Information and Communication in Collective Action Problems*

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Abstract

Incomplete information exacerbates the problems inherent in collective action. Participants cannot efficiently coordinate their actions if they do not know each other's preferences. I investigate when ordinary communication, or cheap talk, may resolve mutual uncertainty in collective action problems. I find that the efficacy of communication depends critically on the relationship between contributions and the value of the joint project. The incentive barriers to honesty are highest when every contribution makes a difference to the project's value. Participants then have a strict incentive to say whatever would induce others to contribute the most, so cheap talk lacks credibility. By contrast, when contributions may be marginally worthless, such as when the project has no value unless contributions hit a certain threshold, communication may help participants avoid wasted effort. Using these findings, I identify which collective action problems in politics might benefit from communication and which require more "expensive" solutions to overcome uncertainty.

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Collective action problems—and how to solve them—are a longstanding concern in political science and political economy. Because of the incentive to free-ride, voluntary contributions to joint projects are likely to be insufficient even in the best of circumstances (Olson 1965). But many collective action problems in politics face an additional hurdle to cooperation that the classical analysis ignores: mutual uncertainty among potential contributors to the common good. For example, a citizen who opposes an autocratic government may be unsure of her fellow citizens' willingness to risk their lives in a protest. Similarly, countries that have a shared security goal may not know how much their allies are willing to mobilize to meet that goal.

Uncertainty makes the hard problem of collective action even harder. A potential contributor cannot calibrate her own actions when she does not fully understand the incentives of other participants. Incomplete information raises two questions that do not arise in the classic collective action problem. First, is it worth contributing at all? In a project that requires everyone's participation to succeed, one player's unwillingness to contribute makes everyone else's contributions worthless. With incomplete information, potential participants may refrain for fear that their partners are insufficiently committed. Second, how should the project's costs be divided among participants? When the participants are fully informed, the equilibrium solution to a standard voluntary collective action problem entails the contributor who values the project most highly taking on a disproportionate share of the effort (Olson and Zeckhauser 1966). With incomplete information, however, players may not know who values the project most and thus may not be able to coordinate on an optimal division of labor (Palfrey, Rosenthal and Roy 2015).

In this paper, I examine the simplest possible mechanism by which participants in collective action could resolve mutual uncertainty without outside involvement—ordinary communication, often called "cheap talk" (Farrell and Rabin 1996). When, if at all, can participants credibly reveal information and thus coordinate their actions through cheap talk? I investigate this question using a simple but general model of collective action under incomplete information. I find that the possibility of meaningful communication depends critically on the relationship between individual contributions and the outcome of collective action. The incentive barriers to honest communication are strongest in continuous collective action problems—those in which all contributions have some marginal benefit, though possibly a small one (e.g., carbon emission reduction). But in threshold problems, in which one participant's contribution may be worthless if her partners do not contribute enough (e.g., building a bridge), at least a limited form of communication is possible in equilibrium.

A simple logic drives the main findings. In any collective action problem, each participant at least weakly prefers greater contributions by her partners. In continuous problems, in which every contribution has positive marginal value, this preference is strict—a player is always better off if others give more rather than less. This strict preference undermines honest communication through cheap talk. In order for cheap talk to work, a player's private information (here, how much she herself is willing to contribute) must affect what she wants the other players to do; otherwise, all "types" of a player will prefer to say the same thing (Aumann 1990). Specifically, in a continuous collective action problem, a player will always want to say whatever would induce her partner to give the most, whether this be by understating her own willingness to contribute (if the players' efforts are substitutes) or by overstating it (if they are complements).

The incentives are similar, yet different in a critical way, in collective action problems where a fixed threshold determines whether the project is achieved. Below the threshold, the marginal benefit of a contribution—one's own or one's partner's—is zero. At the margin, then, a potential contributor may be indifferent whether her partner gives more or less. More to the point, if a player is sure not to contribute enough for the threshold to be met (e.g., because she values the project very little), then she will be indifferent about her partner's contribution. This indifference turns out to be critical for the possibility of communication. It means a participant will be willing to reveal that she will not give enough to meet the threshold, even though this will encourage others to contribute less than they might have otherwise. By contrast, in a collective action problem where all contributions make at least a small difference, a participant would be strictly worse off if she said something that made her partners contribute less.

The analysis speaks to a broad cross-section of political science research, given the ubiquity of collective action problems with incomplete information in politics. One area of application is global public goods problems, such as the ongoing refugee crisis and the effort to reverse climate change. These initiatives are complicated not only by countries' incentives to free-ride, but also by their uncertainty of where each other's breaking points lie. Another application in the international arena is the provision of collective security, a classic collective action problem (Olson and Zeckhauser 1966) that takes place in an environment of high uncertainty (Jervis 1976; Fearon 1995). In domestic politics, perhaps the most prominent collective action problem is revolution or other means of overthrowing the government (Tilly 1978). Potential participants may not know how likely their fellow citizens are to participate, especially in regimes without free media, making coordination difficult without some means of sharing information. Some facets of democratic politics, such as campaign fundraising, also have the features of collective action problems.

The main upshot of my findings is that we should expect uncertainty to impede cooperation in these areas, above and beyond the difficulties inherent in any collective action problem, even if the participants can freely communicate with each other. Uncertainty can only be resolved through mechanisms more "expensive" than mere talk, such as costly signaling of one's intent to participate, direct monitoring by external actors, or binding commitment to a transfer scheme. At best, political actors may reveal through cheap talk when they are so unwilling to contribute that the project is doomed to be worthless. But even this can only take place when there is a fixed, commonly known threshold for contributions below which the project has literally no value, a rarity in political collective action problems.

In the classic collective action setting, with complete information among participants, it is well known that communication has no strategic effect (Ostrom 1998, 6–7).[1] Communication cannot eliminate the incentive to free-ride, which is the core problem of collective action. My analysis of collective action with incomplete information shows that we can take the conventional wisdom further: in a broad set of circumstances, communication also fails to eliminate the auxiliary

[1]Ostrom reviews a set of public goods experiments in which communication alters behavior by building community ties among participants, in effect changing the game. My concern in this paper is with the purely strategic incentives for communication, leaving aside its interaction with identity formation or other unmodeled characteristics.

issues that arise due to uncertainty. If potential contributors do not fully understand their partners' incentives—how highly they value the project, how costly it is for them to contribute—then they cannot even coordinate on the contribution scheme that would prevail under complete information. I show that in continuous collective action problems, cheap talk communication cannot create the common knowledge of preferences that would be necessary for this kind of coordination. In threshold problems, communication may help contributors avoid wasting their effort on hopeless projects, but beyond that is unlikely to lead to a socially optimal division of labor.

This analysis contributes to the political economy literature on collective action and public goods provision. In a seminal study of the finance of public goods, Samuelson (1954) identifies how individual incentives to misrepresent demand for the collective good might undermine efficient taxation and provision. Subsequently, various papers in the mechanism design literature identified transfer schemes by which efficient provision might be achieved (e.g., Clarke 1971; Groves 1973; d'Aspremont and Gérard-Varet 1979). My analysis shows that the consequences of incomplete information for efficient collective action are considerably more severe when we move from a publicly financed public good to the voluntary contribution setting. In fact, in the class of continuous problems I consider in which all contributions have positive marginal value, communication cannot recover even part of the efficiency loss due to incomplete information. Without a central authority that can commit participants to a transfer scheme, as assumed in the mechanism design literature, it will be difficult to achieve coordination through voluntary information revelation. The difference between the centralized and decentralized approaches is particularly important for political scientists who study collective action problems in which there is no central authority (e.g., in international politics) or where the object of collective action is to overthrow that authority (e.g., revolutions or coups).

This paper is perhaps most closely related to a recent set of work in political economy that considers the efficacy of cheap talk in various public goods games (Agastya, Menezes and Sengupta 2007; Barbieri 2012; Costa and Moreira 2012; Palfrey, Rosenthal and Roy 2015). These analyses consider discrete problems, in which the public good is provided if contributions exceed a known threshold and has zero value otherwise.[2] I show that the findings of effective cheap talk in these models does not extend to problems in which the value of the public good increases continuously with contributions, as in environments like that of Bergstrom, Blume and Varian (1986).

The Model

I model a two-player collective action problem with incomplete information and pre-play communication. I look for conditions under which communication is influential—i.e., it reveals some information and affects the selection of contributions.

Contribution Subgame

In the *contribution subgame*, two players, labeled 1 and 2, individually and simultaneously choose how much effort to contribute to a joint project or goal. Throughout the analysis, i denotes a generic player and j her partner. The feasible set of contributions for each player is a compact interval, $X_i = [0, \bar{x}_i]$. The cost of contributing x_i is $c_i(x_i)$, where each c_i is a real-valued, strictly increasing function. A player always has the option to contribute nothing and incur no cost: $c_i(0) = 0$.

Each player's effort affects the value of the joint project or its likelihood of success. Given the contributions x_1 and x_2, the project's value is $p(x_1, x_2) \geq 0$.[3] If we interpret the project as a public good, p is its production function. I assume throughout the analysis that p is non-decreasing in both arguments. As a regularity condition to assure the existence of a pure strategy equilibrium in every contribution subgame, I also assume p is upper semicontinuous and each c_i is lower semicontinuous in x_i. Further conditions on the production and cost functions are enumerated in numbered Assumptions. In particular, I will highlight the difference between when p is strictly increasing in its arguments, so all contributions are valuable, and when p has flat spots, so some contributions may be worthless.

[2]Barbieri (2012) is a partial exception, in that the public good may have arbitrarily low but nonzero value in case of non-completion.

[3]p need not be symmetric in its arguments. Nevertheless, in a mild abuse of notation, $p(x_i, x_j)$ should be understood to denote $p(x_1, x_2)$ when $i = 2$.

Each player's payoff depends on her own contribution and the project's total value. With complete information, utility functions are

$$u_i(x_i, x_j) = p(x_i, x_j) - c_i(x_i). \tag{1}$$

This is the same additively separable payoff structure as in the collective-action game of Myatt and Wallace (2009). Next, I extend the game to include incomplete information and a prior communication stage.

Uncertainty and Communication

Each player has private information about her ability or willingness to contribute to the joint project. A player's *type*, denoted t_i, is drawn by Nature from the type space $T_i \subseteq \mathbb{R}$. I assume each T_i is finite, though possibly large, with N_i elements, ordered $t_{i1} < \cdots < t_{iN_i}$. The prior distribution of t_i is $\pi_i \in \Delta T_i$, which is common knowledge. To avoid trivialities, I assume each $\pi_i \gg 0$.[4]

To obtain stronger results, I sometimes consider the special case of the game with one-sided incomplete information. When I do so, I assume without (further) loss of generality that player 2 is the one whose type is common knowledge; i.e., $N_2 = 1$ and $\pi_{21} = 1$.

As in standard analyses of incomplete information in public goods problems (e.g., Green and Laffont 1977), I assume that a player's private information only affects her own payoff. This rules out, for example, private signals about the nature of the production technology. Therefore, each player only cares about her partner's type insofar as it affects how much her partner will contribute. I model this by assuming each player's type affects only her cost function.[5] Each player's utility

[4]For vectors y and z, I write $y \geq z$ if each $y_n \geq z_n$, $y > z$ if $y \geq z$ and $y_n > z_n$ for some n, and $y \gg z$ if each $y_n > z_n$.

[5]Alternatively, one could imagine a player having private information about how highly she valued the project. We might then think of p as representing the probability of success and write the utility function as $u_i(x_i, x_j, t_i) = \eta_i(t_i)p(x_i, x_j) - \kappa_i(x_i)$. But this is isomorphic to the original game with $c_i(x_i, t_i) = \kappa_i(x_i)/\eta_i(t_i)$. Therefore, the framework here may incorporate private information about one's valuation of the project as well as one's cost of contribution.

function is now

$$u_i(x_i, x_j, t_i) = p(x_i, x_j) - c_i(x_i, t_i). \tag{2}$$

With this payoff structure, the contribution subgame is a Bayesian potential game (Monderer and Shapley 1996; van Heumen et al. 1996) with potential function

$$P(x_1, x_2, t_1, t_2) = p(x_1, x_2) - c_1(x_1, t_1) - c_2(x_2, t_2) \tag{3}$$

(see also Myatt and Wallace 2009).

I model communication with a *messaging subgame* that precedes the contribution subgame. After learning their types, each player simultaneously sends a message m_i from the message space M_i. To avoid trivialities, I assume each $|M_i| \geq 2$. The messages are publicly observed before players choose their contributions. Messages are cheap talk (Crawford and Sobel 1982; Farrell and Rabin 1996), as in other models of political communication (e.g., Austen-Smith 1990; Kydd 2003). Every type of each player may send any message, and the chosen messages have no direct effect on either player's payoff.

