)) & ((N-1)/N)\pi^c\hat{\phi}''(\tilde{c}_X(t;N,\pi^c)) \\ -N\tilde{c}_X(t;N,\pi^c)/(\pi^c)^2 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X(t;N,\pi^c)|} \\
&= \frac{(N-1)\left(\hat{\phi}'(\tilde{c}_X(t;N,\pi^c)) + \tilde{c}_X(t;N,\pi^c)\hat{\phi}''(\tilde{c}_X(t;N,\pi^c))\right)}{\pi^c|\mathbf{J}_X(t;N,\pi^c)|}.
\end{aligned}$$

Therefore, $\partial \tilde{R}_X(t;N,\pi^c)/\partial \pi^c \leq 0$ if and only if

$$\hat{\phi}'(\tilde{c}_X(t;N,\pi^c)) + \tilde{c}_X(t;N,\pi^c)\hat{\phi}''(\tilde{c}_X(t;N,\pi^c)) \geq 0,$$

as claimed. □

The proof of Proposition 6 follows mainly from these lemmas.

Proposition 6. *In the plunder model, the government's equilibrium payoff is increasing in the number of factions, N. If there is a unique equilibrium tax rate t^*, the government's equilibrium payoff is locally increasing in competition effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at the corresponding equilibrium level of internal competition.*

Proof. Throughout the proof I write various equilibrium quantities, including the cutpoints $\hat{t}_0^X$ and $\hat{t}_1^X$, as functions of (N,π^c). Let the government's equilibrium payoff as a function of these parameters be

$$u_G^*(N,\pi^c) = \max_{t\in[0,1]} t \times g(R^*(t;N,\pi^c)) \times X.$$

I begin with the comparative statics on N. First, suppose $t = \hat{t}_0^X(N',\pi^c)$ is an equilibrium for all N' in a neighborhood of N.[17] Then $u_G^*(N',\pi^c) = \hat{t}_0^X(N',\pi^c) \times X$

[17] As in the original game, there cannot be an equilibrium tax rate $t < \hat{t}_0^X$.

in a neighborhood of N', which by Lemma 13 is strictly increasing in N'. Next, suppose there is an equilibrium with $t \in (\hat{t}_0^X(N', \pi^c), \hat{t}_1^X(N', \pi^c))$ for all N' in a neighborhood of N. Then, by the envelope theorem,

$$\frac{\partial u_G^*(N, \pi^c)}{\partial N} = g'(\tilde{R}_X(t; N, \pi^c)) \frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial N} \times X > 0,$$

where the inequality follows from Lemma 14. Finally, suppose $t = 1$ is an equilibrium for all N' in a neighborhood of N.[18] Then $u_G^*(N, \pi^c) = g(\pi^r L) \times X$ is locally constant in N, and thus weakly increasing.

I now consider the comparative statics on π^c. First, suppose $t = \hat{t}_0^X(N, \pi^{c\prime})$ is an equilibrium for all $\pi^{c\prime}$ in a neighborhood of π^c. Then $u_G^*(N, \pi^{c\prime}) = \hat{t}_0^X(N, \pi^{c\prime}) \times X$ in a neighborhood of $\pi^{c\prime}$, which by Lemma 13 is locally increasing at π^c if and only if

$$\hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N} \hat{\phi}''\left(\frac{\pi^c L}{N}\right) \geq 0.$$

Next, suppose there is an equilibrium with $t \in (\hat{t}_0^X(N, \pi^{c\prime}), \hat{t}_1^X(N, \pi^{c\prime}))$ for all $\pi^{c\prime}$ in a neighborhood of π^c. Then, by the envelope theorem,

$$\frac{\partial u_G^*(N, \pi^c)}{\partial \pi^c} = g'(\tilde{R}_X(t; N, \pi^c)) \frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial \pi^c} \times X.$$

This is positive if and only if $\hat{\phi}'(\tilde{c}_X(t)) + \tilde{c}_X(t)\hat{\phi}''(\tilde{c}_X(t)) \geq 0$, per Lemma 14. Finally, suppose $t = 1$ is an equilibrium for all $\pi^{c\prime}$ in a neighborhood of π^c. Then $u_G^*(N, \pi^c) = g(\pi^r L) \times X$ is locally constant in π^c, and thus weakly increasing. $\square$

A.8 Proof of Proposition 7

I begin by characterizing the equilibrium of the conquest game. Throughout the proofs, let $\hat{\chi} = \log \chi$ and $\hat{\psi} = \log \psi$. I will characterize equilibria in terms of the criterion function

$$Q^{ds}(S; N) = \frac{N-1}{N}(\psi(S) + \bar{s}_O)\hat{\chi}'\left(\frac{L-S}{N}\right) - \hat{\psi}'(S)\bar{s}_O, \tag{43}$$

which is strictly increasing in both S and N.

[18] $t = 1$ is the only $t \geq \hat{t}_1^X$ that can be an equilibrium, since resistance is constant above $\hat{t}_1^X$.

Lemma 15. *The conquest game has a unique equilibrium in which each*

$$s_i = \begin{cases} 0 & Q^{ds}(0; N) \geq 0, \\ \tilde{S}(N)/N & Q^{ds}(0; N) < 0, Q^{ds}(L; N) > 0, \\ L/N & Q^{ds}(L; N) \leq 0, \end{cases}$$

and each $d_i = L/N - s_i$*, where* $\tilde{S}(N)$ *is the unique solution to* $Q^{ds}(\tilde{S}(N); N) = 0$.

Proof. Like the original game, the conquest game is log-concave, so a pure-strategy equilibrium exists can be characterized by first-order conditions. In addition, the proof of Lemma 7 carries over to the conquest game, so in equilibrium each $d_i = d_j$ for $i, j \in \mathcal{N}$. The claim then follows from the first-order conditions for maximization of each faction's utility. $\square$

The proof of Proposition 7 follows from this equilibrium characterization.

Proposition 7. *In the conquest model, the probability that the outsider wins is increasing in the number of factions, N.*

Proof. I will prove that the equilibrium value of $\sum_i s_i$ decreases with the number of factions. Let (d, s) and (d', s') be the equilibria at N and N' respectively, where $N' > N$, and let $S = \sum_i s_i$ and $S' = \sum_i s_i'$. If $S = 0$, then $Q^{ds}(0; N) \geq 0$ and thus $Q^{ds}(0; N') \geq 0$, so $S' = 0$ as well. If $S \in (0, L)$, then $S = \tilde{S}(N)$, which implies $Q^{ds}(L; N) > 0$ and thus $Q^{ds}(L; N') > 0$. This in turn implies either $S' = \tilde{S}(N') < \tilde{S}(N)$ or $S' = 0 < \tilde{S}(N)$. Finally, if $S = L$, then it is trivial that $S' \leq S$. $\square$