Communication can only affect outcomes through its effect on the players' beliefs. Each player's message shapes her partner's beliefs about her type, which in turn may affect the equilibrium of the contribution subgame. For example, a player may contribute more if she believes her partner is relatively unwilling to contribute than if she expects her partner to take up most of the burden. For every $m_i \in M_i$, let $\hat{\pi}_i(m_i) = (\hat{\pi}_{i1}(m_i), \ldots, \hat{\pi}_{iN_i}(m_i)) \in \Delta T_i$ represent j's updated beliefs about i's type after receiving m_i. The crucial question is when it is in a player's interest to reveal her type—or even just some information about her type—given how doing so will affect her partner's beliefs.

To summarize, the sequence of play is as follows:

1. Nature privately informs each player of her type, $t_i \in T_i$.

2. Each player simultaneously sends a message, $m_i \in M_i$.

(a) Interval. **(b)** Non-interval.

Figure 1. Illustration of messaging strategy types with $N_1 = 7$.

3. Each player observes her partner's message and updates her beliefs about t_j to $\hat{\pi}_j(m_j)$.

4. Each player simultaneously chooses a contribution, $x_i \in X_i$.

5. The game ends and payoffs are realized.

A *messaging strategy* is a function $\mu_i : T_i \to M_i$ that prescribes a message for each type of each player. A *contribution strategy* is a function $\omega_i : T_i \times M_i \times M_j \to X_i$ that prescribes how much each type contributes after each possible pair of messages. An *assessment* $(\mu_i, \omega_i, (\hat{\pi}_i(m_i))_{m_i \in M_i})_{i=1,2}$ is a tuple containing both players' strategies and belief systems.

At times, to keep the analysis tractable, I will restrict attention to *interval messaging strategies*, in which each player's messaging strategy reveals an interval of the type space in which her type lies. Formally, a messaging strategy μ_i is interval messaging if, for all $t_i, t_i', t_i'' \in T_i$ such that $t_i < t_i' < t_i''$, $\mu_i(t_i) = \mu_i(t_i'') = m_i$ implies $\mu_i(t_i') = m_i$. This type of strategy is common in models of cheap talk communication (e.g., Crawford and Sobel 1982). Figure 1 illustrates the distinction between interval and non-interval messaging. Note that a fully separating messaging strategy, in which μ_i is one-to-one (every type sends a distinct message), is always an interval strategy. So is a "babbling" strategy, in which μ_i is constant (every type sends the same message). Consequently, in the special case where $|T_i| \leq 2$, every messaging strategy for player i is trivially an interval strategy.

Influential Communication

As this is a multistage game of incomplete information with observed actions, the appropriate solution concept is perfect Bayesian equilibrium (Fudenberg and Tirole 1991). An assessment is an *equilibrium* if each player's strategy is sequentially rational given her beliefs and the other player's strategy, and beliefs are updated in accordance with Bayes' rule whenever possible. Throughout the analysis, I restrict attention to pure strategy equilibria. The potential function (3) is upper semicontinuous and thereby attains its maximum, which in turn corresponds to a pure strategy equilibrium (van Heumen et al. 1996, Corollary 5.4).[6]

The interesting question is not just when players might reveal their private information, but when that information sharing can help them coordinate their actions. In the language of cheap talk models, an equilibrium in which a player reveals information and that revelation affects the outcome is an *influential equilibrium*. An equilibrium of this model is influential if at least one type of one player contributes different amounts after different messages that are sent on the path of play. A formal definition follows.

Definition 1. An equilibrium is influential if there is a type $t_i \in T_i$ and types $t_j, t_j' \in T_j$ such that

$$\omega_i(t_i, \mu_i(t_i), \mu_j(t_j)) \neq \omega_i(t_i, \mu_i(t_i), \mu_j(t_j')). \qquad \square$$

An immediate consequence of this definition is that a babbling equilibrium, in which every type of each player sends the same message, cannot be influential. However, not all non-babbling equilibria are influential. For example, an equilibrium in which player 1 fully reveals her type but player 2 always contributes $x_2 = 0$ on the path of play is non-babbling and non-influential.

In cheap talk models like this one, influential equilibria are least likely to exist, though not necessarily impossible (Seidmann 1990; Baliga and Morris 2002), when every type of a player has the same preferences over the other player's actions (see Aumann 1990). In an influential equilibrium, one player's message effectively dictates what the other will do. So for different types to choose different messages, they must prefer different actions by their partner—or at least be indifferent. For example, in the international relations context, this is exactly why cheap talk fails to help resolve crises in the canonical model of crisis bargaining: every type of the informed country wants to get the highest offer possible, regardless of its true resolve (Fearon 1995). In

[6]For a precise existence statement and proof, see Lemma 7 in the Appendix.

situations like this, it is not incentive-compatible for different types to send different messages, and thus communication is ineffective. Therefore, a key task of the present analysis will be to separate those instances of collective action where this uniformity of preference exists from those where it does not.

Continuous Problems

I first consider collective action problems in which all contributions are at least somewhat valuable. These include public goods problems in which the amount of the good produced is a continuous and strictly increasing function of the contributions. All-or-nothing projects whose success is probabilistic also fit the bill (Nitzan and Romano 1990), provided that the probability of success is continuous in the contributions. One example is reducing carbon emissions in order to slow global warming—every reduction has some effect, though perhaps a small one. Military cooperation is another example. Even if a military operation has a discrete goal, such as the removal of a particular regime, the exact level of forces necessary to achieve it probably is not known in advance. Greater deployments simply increase the chance of success.

The main result is that in a wide class of continuous collective action problems, communication either does not occur or has no effect on contribution behavior. The intuition behind the result is simple. If all contributions are valuable, then each player strictly prefers that her partner give as much as possible. Ideally, then, a player would send whichever message induced the greatest contribution by her partner. And since talk is cheap, there is nothing to prevent one from doing so. Therefore, influential communication cannot be sustained as an equilibrium. In order for different types to be willing to send different messages, they must want different things from their partner. But that is not true here—a player always wants the same thing, namely the greatest possible contribution, regardless of her own willingness to give.

I consider a natural set of collective action problems in which there are diminishing marginal returns to contribution and the two players' contributions are substitutes.[7] Concavity and substi-

[7]The main result goes through if the contributions are instead complements, as I discuss near the end of the section.

tutability are common assumptions in Cournot contribution games, a popular class of models of public goods and collective action (see Myatt 2007). The following assumption formalizes the class of games under consideration.

Assumption 1.

(a) *p is strictly increasing, twice continuously differentiable, and strictly concave.*

(b) *x_1 and x_2 are substitutes: $\partial^2 p/\partial x_1 \partial x_2 \leq 0$.*

(c) *Each c_i is twice continuously differentiable and convex in x_i.*

(d) *Each $\partial c_i/\partial x_i$ is strictly increasing in t_i.*

Two relevant properties follow from this assumption. First, because higher types have higher marginal costs of contribution, they are less willing to give to the joint project. Second, because the players' contributions are substitutes, the more a player gives, the less her partner prefers to give. In combination, these imply that the way to induce one's partner to give most is to mimic a high type—in other words, to understate one's willingness to contribute. This incentive to feign unwillingness is what precludes influential communication in equilibrium.

In order to prove that there is no influential equilibrium in interval messaging, I will show that every type of a player contributes weakly more after receiving a "high" message—i.e., one that comes from types that have high costs and thus are relatively unwilling to contribute—than after receiving a "low" message. The proof uses the method of monotone comparative statics (Milgrom and Shannon 1994).[8] Without explicitly solving for equilibria, which would require placing specific functional forms or distributional assumptions limiting the generality of the results, I will show that a player's equilibrium contributions increase monotonically as she moves from believing her partner's type is in a "low" set to a "high" one.

I focus on the substitutes case both because it better fits existing models of continuous public goods and because it is the harder case for the method of proof I employ.

[8]See Ashworth and Bueno de Mesquita (2006) for an introduction aimed at political scientists.

Monotone comparative statics usually arise in games of strategic complementarities, in which a player's best response is an increasing function of the other players' actions. But in the type of collective action problem considered here, we have the opposite—the more a player thinks her partner will give, the less she prefers to give herself. So this is a game of strategic substitutes. To identify monotone comparative statics nonetheless, I transform this into a game of strategic complements by redefining one player's action as the additive inverse of her contribution (see Amir 1996).

Let $\Gamma(\hat{\pi}_1, \hat{\pi}_2)$ denote a contribution subgame in which player 1 believes $t_2 \sim \hat{\pi}_2$, player 2 believes $t_1 \sim \hat{\pi}_1$, and these beliefs are common knowledge.[9] Then let the *transformed contribution subgame*, denoted $\tilde{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$, be a two-player strategic form game with strategy spaces

$$\tilde{X}_1 = \left\{\tilde{x}_1 \in X_1^{N_1} \,|\, \tilde{x}_{11} \leq \cdots \leq \tilde{x}_{1N_1}\right\},$$
$$\tilde{X}_2 = \left\{\tilde{x}_2 \in X_2^{N_2} \,|\, \tilde{x}_{21} \geq \cdots \geq \tilde{x}_{2N_2}\right\}$$

and utility functions

$$\tilde{u}_1(\tilde{x}_1, \tilde{x}_2) = \sum_{n=1}^{N_1} \left[\sum_{m=1}^{N_2} \hat{\pi}_{2m} p(\bar{x}_1 - \tilde{x}_{1n}, \tilde{x}_{2m}) - c_1(\bar{x}_1 - \tilde{x}_{1n}, t_{1n}) \right], \tag{4}$$
$$\tilde{u}_2(\tilde{x}_1, \tilde{x}_2) = \sum_{n=1}^{N_2} \left[\sum_{m=1}^{N_1} \hat{\pi}_{1m} p(\bar{x}_1 - \tilde{x}_{1m}, \tilde{x}_{2n}) - c_2(\tilde{x}_{2n}, t_{2n}) \right]. \tag{5}$$

The transformed contribution subgame differs from the original in a few respects that make it more amenable to monotone comparative statics analysis. First, player 1's action represents the inverse of her contribution, so $\tilde{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$ is a game of strategic complements. The greater $\tilde{x}_i$ is, the greater the optimal choice of $\tilde{x}_j$. Substantively, this means each actor prefers to contribute less the more she expects her partner to contribute (since $\tilde{x}_1$ represents the inverse of player 1's contribution). The following lemma states this result formally.[10]

[9] In what follows, in a mild abuse of notation, I write $x_i : T_i \to X_i$ to denote a strategy for player i in $\Gamma(\hat{\pi}_1, \hat{\pi}_2)$.
[10] All proofs appear in the Appendix.

Lemma 1. *Let Assumption 1 hold.* $\tilde{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$ *is supermodular.*

Second, the transformed subgame represents the Bayesian game in *ex ante* form, with each player choosing a vector of contributions corresponding to each of her own types so as to maximize total payoffs across types. Third, players are restricted to strategies in which contributions weakly decrease by type. Nonetheless, for the purposes of equilibrium analysis, the transformed contribution subgame is isomorphic to the original, as the following lemma states.

Lemma 2. *Let Assumption 1 hold.* $\tilde{x} \in \tilde{X}_1 \times \tilde{X}_2$ *is a Nash equilibrium of* $\tilde{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$ *if and only if the strategy profile defined by*

$$
\begin{aligned}
x_1(t_{1n}) &= \bar{x}_1 - \tilde{x}_{1n}, \qquad && n = 1, \ldots, N_1, \\
x_2(t_{2m}) &= \tilde{x}_{2m}, && m = 1, \ldots, N_2,
\end{aligned}
$$

is a Bayesian Nash equilibrium of $\Gamma(\hat{\pi}_1, \hat{\pi}_2)$.

I have already noted that the contribution subgame is a potential game (Monderer and Shapley 1996), which guarantees the existence of an equilibrium in pure strategies. Under the differentiability and concavity conditions of Assumption 1, the potential function is smooth and strictly concave, so the equilibrium is unique (Neyman 1997), as stated in the following result.

Lemma 3. *Let Assumption 1 hold.* $\tilde{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$ *has a unique equilibrium.*

In cheap talk models like this one, equilibrium uniqueness in the final stage is important for the question of whether influential communication is possible. For example, models in which cheap talk may affect bargaining (e.g., Farrell and Gibbons 1989; Ramsay 2011) rely on multiplicity of equilibria. These models admit both "no trade" equilibria in which bargaining is sure to fail and "trade" equilibria in which serious offers are exchanged. Because of this multiplicity, a player may rationally expect the message "I will stand firm" to lead to more aggressive behavior by some types of her partner (those with whom the no-trade equilibrium is played) and more conciliatory behavior by others (those who proceed to bargain seriously). In the environment I analyze here, by contrast, cheap talk cannot generate a diversity of responses simply by sorting players into different equilibria.

With uniqueness pinned down, the only remaining question for the possibility of influential communication is how each player responds to her partner's messages. In order for influential cheap talk to be incentive-compatible, a message that causes some types to increase their contribution must cause other types to decrease theirs. To see why, consider an influential strategy profile, in which different types of player i send distinct messages m_i and m_i' such that at least one type of player j contributes more after receiving m_i'. Now suppose no type of player j contributes more after receiving m_i. Then this strategy profile cannot be an equilibrium, since every type of player i strictly prefers to send the message m_i'. Sending m_i' guarantees her partner will contribute at least as much if she had sent m_i, and with positive probability her partner will contribute even more.

Unfortunately for the prospects of influential communication, this uniformity of responses is exactly what happens under interval messaging in the broad class of models characterized by Assumption 1. Specifically, in equilibrium, every type of player j contributes weakly less after receiving a "low type" (high willingness) message from player i than after receiving a "high type" (low willingness) message. Figure 2 illustrates this finding,[11] and the following result formalizes it. It considers a range of contribution subgames, holding fixed the distribution of one player's type while varying the relative chance of a lower versus higher type of the other player.[12] As a player's partner becomes more likely to have a high cost of contribution, all types of that player (weakly) increase their own contribution in equilibrium.

Lemma 4. *Let Assumption 1 hold. Let $\hat{\pi}_1^L, \hat{\pi}_1^H \in \Delta T_1$, $\hat{\pi}_2 \in \Delta T_2$, and $\alpha, \beta \in [0,1]$. Assume the*

[11]The parameters used to generate the plot are $p(x_1, x_2) = \log(1 + x_1 + 2x_2) + \log(1 + 2x_1 + x_2)$, each $c_i(x_i, t_i) = x_i t_i$, each $T_i = \{0.25, 0.75, 1.25, 1.75, 2.25, 2.75\}$, and each $\hat{\pi}_i = (1/6, 1/6, 1/6, 1/6, 1/6, 1/6)$.

[12]For convenience, the proposition denotes player 1 as the one whose type distribution varies. Since the player labels are arbitrary, the claim is without loss of generality.

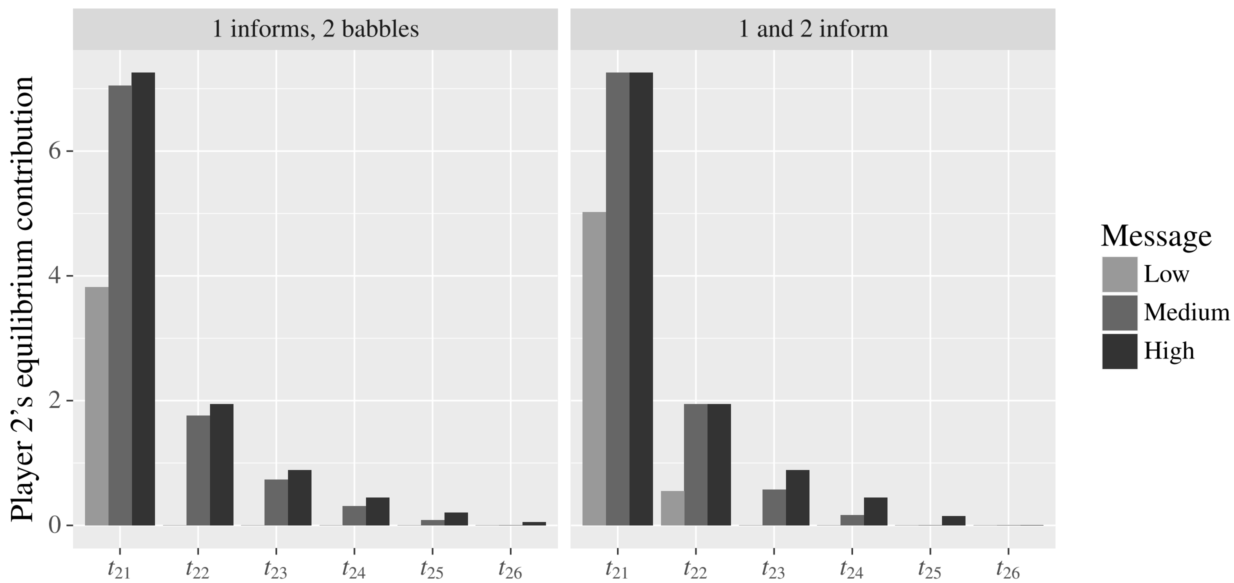

Figure 2. Equilibrium responses by player 2 along the path of play in the contribution subgames when player 1's messaging strategy separates herself into "low," "medium," and "high" types. In the left panel, player 2's messaging strategy is uninformative; in the right panel, she employs the same messaging strategy as player 1. In either case, as stated in Lemma 4, every type of player 2 contributes (weakly) more after receiving the "medium" message than after the "low" message and still more after receiving the "high" message.

support of $\hat{\pi}_1^H$ *consists of higher types than that of* $\hat{\pi}_1^L$*; i.e.,* $\min\{n \,|\, \hat{\pi}_{1n}^H > 0\} > \max\{n \,|\, \hat{\pi}_{1n}^L > 0\}$. *Let*

$$\hat{\pi}_1^\alpha = \alpha \hat{\pi}_1^H + (1 - \alpha) \hat{\pi}_1^L,$$

and define $\hat{\pi}_1^\beta$ *analogously. Let* $\tilde{x}^\alpha$ *and* $\tilde{x}^\beta$ *be the equilibria of* $\tilde{\Gamma}(\hat{\pi}_1^\alpha, \hat{\pi}_2)$ *and* $\tilde{\Gamma}(\hat{\pi}_1^\beta, \hat{\pi}_2)$ *respectively. If* $\alpha \leq \beta$*, then* $\tilde{x}^\alpha \leq \tilde{x}^\beta$.

It is almost immediate from this result that there cannot be influential communication when players use interval messaging strategies and Assumption 1 holds. For example, imagine an equilibrium in which player 1's messaging strategy partitions her type into two intervals ("low cost" and "high cost"). According to Lemma 4, along the path of play, every type of player 2 must contribute weakly more following the "high cost" message than after the "low cost" message. If the equilibrium is influential, then at least one type of player 2 must be contributing strictly more. But then even the low types of player 1 would strictly prefer to send the "high cost" message so as to induce the greatest possible contribution by player 2, contradicting the assumption of equilibrium. Therefore, even if there is an equilibrium in which one player wholly or partly reveals her type via an interval messaging strategy, this revelation does not affect the ultimate choice of contributions. The following proposition summarizes this finding.

Proposition 1. *Let Assumption 1 hold. There is no influential equilibrium in interval messaging.*

This finding, though negative on its face, is important both for the study of collective action problems and that of communication in politics. Even without uncertainty, collective action problems are difficult to overcome, particularly when contributions are voluntary and non-refundable, as in the setting I study here. Private information may exacerbate the problem, as players who do not know how much each other is willing to contribute cannot even coordinate on a second-best division of labor. What Proposition 1 shows is that there is no cheap solution to the additional problems created by uncertainty, at least in the class of games covered by Assumption 1. In order to have information exchange that improves collective action outcomes in these continuous problems, the players must engage in costly signaling, participate in institutions that can independently solicit information about members' willingness to contribute, or use some other costly mechanism.

In terms of the literature on political communication, Proposition 1 speaks to the relationship between the players' interests and the possibility of effective communication through cheap talk. Following the canonical results of Crawford and Sobel (1982) in a principal-agent model, conventional wisdom holds that cheap talk is likely to be effective when the players have overlapping interests. Collective action problems are a classic mixed-motivation game, in which the players' interests partially overlap (both want the project to have high value) and partially conflict (each wants the other to do most of the work). Following the conventional wisdom would lead one to expect that communication would at least be partially effective here, yet Proposition 1 shows the contrary in a wide set of problems. What matters is not merely an overlap of preferences between the two players, but also the interaction between a player's type and her preferences over her partner's actions. It is a major barrier to influential communication if a player has the same preferences over her partner's actions regardless of her own type (Aumann 1990), as in this class of continuous collective action problems.

Before moving on to threshold problems, I briefly state two additional results for the continuous case. First, if the players' contributions were complements instead of substitutes, influential communication would remain impossible in equilibrium. Suppose Assumption 1b were reversed to say $\partial^2 p / \partial x_1 \partial x_2 \geq 0$. Then the (non-transformed) contribution subgame has strategic complements: the more a player expects her partner to give, the more she wants to give. From there all the subsequent results go through. In fact, they would go through if the game were generalized to have an arbitrary number of players. There is still a pervasive incentive to misrepresent that precludes any influential communication. What differs is the nature of the misrepresentation. Whereas before it was profitable to understate one's willingness to chip in, here it would be best to overstate.

Second, the assumptions on the production function can be relaxed even further if only one player has private information about her willingness to contribute. The only condition that is required is that the value of the project be strictly increasing in the contribution of the player whose type is common knowledge.[13] The formal statement of this condition, which is weaker than Assumption 1, is as follows.

Assumption 1W. *p is strictly increasing in x_2.*

With this assumption in hand, it is simple to prove that there cannot be influential messaging with one-sided incomplete information. Imagine an influential strategy profile, in which distinct types of player 1 send distinct messages, leading to distinct responses by player 2. Since there is only one type of player 2, it must be the case that one of these responses is greater—and thereby strictly better for all types of player 1, under Assumption 1W—than the other. But then, by the same logic as in the proof of Proposition 1, every type of player 1 would strictly prefer to send the message that yielded the greater contribution by her partner. Therefore, an influential strategy profile cannot be incentive-compatible. The following result follows immediately from this line of

[13] Recall that in the case of one-sided incomplete information I assume, without loss of generality, that it is player 2 whose type is known.

reasoning.

Proposition 2. *If there is one-sided incomplete information and Assumption 1W holds, there is no influential equilibrium.*

I have identified a broad set of conditions in continuous collective action problems under which communication does nothing to coordinate behavior or improve the efficiency of outcomes. The basic problem is that in the continuous setting, each player *strictly* prefers for her partner to contribute as much as possible. There is accordingly a strict incentive to deviate from an influential messaging strategy, namely to say whatever would get one's partner to contribute the most. In the next section, I consider discrete problems in which these strict preferences do not hold, at least not globally. Although the difference might seem immaterial at a glance, it turns out to be critically important for the possibility of influential communication.

Threshold Problems

The negative findings in the previous section seemingly contradict a recent spate of analyses finding claim cheap talk may help contributors coordinate their actions in the presence of incomplete information (Agastya, Menezes and Sengupta 2007; Barbieri 2012; Costa and Moreira 2012; Palfrey, Rosenthal and Roy 2015). In contrast to the collective action problems I considered above, the models in these articles consider discrete, or threshold, public goods. The value of such a good is zero unless contributions meet a fixed, commonly known threshold. For example, a public works project whose cost is known in advance and that provides no value if not completed (e.g., a bridge) is a threshold good.

In this section, I identify the strategic differences between continuous and threshold problems, and I explain why the possibility of influential communication in the threshold setting does not carry over to the continuous one. In a continuous collective action problem, every contributor strictly prefers greater contributions by her partner. Even if the marginal benefit of your partner's contributions is fairly low, it comes at no cost to you. But in threshold problems, the preference for one's partner to give more becomes weak, at least sometimes. If a player is unwilling to contribute enough for the threshold to be met, then she is indifferent between her partner giving nothing and her partner making a wasted contribution. This indifference is crucial for the possibility of communication, as I illustrate below.

The key feature of the collective action problems I consider in this section is that some contributions are worthless, at least at the margin. For example, in a simple threshold model where the good is provided only if $x_1 + x_2 \geq \theta$, then a contribution $x_i \in (0, \theta)$ is wasted if $x_j < \theta - x_i$. More generally, as stated in the following assumption, we can allow for a set of contribution profiles (not necessarily defined by a summation) for which the project fails and has no value.

Assumption 2. *There exists a set $Z \subseteq X_1 \times X_2$, with $|Z| > 1$, such that $p(x_1, x_2) = 0$ if and only if $(x_1, x_2) \in Z$.*

This assumption encapsulates a wide variety of collective action problems with (marginally) worthless contributions.[14] For example, in the kind of additive threshold problem discussed above, $Z = \{(x_1, x_2) \mid x_1 + x_2 < \theta\}$. Another example would be a public good with a Cobb-Douglas production function, $p(x_1, x_2) = K x_1^{\alpha} x_2^{\beta}$, for which $Z = \{(x_1, x_2) \mid \min\{x_1, x_2\} = 0\}$.

Numerical Example

I will begin with a parameterized example that demonstrates the possibilities—and limitations—of communication in threshold problems. Consider the game in which the action spaces are $X_1 = X_2 = [0, \frac{1}{2}]$, the production function takes the thresholded quadratic form

$$p(x_1, x_2) = \begin{cases} 0 & x_1 + x_2 < \frac{1}{2}, \\ 1 - (1 - x_1 - x_2)^2 & x_1 + x_2 \geq \frac{1}{2}, \end{cases} \tag{6}$$

[14]The condition that $|Z| > 1$ excludes the trivial case where p is strictly increasing with $p(0, 0) = 0$, in which Z is non-empty but no contribution is worthless.

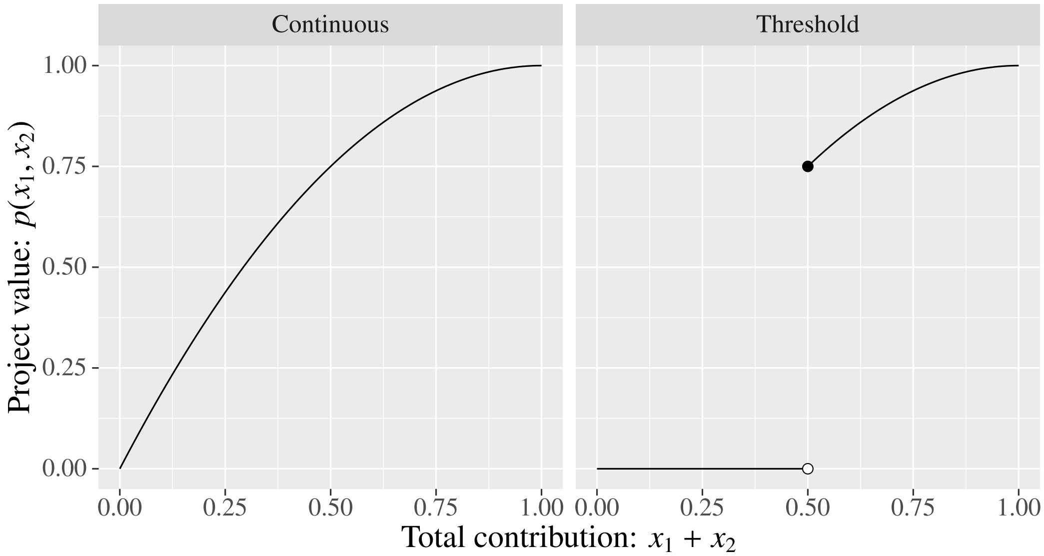

Figure 3. On the left, the function $p(x_1, x_2) = 1 - (1 - x_1 - x_2)^2$. On the right, the thresholded version given in (6).

and the cost functions take the linear form $c_i(x_i, t_i) = x_i t_i$. Figure 3 illustrates the difference between the thresholded production function here and its continuous analogue. The production function here is the kind of thresholded function that Myatt and Wallace (2009) consider.

I assume one-sided incomplete information, both to keep the example simple and to best highlight the contrast with the continuous case (where the impossibility of influential communication does not depend on concavity or other functional form restrictions; see Proposition 2). Specifically, let $t_2 = 3$ and let $T_1 = \{t_1^L, t_1^M, t_1^H\}$ where $\frac{3}{2} < t_1^L < t_1^M \leq 3 < t_1^H$.

In this example, the value of the project at the threshold is $\frac{3}{4}$, and consequently the most a player of type t_i is willing to spend in order to make the difference between meeting the threshold or not is

$$\bar{x}(t_i) = \begin{cases} \frac{1}{2} & t \leq \frac{3}{2}, \\ \frac{3}{4t_i} & t > \frac{3}{2}. \end{cases}$$

Past the threshold, the marginal benefit of a contribution is 1 or less, so only a player of type $t_i < 1$ would spend beyond that point.

This game has an influential equilibrium[15] in which the two players coordinate to meet the

[15]In fact, if $t_1^M < 3$, there is a continuum of similar equilibria in which player 2 contributes $x_2 \in [\frac{1}{2} - \bar{x}(t_1^M), \frac{1}{4}]$

threshold if and only if it is individually rational to do so. In the messaging stage, player 1 sends the "low" message m_1^L if $t_1 \in \{t_1^L, t_1^M\}$ and the "high" message $m_1^H \neq m_1^L$ if $t_1 = t_1^H$. In the contribution subgame, following the "low" message, either type of player 1 contributes $x_1 = \bar{x}(t_1^M) \geq \frac{1}{4}$ and player 2 contributes $x_2 = \frac{1}{2} - \bar{x}(t_1^M)$, so the threshold is met exactly.[16] Following the "high" message, both players contribute $x_1 = x_2 = 0$, so the project fails but no effort is wasted.

It is trivial to confirm that the proposed contribution strategies form Bayesian Nash equilibria in their respective subgames. It is also easy to see why t_1^L and t_1^M prefer not to deviate from sending m_1^L, as doing so would result in a lower contribution by their partner. What is more surprising is that it is not profitable for t_1^H to deviate to sending the low message. Doing so would yield a greater contribution by player 2, but this contribution has no value to the high type of player 1. The most this type is willing to spend to meet the threshold is

$$\bar{x}(t_1^H) = \frac{3}{4t_1^H} < \frac{1}{4},$$

which, when combined with player 2's contribution of $x_2 \leq \frac{1}{4}$, is not enough. Therefore, the high type is indifferent between her own messaging strategy and a deviation. This indifference, in contrast to the strict preference for a greater contribution by the other player in the continuous case, enables influential communication.

Not only is influential communication possible, it may also provide an efficiency gain over the no-communication equilibrium. In the game without communication, as the prior probability of t_1^H approaches one, the most player 2 is willing to contribute approaches zero. So when this prior probability is sufficiently high, even the low type of player 1 will be unwilling to make up the difference. (Recall that $t_1^L > \frac{3}{2}$, so the low type is not willing to meet the threshold on her own.) Consequently, the project will sometimes fail even though it would have been feasible had player 2 known $t_1 \leq t_1^M$.

Nonetheless, in this equilibrium, some potential social welfare gains are being left on the table.

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following the low message. I focus on the equilibrium in the text since it has the minimal social cost among the equilibria in this class.

[16]If t_1^H were to deviate off the equilibrium path and reach this subgame, she would contribute $x_1 = 0$.

If the players' types were common knowledge, the contribution scheme that would maximize social welfare is

$$(x_1, x_2) = \begin{cases} (\bar{x}(t_1^L), \frac{1}{2} - \bar{x}(t_1^L)) & t_1 = t_1^L, \\ (\bar{x}(t_1^M), \frac{1}{2} - \bar{x}(t_1^M)) & t_1 = t_1^M, \\ (0, 0) & t_1 = t_1^H. \end{cases}$$

But consider a strategy profile in which player 1's message reveals her type exactly, and then the two players coordinate on this contribution scheme. Such a strategy cannot be an equilibrium, as t_1^L has a strict incentive to mimic t_1^M in the messaging stage. By doing so, t_1^L could lower her own contribution to $\bar{x}(t_1^M)$ yet still have the project succeed—the same benefit at less cost. Even when communication recovers some of the efficiency loss due to incomplete information in a threshold collective action problem, it may still not allow players to achieve what they could have under full information.

The upshot of this example is twofold. First, in threshold collective action problems, cheap talk communication may help players reveal information so as to better coordinate their contributions. This possibility arises because, unlike in the class of continuous problems studied in the previous section, a player may be indifferent at the margin about whether her partner gives more or less. More to the point, a player who is unwilling to give enough to meet the threshold does not lose anything by revealing this. This brings us to the second takeaway, which is that even when influential cheap talk is possible in threshold problems, its efficacy may still be limited (Costa and Moreira 2012; Palfrey, Rosenthal and Roy 2015). Once a player expects the threshold to be met, she once again strictly benefits from greater contributions by her partner. A player may honestly reveal whether she is willing to contribute at all, but there are incentive barriers to more detailed revelations.

Formal Results

I now turn to some more general findings on communication and contributions in collective action problems characterized by Assumption 2. I focus on results that reinforce the lessons of the numerical example above. This is a wide class of games, so it is difficult to pin down more than broad patterns. The previous literature on cheap talk in threshold games contains more detailed findings on specific models within this class (Agastya, Menezes and Sengupta 2007; Barbieri 2012; Costa and Moreira 2012; Palfrey, Rosenthal and Roy 2015).

First, I will establish that it is incentive-compatible for a player to reveal that she is unwilling to contribute enough to meet the threshold. What complicates this is that how much a player is willing to contribute, and whether that is enough to meet the threshold, may depend at least partly on what she expects her partner to do. To accommodate this, I use an iterated strict dominance criterion to define unwillingness to meet the threshold. Let each $\mathcal{X}_i^0(t_i) = X_i$. Then for each $k \in \mathbb{N}_+$, let $\mathcal{X}_i^k(t_i) \subseteq X_i$ be the set of contributions that are not dominated for t_i given that player j makes a contribution that is not dominated for all types in the $k-1$'th iteration: $x_i \in \mathcal{X}_i^k(t_i)$ if and only if there is no $x_i' \in X_i$ such that $u_i(x_i', x_j, t_i) > u_i(x_i, x_j, t_i)$ for all $x_j \in \bigcup_{t_j \in T_j} \mathcal{X}_j^{k-1}(t_j)$. Let $\mathcal{X}_i^\infty(t_i) = \bigcap_{k=0}^\infty \mathcal{X}_i^k(t_i)$ denote the set of contributions that survive iterated strict dominance for t_i.

I call type t_i of player i *unwilling to meet the threshold* if $(x_i, x_j) \in Z$ for all $x_i \in \mathcal{X}_i^\infty(t_i)$ and all $x_j \in \bigcup_{t_j \in T_j} \mathcal{X}_j^\infty(t_j)$. In other words, t_i is unwilling to meet the threshold if it is strictly dominated for her to make up the difference between the threshold and any rationalizable contribution by her partner. In the numerical example above, t_1^H is unwilling to meet the threshold. To see why, notice that $\mathcal{X}_2^1(t_2) = [0, \bar{x}(t_2)] = [0, \frac{1}{4}]$. We then have $u_1(x_1, x_2, t_1^H) < u_1(0, x_2, t_1^H)$ for all $x_1 \in X_1 \setminus \{0\}$ and $x_2 \in \mathcal{X}_2^1(t_2)$, so $\mathcal{X}_1^2(t_1^H) = \{0\}$ and thus $\mathcal{X}_1^\infty(t_1^H) = \{0\}$. This implies $x_1 + x_2 < \frac{1}{2}$ for all $x_1 \in \mathcal{X}_1^\infty(t_1^H)$ and $x_2 \in \mathcal{X}_2^\infty(t_2) \subseteq [0, \frac{1}{4}]$, so t_1^H is unwilling to meet the threshold.

If a player is unwilling to meet the threshold, there is no downside to revealing this through cheap talk. In the end, the project will have zero value either way; she is indifferent between whether this occurs with her partner giving nothing or making a wasted contribution. At the same time, if a player is willing to meet the threshold, it is obviously beneficial to reveal this, allaying the other player's fear of making a contribution that will go to waste. Therefore, as the following proposition states, threshold collective action games have equilibria in which players' messages reveal whether they are willing to meet the threshold.

Proposition 3. *Let Assumption 2 hold. For each player i, let $T_i^U \subseteq T_i$ be the set of types that are unwilling to meet the threshold. For any distinct $m_i^U, m_i^W \in M_i$, there exists an equilibrium in which each player's message reveals whether she is unwilling to meet the threshold:*

$$\mu_i(t_i) = \begin{cases} m_i^U & t_i \in T_i^U, \\ m_i^W & t_i \notin T_i^U. \end{cases}$$

Along the path of play, in any contribution subgame in which either $m_i = m_i^U$, both players contribute $x_i = 0$.

A couple of caveats are in order. First, unwillingness to meet the threshold, as I have defined it here, is a strong condition. In order for a type to be unwilling to meet the threshold, any rationalizable contribution under any set of beliefs by that type must result in failure. There may be other equilibria in which high-cost types that do not quite meet this stringent condition separate themselves, but Proposition 3 does not speak to them. Second, the equilibrium described in the proposition need not be influential. Depending on the parameters of the game, including the prior distribution of types, the equilibrium in the subgame that follows both players sending m_i^W may still entail zero contributions.

Despite these caveats, Proposition 3 captures a substantive difference between continuous and threshold collective action problems. In continuous problems, each player strictly prefers greater contributions by her partner, regardless of her own type. There is consequently a major incentive barrier to information revelation through cheap talk, as each player simply prefers to say whatever would induce her partner to give as much as possible. In threshold problems, by contrast, some types—namely, those who value the project so little that they are unwilling to give enough for it to have a chance of completion—are indifferent about how much their partner gives. This indifference enables meaningful communication, which may allow players to coordinate to avoid wasted contributions.

There is more room for communication in collective action problems with known thresholds than in the continuous case. But there are still limits (Costa and Moreira 2012; Palfrey, Rosenthal and Roy 2015). In the example above, cheap talk could help avert wasted contributions in equilibrium, but it could not lead to coordination on a socially efficient contribution scheme. Once a player expects the threshold to be met, she once again strictly prefers for her partner to give more, raising the same incentive problems as in the continuous case.

Proposition 3 shows that cheap talk may in equilibrium reveal who is willing or unwilling to meet the threshold. I now characterize a class of threshold problems in which this essentially is all cheap talk can do—there is no coordination beyond whether to contribute at all. To make this analysis tractable, I focus on one-sided incomplete information (player 2's type is common knowledge). I also must impose the condition that the value of the project strictly increases with contributions once the threshold is met. The following assumption, which is stronger than Assumption 2, formalizes this condition.

Assumption 2S. *There exists a set $Z \subseteq X_1 \times X_2$, with $|Z| > 1$, such that $p(x_1, x_2) = 0$ if and only if $(x_1, x_2) \in Z$. If $(x_1, x_2) \notin Z$, $x_1' \geq x_1$, and $x_2' > x_2$, then $p(x_1', x_2') > p(x_1, x_2)$.*

Under these conditions, any influential equilibrium takes a very specific form. The player whose type is private information sends a message of "unwilling" or "willing," revealing nothing further about her type. After the unwilling message, both players give nothing. After the willing message, the player whose type is common knowledge makes a contribution that is insufficient to meet the threshold on its own.

Proposition 4. *If there is one-sided incomplete information and Assumption 2S holds, then in any influential equilibrium:*

(a) *Player 2 always contributes* $x_2 = 0$ *or* $x_2 = \tilde{x}_2$ *on the path of play, where* $\tilde{x}_2 > 0$ *and* $p(0, \tilde{x}_2) = 0$.

(b) *On the path of play, if* t_1*'s message induces player 2 to contribute* $x_2 = 0$*, then* t_1 *spends* $x_1 = 0$.

Communication is more likely to have some effect in a threshold problem than when all contributions have positive marginal value, but it is not a silver bullet. A player will only pass up the chance for her partner to give more if she knows the greater contribution would be worthless anyway.

Discussion

The main finding of the analysis is that the possibility for communication to affect collective action outcomes depends critically on the structure of the collective action problem. In a wide class of continuous problems, in which all contributions are valuable, strong incentives to misrepresent hinder effective communication. There is a greater chance of influential communication in threshold problems, in which contributions may have zero marginal value, though even here the scope of coordination brought on by communication may be limited. I now consider the implication of these findings for collective action problems in various arenas of politics.

International cooperation. An enduring question in the study of international politics is how to facilitate the provision of global public goods. In the anarchical international system, there is no central authority to enforce contracts or mandate cooperation (Waltz 1979). States' uncertainty about each other's willingness to contribute to the common good only exacerbates the free-rider problem (Jervis 1976; Keohane 1984, 92–95). The most prominent global collective action problems, such as reducing carbon emissions and resettling refugees, are continuous rather than threshold problems; every contribution has some value, though perhaps a small one. Therefore, we should expect cheap talk to do relatively little to coordinate states' efforts in these areas. A possible exception would be international vaccination campaigns, if the threshold number of vaccinations for herd immunity is known in advance.

International organization theorists have focused on the role of international institutions in promoting cooperation (Keohane 1984; Martin 1992; Martin and Simmons 1998). The inefficacy of cheap talk is itself an argument for institutional involvement—if states could resolve uncertainty through talk alone, then they need only use traditional diplomatic channels. By the same token, the role of institutions is not merely to bring states together; the "forum effects" of institutional deliberation on global public goods provision should be minimal.[17] To be most effective, an institution must have a degree of autonomy from its member states (see Abbott and Snidal 1998), enough to prioritize overall provision over individual members' distributive concerns. Though it is unlikely any international institution would have the power to implement a binding transfer scheme like those discussed in the mechanism design literature (e.g., d'Aspremont and Gérard-Varet 1979), one could at least enhance provision by independently collecting information and publicizing its findings to its members.

International conflict. There is a long tradition of modeling military coalitions as collective action problems (Olson and Zeckhauser 1966). When nations share security goals, the military effort of one works to the benefit of all. Once again, these are typically continuous problems—even if there is a discrete goal, such as destroying a particular weapons program, the exact amount of military effort required to accomplish it usually is unknown in advance. Success is a probabilistic function of contributions, with additional effort increasing the likelihood of victory at the margin.

A common assumption in the alliance literature is that partners freely share information with one another (Bearce, Flanagan and Floros 2006; Konrad 2012). My findings cast doubt on this assumption, or at least they qualify the mechanism by which it operates. Shared interest in military success is not a sufficient condition for allies (or coalition partners more generally) to reveal information that will aid the effort. The Libyan crisis of 2011, in which confusion prevailed over how

[17] Alternatively, forum effects may occur if the act of talking changes participants' payoff structures (Mitzen 2005), which is outside the scope of this analysis.

much NATO members were willing to contribute and who would lead the joint effort, is one example (Michaels 2013). If military alliances facilitate information sharing, they must do so through some means separate from ordinary diplomatic communication.

Revolt and regime change. Revolt is a classic example of a collective action problem (Tilly 1978). The model results show that private information about willingness to partake in revolt may be a major impediment to revolutionary activity, even if citizens are allowed to communicate freely with each other. If individuals' efforts in revolt are complementary—i.e., one's own incentive to participate increases with the number of others involved—then there is an incentive to overstate one's own likelihood of participation (Barberà and Jackson 2016). Credible signaling might require costly early action by revolutionary entrepreneurs (Bueno de Mesquita 2010).

However, two factors might enable credible communication among revolutionaries. The first is if potential participants have prosocial preferences, meaning they at least partially internalize the costs of others. In the model here, where players only internalize their own costs, there is no downside to fooling one's partner into expending effort that just barely increases the chance of success. But if a player prefers that her partner not expend costs for little benefit, the incentive to overstate disappears. The second exception is when a certain player's participation is strictly necessary for the movement to have any chance of success, in which case the threshold model applies. Here a coup might be a better example than a mass revolution.[18] If the effort cannot succeed without the involvement of a particular official, and that official does not intend to get involved, it is (weakly) in her interest to say so.

Lobbying and campaign finance. Lobbying entails spending time and money in hopes of having favorable policies enacted (Tullock 1980). When multiple interest groups share a policy goal, their lobbying efforts are a kind of collective action problem. Typically this is a continuous problem—lobbying increases the chance of a favorable outcome, but there is no exact threshold.[19]

[18] See also Casper and Tyson (2014) and Little (2017) on the information economics of coups.

[19] A possible exception would be literal bribery—e.g., a politician demanding a specific donation in order to vote a certain way.

Therefore, according to the results of the model, even if special interest groups have identical policy goals, we should not expect coordination among them to be seamless in the presence of incomplete information. To able to coordinate effectively, groups must have some means besides cheap talk of signaling their priorities and abilities, or they must be able to make binding commitments to each other. We should expect informational impediments to lobbying coordination to be most severe when legislation of interest arises suddenly, preventing signaling through sequential contributions (Barbieri 2012), or when issues create temporary alliances between groups, in which case reputational concerns (Sartori 2002) cannot support honest communication.

Like lobbying, campaign spending entails exerting costly effort in the service of a particular political goal (Meirowitz 2008). In contemporary American politics, multiple independent actors are involved in the finance of major campaigns—the candidates themselves, the parties that support them, and outside groups like super PACs. An important question is whether rules preventing candidates and super PACs from explicit coordination have any electoral effect. This analysis suggests one way that they may. If candidates and outside groups cannot engage in the kinds of activities that reduce informational asymmetries within an organization, cheap talk will not help make up the gap. Despite their joint electoral interest with a super PAC, candidates have an incentive to downplay or exaggerate their own fundraising ability, so as to encourage the outside group to take up more of the burden. Fundraising disclosure rules, by providing a verifiable source of information, might alleviate the coordination problem between candidates and outside groups by creating common knowledge of ability and willingness to raise money. The transparency benefits of such rules may thus come at the cost of undermining the independence of candidates' and outside groups' expenditures.

Conclusion

I have shown that the efficacy of communication at resolving uncertainty in collective action problems depends critically on the nature of contributions. If all contributions are valuable at the margin, as in continuous collective action problems, then there is an incentive to misrepresent that undermines effective communication. Each player wants to say whatever would get her partner to contribute the most, whether that be by understating her own willingness to contribute or by overstating it. On the other hand, if there is the possibility of worthless contributions, as in threshold problems, at least a limited form of meaningful communication is possible. Participants will be willing to signal whether their costs of contributing are so high that the joint project is worthless. But communication is unlikely to lead to further coordination or efficiency gains among those who are willing to meet the threshold.

An avenue for future research is to study how to credibly transmit information in collective action problems when contributions are voluntary and there is no central authority to enact transfers. This is closely related to the longstanding problem of how to foster political cooperation (Olson 1965; Axelrod 1984; Taylor 1987). Given that uncertainty usually undermines collective action (Menezes, Monteiro and Temimi 2001), how can we structure political actors' incentives so as to encourage honest information sharing? Since cheap talk is ineffective in the one-shot setting, it may be worthwhile to follow the classical literature on cooperation by examining whether the shadow of the future helps enable honest communication and efficient coordination where it otherwise would have been impossible.

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Appendix

This appendix contains proofs and two brief extensions to the model.

Proof of Lemma 1

I begin with an intermediate result on the lattice structure of the strategy spaces in the transformed contribution subgame.

Lemma 5. *$\tilde{X}_1 \times \tilde{X}_2$ with the component-wise order $\geq$ is a complete sublattice.*

Proof. I begin by showing that $\tilde{X}_1$ is closed under the meet and join operations and is thereby a sublattice. Take any $\tilde{x}, \tilde{x}' \in \tilde{X}_1$ and any $n = 2, \ldots, N_1$. Without loss of generality, suppose $\tilde{x}_n \geq \tilde{x}'_n$. We have

$$(\tilde{x} \wedge \tilde{x}')_n = \min\{\tilde{x}_n, \tilde{x}'_n\} = \tilde{x}'_n \geq \tilde{x}'_{n-1} \geq \min\{\tilde{x}_{n-1}, \tilde{x}'_{n-1}\} = (\tilde{x} \wedge \tilde{x}')_{n-1},$$

so $\tilde{x} \wedge \tilde{x}' \in \tilde{X}_1$, and

$$(\tilde{x} \vee \tilde{x}')_n = \max\{\tilde{x}_n, \tilde{x}'_n\} \geq \max\{\tilde{x}_{n-1}, \tilde{x}'_{n-1}\} = (\tilde{x} \vee \tilde{x}')_{n-1},$$

so $\tilde{x} \vee \tilde{x}' \in \tilde{X}_1$. The proof that $\tilde{X}_2$ is a sublattice is analogous. Therefore, $\tilde{X}_1 \times \tilde{X}_2$ is a sublattice. Finally, as $\tilde{X}_1 \times \tilde{X}_2$ is compact, it is a complete sublattice (Topkis 1998, Theorem 2.3.1). □

With this technical condition in hand, I move on to the proof of Lemma 1.

Lemma 1. *Let Assumption 1 hold. $\tilde{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$ is supermodular.*

Proof. Each player's strategy space is a complete sublattice (Lemma 5), and each player's utility function is twice continuously differentiable (Assumption 1). For any $i \in \{1, 2\}$ and distinct $n, m \in \{1, \ldots, N_i\}$,

$$\frac{\partial^2 \tilde{u}_i(\tilde{x}_i, \tilde{x}_j)}{\partial \tilde{x}_{in} \partial \tilde{x}_{im}} = 0.$$

For any $n \in \{1, \ldots, N_1\}$ and $m \in \{1, \ldots, N_2\}$,

$$\frac{\partial^2 \tilde{u}_1(\tilde{x}_1, \tilde{x}_2)}{\partial \tilde{x}_{1n} \partial \tilde{x}_{2m}} = -\hat{\pi}_{2m} \frac{\partial p(\bar{x}_1 - \tilde{x}_{1n}, \tilde{x}_{2m})}{\partial x_1 \partial x_2} \geq 0,$$
$$\frac{\partial^2 \tilde{u}_2(\tilde{x}_1, \tilde{x}_2)}{\partial \tilde{x}_{1n} \partial \tilde{x}_{2m}} = -\hat{\pi}_{1n} \frac{\partial p(\bar{x}_1 - \tilde{x}_{1n}, \tilde{x}_{2m})}{\partial x_1 \partial x_2} \geq 0.$$

Therefore, by Milgrom and Roberts (1990, Theorem 4), $\tilde{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$ is supermodular. □

Proof of Lemma 2

The claim is that the equilibria of the transformed contribution subgame exactly coincide with those of the original contribution subgame with the same beliefs. I first prove a lemma showing that best responses in the original contribution subgame are monotone decreasing in type. Taking player j's strategy as given, denote player i's interim expected utility function in the contribution subgame by

$$Eu_i(x_i, t_i) = \sum_{n=1}^{N_j} \hat{\pi}_{jn} p(x_i, x_j(t_{jn})) - c_i(x_i, t_i). \tag{7}$$

Lemma 6. *Let Assumption 1 hold. For any strategy by player j in* $\Gamma(\hat{\pi}_1, \hat{\pi}_2)$*, each type* t_{in} *of player i has a unique best response* x_{in}*. Moreover,* $m > n$ *implies* $x_{im} \leq x_{in}$*.*

Proof. Let player j's strategy be fixed. The strict concavity of the interim expected utility function (7) and compactness of the strategy space imply the existence and uniqueness of a best response x_{in} for each type t_{in} of player i. Player i's payoff has decreasing differences in her own contribution and type: for all $x_i \in X_i$ and $t_i' > t_i$,

$$\frac{\partial Eu_i(x_i, t_i')}{\partial x_i} - \frac{\partial Eu_i(x_i, t_i)}{\partial x_i} = \frac{\partial c_i(x_i, t_i)}{\partial x_i} - \frac{\partial c_i(x_i, t_i')}{\partial x_i} < 0,$$

where the inequality follows from Assumption 1d. Therefore, $m > n$ implies $x_{im} \leq x_{in}$ (Milgrom and Shannon 1994, Theorem 5). □

This implies that the restriction placed on the strategy spaces in the transformed game is without loss of generality for equilibrium analysis. I can now prove Lemma 2.

Lemma 2. *Let Assumption 1 hold.* $\tilde{x} \in \tilde{X}_1 \times \tilde{X}_2$ *is a Nash equilibrium of* $\tilde{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$ *if and only if the strategy profile defined by*

$$\begin{aligned} x_1(t_{1n}) &= \bar{x}_1 - \tilde{x}_{1n}, \qquad & n &= 1, \ldots, N_1, \\ x_2(t_{2m}) &= \tilde{x}_{2m}, & m &= 1, \ldots, N_2, \end{aligned}$$

is a Bayesian Nash equilibrium of $\Gamma(\hat{\pi}_1, \hat{\pi}_2)$*.*

Proof. The "if" direction is trivial. To prove the "only if" direction, let $\tilde{x}$ be an equilibrium of $\tilde{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$, and let (x_1, x_2) be the corresponding strategy profile in $\Gamma(\hat{\pi}_1, \hat{\pi}_2)$. Suppose this is not an equilibrium, so x_i is not a best response to x_j for some i and j. By Lemma 6, there exists a best response $x_i' \neq x_i$ such that the corresponding $\tilde{x}_i' \in \tilde{X}_i'$. This implies $\tilde{x}$ is not an equilibrium of $\tilde{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$, a contradiction. □

Proof of Lemma 3

I first prove a lemma that addresses equilibrium existence in the contribution subgame in the general case and its uniqueness under Assumption 1. Let $\hat{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$ denote the *ex ante* game[20] that

[20]Unlike $\tilde{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$, this is a proper *ex ante* game, insofar as the sum in a player's utility function is weighted by the probability of her own type. Therefore, unlike the transformed contribution subgame, this game admits an exact potential function, which is useful for applying potential game results. The downside—and the reason I do not use this form of the *ex ante* game in the rest of the analysis—is that then a player is indifferent over all actions by her own types on which her partner's belief places zero probability. But perfect Bayesian equilibrium analysis requires that we specify optimal actions for every type, even off the equilibrium path.

corresponds to $\Gamma(\hat{\pi}_1, \hat{\pi}_2)$, in which each player chooses $\hat{x}_i \in X_i^{N_i}$ to maximize the weighted utility function

$$\hat{u}_i(\hat{x}_i, \hat{x}_j) = \sum_{n=1}^{N_i} \hat{\pi}_{in} \left[\sum_{m=1}^{N_j} \hat{\pi}_{jm} p(\hat{x}_{in}, \hat{x}_{jm}) - c_i(\hat{x}_{in}, t_{in}) \right]. \tag{8}$$

Lemma 7. $\Gamma(\hat{\pi}_1, \hat{\pi}_2)$ *has a pure strategy Bayesian Nash equilibrium. If Assumption 1 holds, the equilibrium is unique.*

Proof. $\hat{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$ is a potential game with exact potential function

$$\hat{P}(\hat{x}_1, \hat{x}_2) = \sum_{n=1}^{N_1} \sum_{m=1}^{N_2} \hat{\pi}_{1n} \hat{\pi}_{2m} \left[p(\hat{x}_{1n}, \hat{x}_{2m}) - c_1(\hat{x}_{1n}, t_{1n}) - c_2(\hat{x}_{2m}, t_{2m}) \right]. \tag{9}$$

Because p is upper semicontinuous and each c_i is lower semicontinuous in x_i, $\hat{P}$ is upper semicontinuous and therefore attains its maximum on the compact set $X_1^{N_1} \times X_2^{N_2}$ (Sundaram 1996, Theorem 9.13).

Call the type t_{in} active if $\hat{\pi}_{in} > 0$ and inactive otherwise. I first consider the case in which each $\hat{\pi}_i \gg 0$, so every type is active. Let $\hat{x}^* = (\hat{x}_1^*, \hat{x}_2^*)$ be a maximizer of $\hat{P}$. $\hat{x}^*$ is a Nash equilibrium of $\hat{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$ (Monderer and Shapley 1996, Lemma 2.1). Therefore, the strategy profile of $\Gamma(\hat{\pi}_1, \hat{\pi}_2)$ defined by $x_i(t_{in}) = x_{in}^*$ is a Bayesian Nash equilibrium (Harsanyi 1968, Theorem 1; van Heumen et al. 1996, Theorem 5.3). If in addition Assumption 1 holds, then $\hat{P}$ is C^1 and strictly concave, so this equilibrium is unique (Neyman 1997, Theorem 2).

Now suppose there are inactive types. Let $(\hat{x}_{in}^*)_{in:\hat{\pi}_{in}>0}$ be a maximizer of $\hat{P}$ with respect to the active types' contributions. Take a strategy profile of $\Gamma(\hat{\pi}_1, \hat{\pi}_2)$ such that $x_i(t_{in}) = x_{in}^*$ if $\hat{\pi}_{in} > 0$ and

$$x_i(t_{in}) \in \arg\max_{x_i \in X_i} \left\{ \sum_{m:\hat{\pi}_{jm}>0} \hat{\pi}_{jm} p(x_i, \hat{x}_{jm}^*) - c_i(x_i, t_{in}) \right\}$$

if $\hat{\pi}_{in} = 0$. The maximand in the expression above is upper semicontinuous, so such a strategy profile exists. Since a player's expected utility does not depend on the actions of inactive types of her partner, this is a Bayesian Nash equilibrium of $\Gamma(\hat{\pi}_1, \hat{\pi}_2)$ by the same arguments as above. If in addition Assumption 1 holds, then there cannot be an equilibrium in which the active types employ different strategies, per the argument above. Moreover, each individual type's expected utility is strictly concave in her own contribution, so the active types' strategies pin down unique best responses for the inactive types. Therefore, there can be no other equilibrium of $\Gamma(\hat{\pi}_1, \hat{\pi}_2)$. □

Lemma 3 follows essentially as a corollary from the above.

Lemma 3. *Let Assumption 1 hold.* $\tilde{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$ *has a unique equilibrium.*

Proof. Let Assumption 1 hold. By Lemma 7, $\Gamma(\hat{\pi}_1, \hat{\pi}_2)$ has a unique Bayesian Nash equilibrium. By Lemma 2, the corresponding strategy profile of $\tilde{\Gamma}(\hat{\pi}_1, \hat{\pi}_2)$ is its unique Nash equilibrium. □

Proof of Lemma 4

Lemma 4 concerns how the equilibrium of the transformed contribution subgame changes with the relative weight placed on a high versus low set of types of player 1. The proof relies on monotone comparative statics (Milgrom and Shannon 1994). As a sufficient condition for the single-crossing property from which monotone comparative statics follow, I will show that each player's utility function in the transformed contribution subgame has increasing differences in her own contribution and the aforementioned weight on high types of player 1. The following intermediate lemma states this result.

Lemma 8. *Let $\hat{\pi}_1^L, \hat{\pi}_1^H \in \Delta T_1$, $\hat{\pi}_2 \in \Delta T_2$, and $\alpha \in [0,1]$. If $\min\{n \,|\, \hat{\pi}_{1n}^H > 0\} > \max\{n \,|\, \hat{\pi}_{1n}^L > 0\}$, then each player's utility function in $\tilde{\Gamma}(\alpha\hat{\pi}_1^H + (1-\alpha)\hat{\pi}_1^L, \hat{\pi}_2)$ has increasing differences in $\tilde{x}_i$ and α.*

Proof. Let $n^L = \max\{n \,|\, \hat{\pi}_{1n}^L > 0\}$ and $n^H = \min\{n \,|\, \hat{\pi}_{1n}^H > 0\}$, and assume $n^H > n^L$. The claim of the lemma holds trivially for player 1, since $\hat{\pi}_1$ and thus α does not enter into her utility function (4). For any type $m \in \{1, \dots, N_2\}$ of player 2, we have

$$\begin{aligned}
\frac{\partial \tilde{u}_2(\tilde{x}_1, \tilde{x}_2)}{\partial \tilde{x}_{2m}} &= \sum_{n=1}^{N_1} \left[\alpha\hat{\pi}_{1n}^H + (1-\alpha)\hat{\pi}_{1n}^L\right] \frac{\partial p(\bar{x}_1 - \tilde{x}_{1n}, \tilde{x}_{2m})}{\partial x_2} - \frac{\partial c_2(\tilde{x}_{2m}, t_{2m})}{\partial x_2} \\
&= (1-\alpha)\sum_{n=1}^{n^L} \hat{\pi}_{1n}^L \frac{\partial p(\bar{x}_1 - \tilde{x}_{1n}, \tilde{x}_{2m})}{\partial x_2} + \alpha \sum_{n=n^H}^{N_1} \hat{\pi}_{1n}^H \frac{\partial p(\bar{x}_1 - \tilde{x}_{1n}, \tilde{x}_{2m})}{\partial x_2} \\
&\quad - \frac{\partial c_2(\tilde{x}_{2m}, t_{2m})}{\partial x_2}
\end{aligned}$$

and thus

$$\begin{aligned}
\frac{\partial^2 \tilde{u}_2(\tilde{x}_1, \tilde{x}_2)}{\partial x_{2m}\partial\alpha} &= \sum_{n=n^H}^{N_1} \hat{\pi}_{1n}^H \frac{\partial p(\bar{x}_1 - \tilde{x}_{1n}, \tilde{x}_{2m})}{\partial x_2} - \sum_{n=1}^{n^L} \hat{\pi}_{1n}^L \frac{\partial p(\bar{x}_1 - \tilde{x}_{1n}, \tilde{x}_{2m})}{\partial x_2} \\
&\geq \min_{n \in \{n^H, \dots, N_1\}} \frac{\partial p(\bar{x}_1 - \tilde{x}_{1n}, \tilde{x}_{2m})}{\partial x_2} - \max_{n \in \{1, \dots, n^L\}} \frac{\partial p(\bar{x}_1 - \tilde{x}_{1n}, \tilde{x}_{2m})}{\partial x_2} \\
&= \frac{\partial p(\bar{x}_1 - \tilde{x}_{1n^H}, \tilde{x}_{2m})}{\partial x_2} - \frac{\partial p(\bar{x}_1 - \tilde{x}_{1n^L}, \tilde{x}_{2m})}{\partial x_2} \\
&\geq 0,
\end{aligned}$$

establishing the claim. □

With this result in hand, the proof of Lemma 4 follows almost immediately from the well-known properties of supermodular games.

Lemma 4. *Let Assumption 1 hold. Let $\hat{\pi}_1^L, \hat{\pi}_1^H \in \Delta T_1$, $\hat{\pi}_2 \in \Delta T_2$, and $\alpha, \beta \in [0,1]$. Assume the support of $\hat{\pi}_1^H$ consists of higher types than that of $\hat{\pi}_1^L$; i.e., $\min\{n \,|\, \hat{\pi}_{1n}^H > 0\} > \max\{n \,|\, \hat{\pi}_{1n}^L > 0\}$. Let*

$$\hat{\pi}_1^\alpha = \alpha\hat{\pi}_1^H + (1-\alpha)\hat{\pi}_1^L,$$

and define $\hat{\pi}_1^\beta$ analogously. Let $\tilde{x}^\alpha$ and $\tilde{x}^\beta$ be the equilibria of $\tilde{\Gamma}(\hat{\pi}_1^\alpha, \hat{\pi}_2)$ and $\tilde{\Gamma}(\hat{\pi}_1^\beta, \hat{\pi}_2)$ respectively.

If $\alpha \le \beta$, then $\tilde{x}^{\alpha} \le \tilde{x}^{\beta}$.

Proof. $\{\tilde{\Gamma}(\xi\hat{\pi}_1^H + (1-\xi)\hat{\pi}_1^L, \hat{\pi}_2)\}_{\xi\in[0,1]}$ is a family of supermodular games (Lemma 1), each of which has a unique equilibrium (Lemma 3). Under the hypothesis of the lemma, each player's utility function has increasing differences in $\tilde{x}_i$ and ξ (Lemma 8). Therefore, the equilibrium is nondecreasing in ξ (Milgrom and Roberts 1990, Theorem 6). □

Proof of Proposition 1

Proposition 1. *Let Assumption 1 hold. There is no influential equilibrium in interval messaging.*

Proof. Consider an equilibrium in interval messaging in which player i sends distinct messages m_i^L, m_i^H on the equilibrium path. Without loss of generality, let $i = 1$ and let m_1^L be the message that corresponds to lower types of player 1. Take any $t_2 \in T_2$, let $\tilde{x}^L$ denote the equilibrium of $\tilde{\Gamma}(\hat{\pi}_1(m_1^L), \hat{\pi}_2(\mu_2(t_2)))$ (whose existence and uniqueness are proven in Lemma 3), and define $\tilde{x}^H$ analogously. Under interval messaging and the equilibrium requirement of consistent beliefs, we have $\min\{n \,|\, \hat{\pi}_{1n}^H > 0\} > \max\{n \,|\, \hat{\pi}_{1n}^L > 0\}$. Therefore, by Lemma 4, $\tilde{x}_2^H \ge \tilde{x}_2^L$. It then follows from Lemma 2 that

$$\omega_2(t_2, \mu_2(t_2), m_1^H) \ge \omega_2(t_2, \mu_2(t_2), m_1^L). \tag{10}$$

Since t_2 was chosen arbitrarily, this holds for all $t_2 \in T_2$. If the equilibrium is influential, then (10) holds strictly for some $t_2' \in T_2$. But then every type of player 1 would strictly prefer sending m_1^H over sending m_1^L, contradicting the assumption of equilibrium. Therefore, the equilibrium is not influential. □

Proof of Proposition 3

Proposition 3. *Let Assumption 2 hold. For each player i, let $T_i^U \subseteq T_i$ be the set of types that are unwilling to meet the threshold. For any distinct $m_i^U, m_i^W \in M_i$, there exists an equilibrium in which each player's message reveals whether she is unwilling to meet the threshold:*

$$\mu_i(t_i) = \begin{cases} m_i^U & t_i \in T_i^U, \\ m_i^W & t_i \notin T_i^U. \end{cases}$$

Along the path of play, in any contribution subgame in which either $m_i = m_i^U$, both players contribute $x_i = 0$.

Proof. Let d_{in} be an indicator for $t_{in} \in T_i^U$. Let each μ_i be defined as in the proposition, and let each player's belief system be defined by

$$\hat{\pi}_{in}(m_i) = \begin{cases} d_{in}\pi_{in} / \sum_{m=1}^{N_i} d_{im}\pi_{im} & m_i = m_i^U, \\ (1-d_{in})\pi_{in} / \sum_{m=1}^{N_i}(1-d_{im})\pi_{im} & m_i = m_i^W, \end{cases}$$

for each $n = 1, \ldots, N_i$. In case either T_i^U is empty or is not a proper subset of T_i, so that the above expression results in division by zero, let $\hat{\pi}_i(m_i^U) = \hat{\pi}_i(m_i^W) = \pi_i$. To complete the belief systems, let any off-the-path message $m_i \notin \{m_i^U, m_i^W\}$ induce the same beliefs and contribution strategies as m_i^U, so that off-the-path deviations in the messaging stage can be ignored. These beliefs are consistent with Bayes' rule wherever possible.

For each contribution subgame following the message pair (m_1, m_2), define each $\omega_i(\cdot, m_i, m_j)$ to correspond to an equilibrium of $\Gamma(\hat{\pi}_1(m_1), \hat{\pi}_2(m_2))$. Lemma 7 guarantees the existence of such an equilibrium. These contribution strategies are sequentially rational by construction.

Suppose a contribution subgame in which $m_i = m_i^U$ is reached along the path of play. Take any $t_i \in T_i^U$; if m_i^U is sent on the path of play, at least one such type must exist. Under the assumption of equilibrium, neither player may choose an iteratively strictly dominated action, so we have

$$p(\omega_i(t_i, m_i^U, m_j), \omega_j(t_j, m_j, m_i^U)) = 0$$

for all $t_j \in T_j$ and $m_j \in M_j$. Sequential rationality then implies $\omega_i(t_i, m_i^U, m_j) = 0$. By the same token, since player j believes for sure after receiving t_i^U that the threshold will not be met, we must have $\omega_j(t_j, m_j, m_i^U) = 0$ for all $t_j \in T_j$ and $m_j \in M_j$. This proves the claim that both players contribute zero along the path of play in any contribution subgame following $m_i = m_i^U$.

The last step to prove that this is an equilibrium is to show that no type strictly prefers to deviate from her prescribed message. It is obviously unprofitable for $t_i \notin T_i^U$ to deviate to sending m_i^U, since doing so would ensure her partner contributes nothing. The only remaining question is whether $t_i \in T_i^U$ would strictly prefer to send t_i^W. On the equilibrium path, such types receive a payoff of zero. Once again, since neither player may employ an iteratively strictly dominated action in the contribution subgame equilibrium, we have

$$p(\omega_i(t_i, m_i^W, \mu_j(t_j)), \omega_j(t_j, \mu_j(t_j), m_i^W)) = 0$$

for all $t_j \in T_j$. Therefore, deviating to m_i^W in the messaging stage is weakly unprofitable. □

Proof of Proposition 4

Proposition 4. *If there is one-sided incomplete information and Assumption 2S holds, then in any influential equilibrium:*

(a) Player 2 always contributes $x_2 = 0$ or $x_2 = \tilde{x}_2$ on the path of play, where $\tilde{x}_2 > 0$ and $p(0, \tilde{x}_2) = 0$.

(b) On the path of play, if t_1's message induces player 2 to contribute $x_2 = 0$, then t_1 spends $x_1 = 0$.

Proof. Assume the conditions of the proposition and consider an influential equilibrium. Under one-sided incomplete information, let $T_2 = \{t_2\}$ and denote $m_2 = \mu_2(t_2)$. To cut down on notational clutter, let $\tilde{\omega}_2(m_1) = \omega_2(t_2, m_2, m_1)$.

To prove statement (a), suppose $\tilde{\omega}_2(m_1') > \tilde{\omega}_2(m_1) > 0$, where m_1 and m_1' are both sent on the path of play. In order for it to be a best response by player 2 to make a positive contribution after receiving m_1, there must be a type t_1 such that $\mu_1(t_1) = m_1$ and $(\omega_1(t_1, m_1, m_2), \tilde{\omega}_2(m_1)) \notin Z$; otherwise, it would be profitable for player 2 to deviate to spending nothing in this subgame. But this means t_1 could raise her payoff by sending the message m_1' and then spending the same amount as when she sends m_1:

$$\begin{aligned}u_1(\omega_1(t_1, m_1, m_2), \tilde{\omega}_2(m_1'), t_1) &= p(\omega_1(t_1, m_1, m_2), \tilde{\omega}_2(m_1')) - c_1(\omega_1(t_1, m_1, m_2), t_1) \\ &> p(\omega_1(t_1, m_1, m_2), \tilde{\omega}_2(m_1)) - c_1(\omega_1(t_1, m_1, m_2), t_1) \\ &= u_1(\omega_1(t_1, \mu_1(t_1), m_2), \tilde{\omega}_2(\mu_1(t_1)), t_1).\end{aligned}$$

This contradicts the assumption of equilibrium. By the same token, it must be the case that $(0, \tilde{x}_2) \in Z$; otherwise, it would be strictly profitable for any type of player 1 whose message yields zero contribution to sending one that results in $\tilde{x}_2$.

To prove statement (b), suppose there is $t_1 \in T_1$ such that $\tilde{\omega}_2(\mu_1(t_1)) = 0$ and $\omega_1(t_1, \mu_1(t_1), m_2) = x_1 > 0$. This is a best response for t_1 only if $p(x_1, 0) > 0$, which implies $(x_1, 0) \notin Z$. Therefore, it would be profitable for t_1 to deviate to sending a message m_1 such that $\tilde{\omega}_2(m_1) = \tilde{x}_2 > 0$ and then spending the same amount in the contribution stage:

$$\begin{aligned}u_1(x_1, \tilde{\omega}_2(m_1), t_1) &= p(x_1, \tilde{x}_2) - c_1(x_1, t_1) \\ &> p(x_1, 0) - c_1(x_1, t_1) \\ &= u_1(\omega_1(t_1, \mu_1(t_1), m_2), \tilde{\omega}_2(\mu_1(t_1)), t_1).\end{aligned}$$

This contradicts the assumption of equilibrium. □

Extensions

I now briefly present two extensions to the model of communication and collective action. The first considers whether influential communication might be possible in a special case of the continuous model if a mediator were involved. The findings are negative. The second shows that cheap talk may influence outcomes under a broad set of conditions if there is more than one way to contribute and a player's private information concerns her comparative advantage among them.

Mediated Communication

I return to the continuous collective action setting, in which bilateral cheap talk was shown to be ineffective in a broad class of problems. The basic incentive problem is that a player does not want to reveal that her costs of contribution are low, as doing so induces her partner to contribute less than she would otherwise. What if a player could be compensated for revealing undesirable information (i.e., that her costs of contribution are low) by learning more about her partner? A mediator can facilitate this type of information exchange. Mediated communication has been shown to be effective in oligopolistic competition, in which ordinary cheap talk is ineffective because each firm always prefers that its competitors produce less (Goltsman and Pavlov 2014).

I investigate the same mediated communication scheme that Goltsman and Pavlov (2014) use in the oligopoly setting. There are two types of player, a low-cost (more willing) and high-cost (less willing) type. Each player makes a private, unverifiable report of her type to the mediator. If both players report low, then the mediator publicly announces L. Otherwise, if either reports high, the mediator announces H. In unmediated cheap talk, each player would always prefer to pretend to be the high-cost type, as shown above in Proposition 1. However, under the mediated scheme, there is a potential benefit to revealing low costs—a player learns her partner's report if and only if she reports a low type. I call this the *mediated information exchange mechanism*, which is *incentive compatible* if it is an equilibrium for each player to always report her type honestly.

For tractability in the analysis, I assume cost functions are linear and the production function is quadratic. With quadratic utility, players benefit from certainty about their partner's strategy, as is required for the mediated information exchange mechanism to be sensible. The following assumption formalizes these conditions.[21]

Assumption 3. *Each* $c_i(x_i, t_i) = t_i x_i$*, and* $p(x_1, x_2) = q(x_1 + x_2)$*, where*

$$q(x) = Ax^2 + Bx + C$$

with $A < 0$ *and* $\min(T_1 \cup T_2) < B \leq \min T_1 + \min T_2$.

The conditions on A and B are the weakest assumptions that guarantee that p is strictly concave and strictly increasing on the set of contribution profiles that are not strictly dominated, and that at least one type contributes $x_i > 0$ in equilibrium.

Ultimately, mediated information exchange does not solve the incentive problems that impede communication among actors with a common goal. As the following proposition states, the scheme is not incentive compatible—the certainty premium is not worth the cost of revealing that one is highly willing to contribute.

Proposition 5. *If Assumption 3 holds,* $T_1 = T_2 = \{t_L, t_H\}$ $(t_L < t_H)$*, and* $\Pr(t_1 = t_L) = \Pr(t_2 = t_L) = \pi \in (0, 1)$*, then the mediated information exchange mechanism is not incentive compatible.*

Proof. For each $t \in \{t_L, t_H\}$ and $m \in \{L, H\}$, let $\omega(t, m)$ denote the action of type t following announcement m. In addition, let x_L^* denote the standalone contribution of type t_L (i.e., what t_L would contribute if she expected her partner to contribute nothing; see Bag and Roy 2011),

$$x_L^* = \frac{t_L - B}{2A},$$

where $x_L^* > 0$ by Assumption 3. Similarly, let

$$x_H^* = \max\left\{0, \frac{t_H - B}{2A}\right\}.$$

Best responses to the mediated information exchange mechanism are as follows. After receiving message L, each player knows her partner is the low type, so $\omega(t_L, L) = x_L^*/2$.[22] After receiving message H, low types know their partner is the high type and therefore contribute $\omega(t_L, H) = x_L^* - \omega(t_H, H)$. High types still believe their partner is low with probability π and high

[21]Technically, Assumption 3 is not stronger than Assumption 1, because p is not strictly concave (its Hessian matrix is indefinite).

[22]The result would still hold if responses to the low message were asymmetric. Specifically, the relevant inequalities would hold for whichever player spent more.

with probability $1-\pi$; their first-order condition gives

$$\omega(t_H, H) = \max\left\{\frac{t_H - \pi t_L - (1-\pi)B}{4A(1-\pi)}, 0\right\}.$$

First suppose $t_H \geq \pi t_L + (1-\pi)B$, so that $\omega(t_H, H) = 0$. The expected utility to a low type for honest reporting is

$$U_L = q(x_L^*) - \left(1 - \frac{\pi}{2}\right) t_L x_L^*.$$

If a low type deviated in the messaging stage, misreporting her type as t_H, the mediator would send H for sure. Under the low type's best reply of $x_L' = (1-\pi)x_L^*$, her expected utility from deviating would be

$$U_L' = \pi q((2-\pi)x_L^*) + (1-\pi)q((1-\pi)x_L^*) - (1-\pi)t_L x_L^*.$$

We have

$$\begin{aligned}
U_L' - U_L &= \pi q((2-\pi)x_L^*) + (1-\pi)q((1-\pi)x_L^*) - q(x_L^*) + \frac{\pi}{2} t_L x_L^* \\
&= A(x_L^*)^2(1-\pi)\pi + \frac{\pi}{2} t_L x_L^* \\
&= \pi x_L^* \left(\frac{(2-\pi)t_L - (1-\pi)B}{2}\right) \\
&\geq \pi x_L^* \left(\frac{(2-\pi)\frac{B}{2} - (1-\pi)B}{2}\right) \\
&= \frac{B\pi^2 x_L^*}{4} \\
&> 0,
\end{aligned}$$

so the mechanism is not incentive compatible.

Now suppose $t_H < \pi t_L + (1-\pi)B$, so that $\omega(t_H, H) > 0$. For ease of exposition, let $\tilde{x}_L = \omega(t_L, H)$ and $\tilde{x}_H = \omega(t_H, H)$. The expected utility to a low type for honest reporting is now

$$U_L = q(x_L^*) - t_L\left(\pi \frac{x_L^*}{2} + (1-\pi)\tilde{x}_L\right).$$

If the low type deviated, her best reply would be $x_L' = \pi \tilde{x}_H + (1-\pi)\tilde{x}_L$ and her expected utility would be

$$U_L' = \pi q(x_L' + \tilde{x}_L) + (1-\pi)q(x_L' + \tilde{x}_H) - t_L x_L'.$$

We have

$$x_L^* - 2\tilde{x}_H = \frac{t_L - t_H}{2A(1-\pi)} > 0$$

and thus

$$\begin{aligned}
U_L' - U_L &= \pi q(x_L' + \tilde{x}_L) + (1-\pi) q(x_L' + \tilde{x}_H) - q(x_L^*) \\
&\quad + t_L \left(\frac{\pi}{2} x_L^* + (1-\pi)\tilde{x}_L - x_L' \right) \\
&= A\pi(1-\pi)(x_L^* - 2\tilde{x}_H)^2 + t_L \frac{\pi}{2}(x_L^* - 2\tilde{x}_H) \\
&= \frac{\pi(x_L^* - 2\tilde{x}_H)}{2}(2t_L - t_H) \\
&> \frac{\pi(x_L^* - 2\tilde{x}_H)}{2}((2-\pi)t_L - (1-\pi)B) \\
&> 0,
\end{aligned}$$

so the mechanism is not incentive compatible. □

The barriers to communication in continuous collective action problems, as characterized in the main text, do not disappear with the introduction of a mediator. Whether a player is speaking directly to her partner or indirectly through a mediator, the incentive to misrepresent remains. To the extent that particular political institutions (e.g., international organizations) facilitate collective action, it is not through their roles in mediating cheap talk. Rather, it is because they can gather and disseminate information independently, or because they can make it costly to lie (see also Fey and Ramsay 2010).

Multiple Methods of Contribution

Up to now, I have considered how communication can (or cannot) allow participants in a collective action problem to coordinate their efforts when they are unsure of each other's overall willingness to give. In this section, I study a different kind of uncertainty coordination problem—how to distribute effort across diverse means of contributing to the same project when participants are unsure who is better suited to which task. For example, a military operation might depend on both air power and naval support. Allies would want to coordinate their efforts to ensure that both are adequately supplied. The most efficient way to do this is for each partner to specialize according to its comparative advantage. If one's relative ability to supply the various components is private information, efficient coordination requires prior communication.

I consider a model with two avenues of contribution and uncertainty about the players' comparative advantages. I assume all contributions are valuable, as in the continuous case of the baseline model—a setting where the barriers to influential cheap talk are highest.[23] Nevertheless, in this multimethod extension, I find that communication may aid coordination. The key, once again, is that a player's private information affect what she wants from her partner. A country with an excellent air force would prefer for its ally to supply the boats, and vice versa for a country with a naval advantage. The setup and results of the extension mirror Trager's (2011) analysis of multi-dimensional private information in crisis diplomacy.

[23] I also assume one-sided incomplete information, which is a hard case for influential communication when all contributions are valuable (Proposition 2).

1 \ 2	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
$(0,0)$	q_{00} q_{00}	q_{10} $q_{10} - c_2^A$	q_{01} $q_{01} - c_2^B$	q_{11} $q_{11} - c_2^A - c_2^B$
$(1,0)$	$q_{10} - c_1^A$ q_{10}	$q_{20} - c_1^A$ $q_{20} - c_2^A$	$q_{11} - c_1^A$ $q_{11} - c_2^B$	$q_{21} - c_1^A$ $q_{21} - c_2^A - c_2^B$
$(0,1)$	$q_{01} - c_1^B$ q_{01}	$q_{11} - c_1^B$ $q_{11} - c_2^A$	$q_{02} - c_1^B$ $q_{02} - c_2^B$	$q_{12} - c_1^B$ $q_{12} - c_2^A - c_2^B$
$(1,1)$	$q_{11} - c_1^A - c_1^B$ q_{11}	$q_{21} - c_1^A - c_1^B$ $q_{21} - c_2^A$	$q_{12} - c_1^A - c_1^B$ $q_{12} - c_2^B$	$q_{22} - c_1^A - c_1^B$ $q_{22} - c_2^A - c_2^B$

Figure 4. Payoff matrix for the contribution stage of the two-method game.

The *two-method game* is defined as follows. Each player chooses a pair of actions, $x_i = (x_i^A, x_i^B)$. It may be helpful to think of these as *A*irplanes and *B*oats. Each strategy space $X_i = \{0, 1\}$, so each choice is binary.[24] The production function for the project depends only on the total contributed to each method:

$$p(x_1, x_2) = q(x_1^A + x_2^A, x_1^B + x_2^B), \tag{11}$$

where $q : \{0, 1, 2\}^2 \to \mathbb{R}$ is strictly increasing. For brevity, let $q_{ab} = q(a, b)$. A player pays a cost c_i^A for contributing method A and c_i^B for B. Under complete information, a player's utility function is

$$u_i(x_i, x_j) = p(x_i, x_j) - c_i^A x_i^A - c_i^B x_i^B. \tag{12}$$

Figure 4 shows the payoff matrix.

Let player 1's cost of method B be private information, so her cost function is $c_1(x_1, t_1) = c_1^A x_1^A + t_1 x_1^B$. Assume $T_1 = \{\underline{c}_1^B, \bar{c}_1^B\}$, with $\underline{c}_1^B < c_1^A < \bar{c}_1^B$. The low type, $\underline{c}_1^B$, can supply method B relatively cheaply, so call her "B-advantaged." By the same token, call type $\bar{c}_1^B$ "A-advantaged." Let the prior probability of a B-advantaged type be $\pi \in (0, 1)$. Let all other cost parameters be common knowledge. To simplify the statements of the equilibrium conditions, I assume throughout that it is never in a player's interest to contribute a method she knows her partner is also supplying:

$$q_{2b} - q_{1b} < \min\{c_1^A, c_2^A\} \quad \text{for all } b \in \{0, 1, 2\};$$
$$q_{a2} - q_{a1} < \min\{\underline{c}_1^B, c_2^B\} \quad \text{for all } a \in \{0, 1, 2\}.$$

As before, player 1 may send a message about her type before the contribution stage. Let the message space $M_1 = \{A, B\}$. I will look for equilibria in which player 1 honestly reveals her comparative advantage, which in turn allows the players to efficiently coordinate their effort. In a *division of labor equilibrium*,

1. Player 1 sends $m_1 = A$ if she is A-advantaged and $m_1 = B$ if she is B-advantaged.

[24]This is for simplicity; the main results would still hold with continuous action spaces.

2. Along the path of play, if player 1 is A-advantaged, she contributes $x_1 = (1, 0)$ and player 2 contributes $x_2 = (0, 1)$.

3. Along the path of play, if player 1 is B-advantaged, she contributes $x_1 = (0, 1)$ and player 2 contributes $x_2 = (1, 0)$.

A division of labor equilibrium is, by definition, influential.

Before proving the proposition on the existence of a division of labor equilibrium, I state a lemma about when the proposed strategies constitute Nash equilibria of the contribution subgame.

Lemma 9. *Consider the contribution stage of the two-method game, and let it be common knowledge that $t_1 = c_1^B$. The strategy profile $(x_1, x_2) = ((1, 0), (0, 1))$ is a Nash equilibrium if and only if*

$$q_{11} - q_{01} \geq c_1^A, \tag{13}$$

$$q_{11} - q_{10} \geq c_2^B. \tag{14}$$

The strategy profile $(x_1, x_2) = ((0, 1), (1, 0))$ is a Nash equilibrium if and only if

$$q_{11} - q_{01} \geq c_2^A, \tag{15}$$

$$q_{11} - q_{10} \geq c_1^B. \tag{16}$$

Proof. I will prove the first statement; the proof of the second is analogous. Under the given strategy profile, since $x_2^B = 1$, it cannot be profitable for player 1 to deviate to any strategy with $x_1^B = 1$. Therefore, the only deviation we must consider for player 1 is $x_1 = (0, 0)$. This deviation is unprofitable if and only if (13) holds. Similarly, because $x_1^A = 1$ under the given strategy profile, the only deviation we must consider for player 2 is $x_2 = (0, 0)$. This is unprofitable if and only if (14) holds. □

The two-method game preserves the essential features of the baseline model. Effort is jointly beneficial but individually costly, creating an incentive to free-ride. Method by method, each player still strictly prefers for her partner to contribute rather than not. However, if player 2 is only going to supply one method or the other, what player 1 prefers may depend on her type. Under broad conditions, each type would prefer that player 2 supply whichever method player 1 is disadvantaged in. These conditions, which are summarized in the following proposition, enable influential communication in a division of labor equilibrium. Such an equilibrium is possible if the marginal effects of both types of contribution are sufficiently strong.

Proposition 6. *In the two-method game, if*

$$q_{11} - q_{01} \geq \max\left\{c_1^A, \underline{c}_1^B, c_2^A\right\}, \tag{17}$$

$$q_{11} - q_{10} \geq \max\left\{c_1^A, \underline{c}_1^B, c_2^B\right\}, \tag{18}$$

then there is a division of labor equilibrium.

Proof. I begin by constructing the claimed equilibrium. Let the messaging strategy μ_1 be fully separating, so $\mu_1(t_1) = t_1$ for each $t_1 \in \{\underline{c}_1^B, \bar{c}_1^B\}$. Let player 2's strategy be

$$\omega_2(m_1) = \begin{cases} (1,0) & m_1 = \underline{c}_1^B, \\ (0,1) & m_1 = \bar{c}_1^B. \end{cases}$$

Along the path of play, let player 1's strategy be

$$\omega_1(t_1, t_1) = \begin{cases} (0,1) & t_1 = \underline{c}_1^B, \\ (1,0) & t_1 = \bar{c}_1^B. \end{cases}$$

Off the path of play (i.e., in subgames in which $m_1 \neq t_1$), let player 1's strategy be any best response to $\omega_2(m_1)$; as the game is finite, a best response exists. Finally, let player 2's updated beliefs after each m_1 place probability 1 on $t_1 = m_1$.

I proceed by showing that the proposed contribution strategies are sequentially rational. On the path of play when $t_1 = \underline{c}_1^B$, (17) implies (15) and (18) implies (16), so the proposed strategies form a Nash equilibrium of the contribution subgame (Lemma 9). Similarly, on the path when $t_1 = \bar{c}_1^B$, (17) implies (13) and (18) implies (14), so the proposed strategies are again an equilibrium. In the subgames off the equilibrium path, player 1's actions are best responses by construction.

Next, I show that player 1's messaging strategy is incentive compatible: neither type can strictly improve her payoff by deviating from her messaging strategy. Consider the maximal payoff to the B-advantaged type after falsely reporting $\bar{c}_1^B$. Since player 2 contributes $x_2^B = 1$ after receiving the A-advantaged message, we only need to consider actions with $x_1^B = 0$. The condition for the deviation to be unprofitable is thus

$$q_{11} - \underline{c}_1^B \geq \max\left\{q_{01}, q_{11} - c_1^A\right\},$$

which follows from (17) and $\underline{c}_1^B < c_1^A$. Similarly, the condition for it to be unprofitable for the A-advantaged type to deviate in the messaging stage is

$$q_{11} - c_1^A \geq \max\left\{q_{10}, q_{11} - \bar{c}_1^B\right\},$$

which follows from (18) and $\bar{c}_1^B > c_1^A$.

The final step is to confirm that player 2's updated beliefs are consistent with Bayes' rule wherever possible, which is true by construction. □

The incentive to misrepresent that impedes communication in the baseline continuous case does not show up here, as player 1's comparative advantage dictates what she wants from player 2. Under the stated conditions, an A-advantaged player wants her partner to contribute B, and the B-advantaged type wants her partner to contribute A. Therefore, neither type has an incentive to mimic the other. The result suggests that uncertainty about comparative advantages across contribution methods should not be a major hindrance to joint political projects, even in contexts where actors cannot make verifiable statements or binding commitments to each other. As long as each avenue of contribution is reasonably effective, it is in a player's interest to be honest with her partners about her comparative advantage.

A division of labor equilibrium is also more robust than the influential equilibria of threshold games characterized in the main text. If the inequalities in Proposition 6 hold strictly, then each type of player 1 *strictly* prefers telling the truth over mimicking the other type. Therefore, unlike in threshold games, small perturbations to the production or cost functions cannot break the possibility of influential communication.